A Review on M(atrix) Theory

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November 25, 2017

Content

- History& Introduction.
- Formulating Matrix Theory.
- Rebuilding M-theory's objects.
- Other development and conclusion.

(Perturbative) String Theory

STRINGS

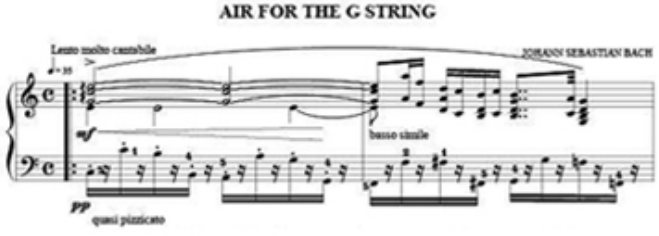
VIBRATE

VARIOUS MODES

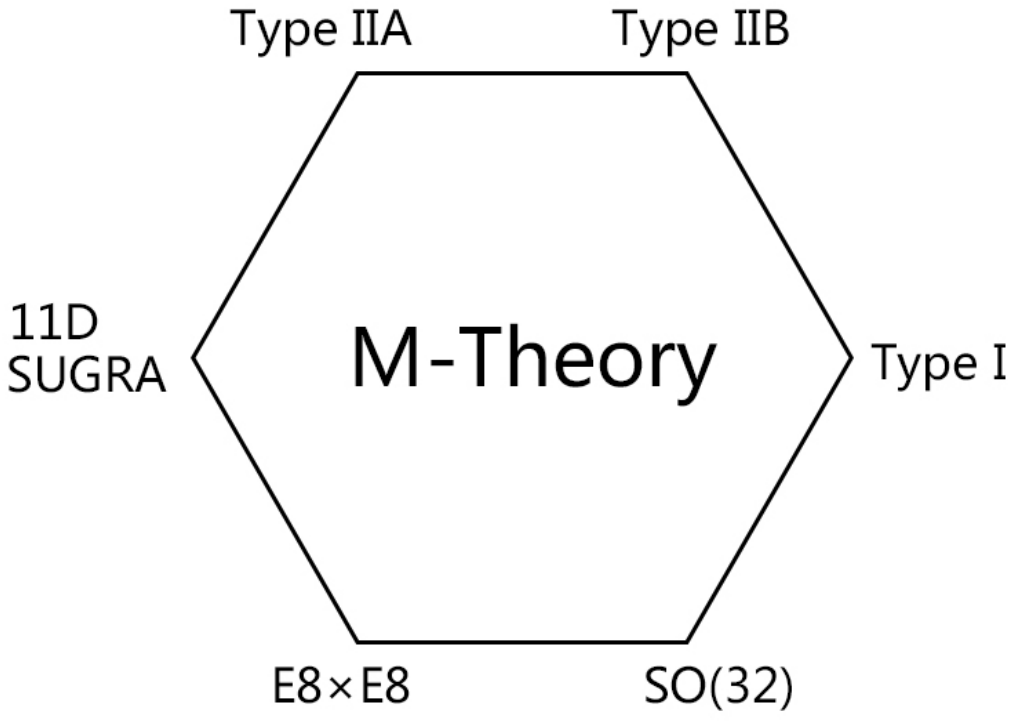
(PARTICLES)

440HZ~880HZ

OUR WORLDS



’M’ Stands For Mother



String theories are special limiting cases of M-theory

- Unify string theories.
- (Maximal) 11 dimensional quantum gravity theory.

'M' Stands For Mystery

- Q:What is exactly M-theory?
- A:We do not know.

$\rightarrow$M-theory is not yet fully formulated.

- Q:Is there a perturbative method?
- A:No.

$\rightarrow$No dimensionless parameters in M-theory.

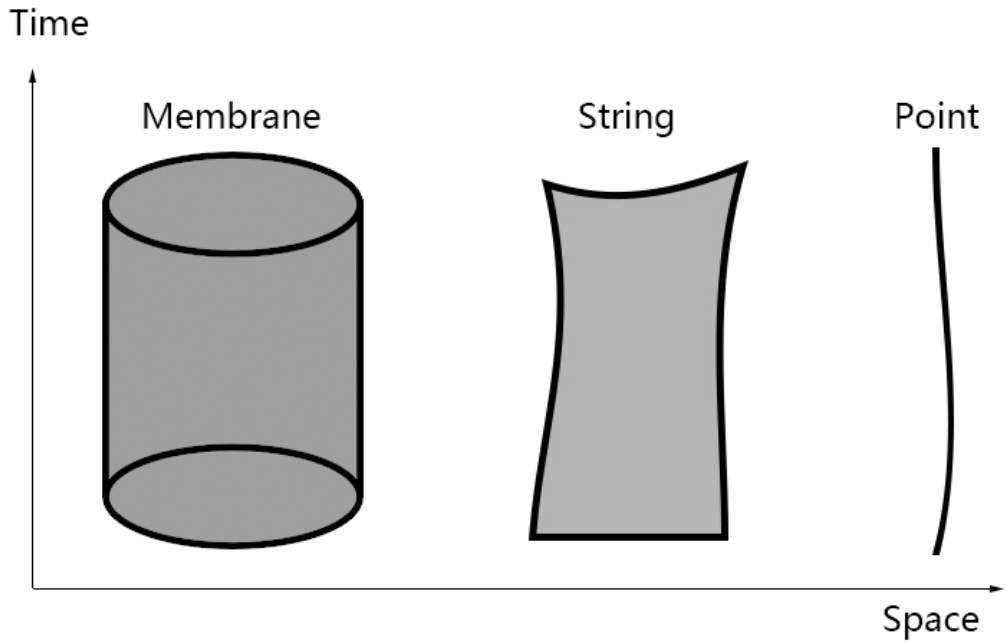
- Q:Is the low-energy limit known?
- A:Yes:

$$\rightarrow \text{11D supergravity} \begin{cases} \text{Gravitons} : g_{\mu\nu} \\ \text{3-form} : A_{\mu\nu\rho} \\ \text{Gravitino} : \psi^a_\mu \end{cases}$$

Dynamical objects in M-theory

A (1+2)-dim membrane (M2-brane) charged under the 3-form field:

$$e \int A_\mu \frac{dx^\mu}{dt} dt \to \mu_2 \int A_{\mu_1\mu_2\mu_3} \frac{\partial x^{\mu_1}}{\partial \tau} \frac{\partial x^{\mu_2}}{\partial \sigma^1} \frac{\partial x^{\mu_3}}{\partial \sigma^2} \mathrm{d}^3\sigma \tag{1}$$



Trajectory of classical point, string and membrane

The 3-form also couples (magnetically) to a 6-dim branes (M5-brane):

$$F_4 = \mathrm{d}A_3 \xrightarrow{\text{Hodge dual}} \tilde{F}_7 = \mathrm{d}\tilde{A}_6 \tag{2}$$

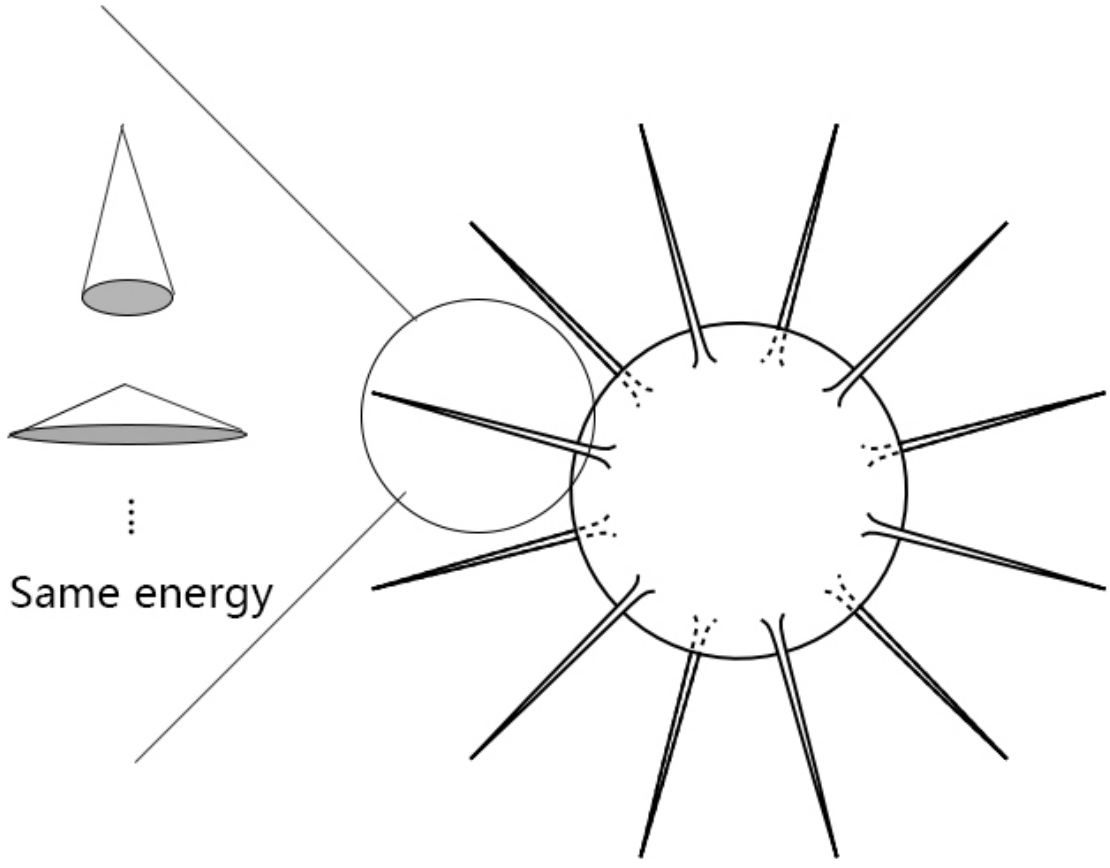
Membrane Quantization[1]

- Quantize strings $\rightarrow$ (Perturbative) String Theory.
- We obtain:
  - A discrete spectrum.
  - Massless modes combined to supergravity particles.
- Quantize M2-brane $\rightarrow$ Matrix Theory.
- We expect:
  - A discrete spectrum.
  - Massless excitation contains graviton, 3-form and gravitino.

But this is not true for supersymmetric membrane.

[1] de Wit, Hoppe, Nicolai, 1988

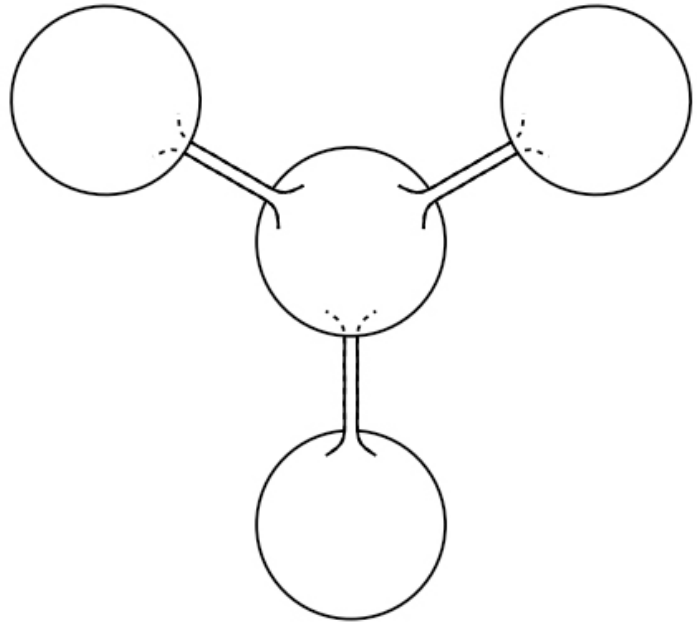
Reason:Membrane is instable.



Classical membrane is unstable to fluctuations

Highly degenerated configurations $\rightarrow$ Continuous spectrum.

Correct pictures [2]



Multi-membranes connected by narrow tubes

- A second quantized theory. [3]
- Limit of small tubes $\rightarrow$ multiple membranes.

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[2] Banks, Fischler, Shenker, Susskind, 1997. Almost ten years later.

[3] By 'second quantized' we mean it is a many-body theory.

Summary

- String Theories $\subset$ M-Theory.
- M2/5-branes $\in$ M-theory.
- Membrane quantization $\rightarrow$ Matrix theory.
- Second quantized nature.

Next

- What is exactly Matrix Theory.
- Meaning of the matrix.

The BFSS Conjecture [4]

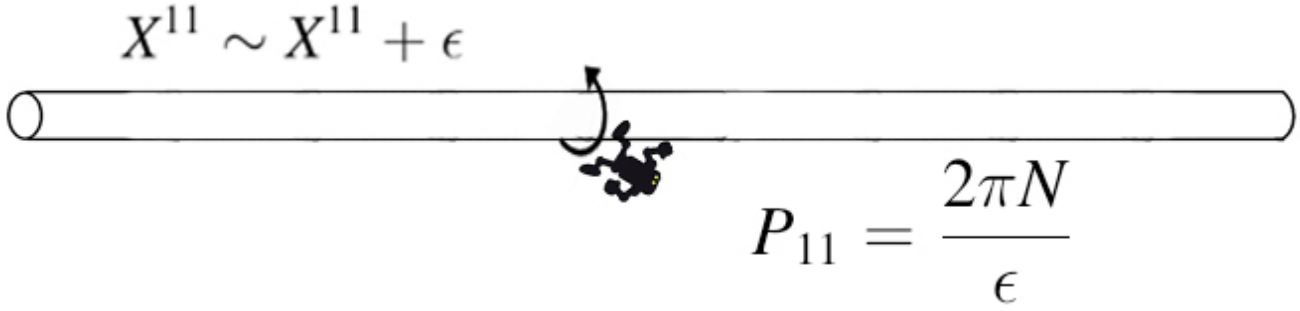
M-theory compactified on a light-like circle with N units of compact momenta is equivalent to the low energy dynamics of N D0-branes.

- What are D0-branes?
- Why is the conjecture true?
- Where is matrix theory?

[4] Original one is proposed by Banks, Fischler, Shenker, and Susskind, 1997. This is a modified one by Susskind later.

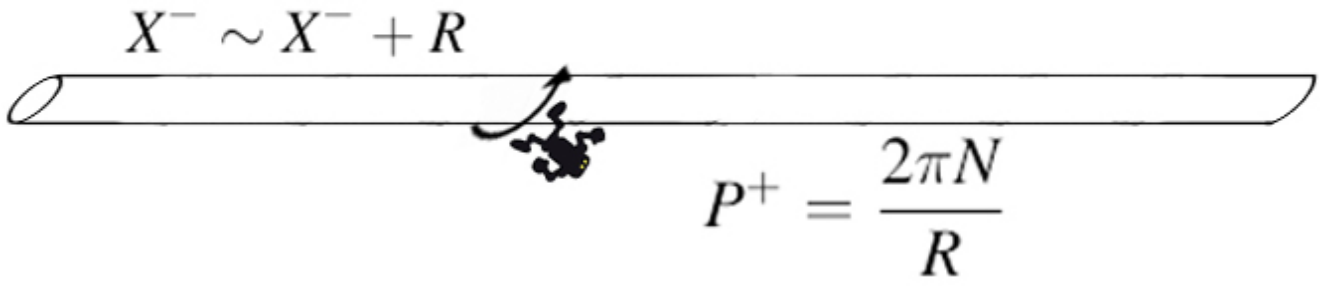
Heuristic understanding [5]

We consider string theory limit of M-theory:



- $N = 1 \rightarrow E^2 = \vec{P}_{10}^2 + \left(\frac{2\pi}{\epsilon}\right)^2 \rightarrow$ D0-brane.
- N D0-branes in IIA String Theory.

After infinite Lorentz boost in X^{11} direction:



$\rightarrow$ M-theory on light-like circle, with N unit of momenta.

[5] Seiberg,1997;Sen,1998

'M' stands for 'Matrix'

Low energy dynamics of N D0-branes in IIA theory:

$$H = \frac{R}{2}\mathrm{Tr}\left(P^i P^i - \frac{1}{2}[X^i, X^j][X^i, X^j] + \theta^T \gamma_i [X^i, \theta]\right)$$

- Matrix-valued $(N \times N)$, supersymmetric quantum mechanics.
- $i = 1, .., 9$ and θ is 16-dim SO(9) real spinor.
- U(N) symmetry.
- Identical to matrix theory.
- Describe a sector of M-theory on Minkowski background. [6]

[6] Compactified on $x^- \sim x^- + 2\pi R$, and sector $p^+ = N/R$.

Second quantized theory

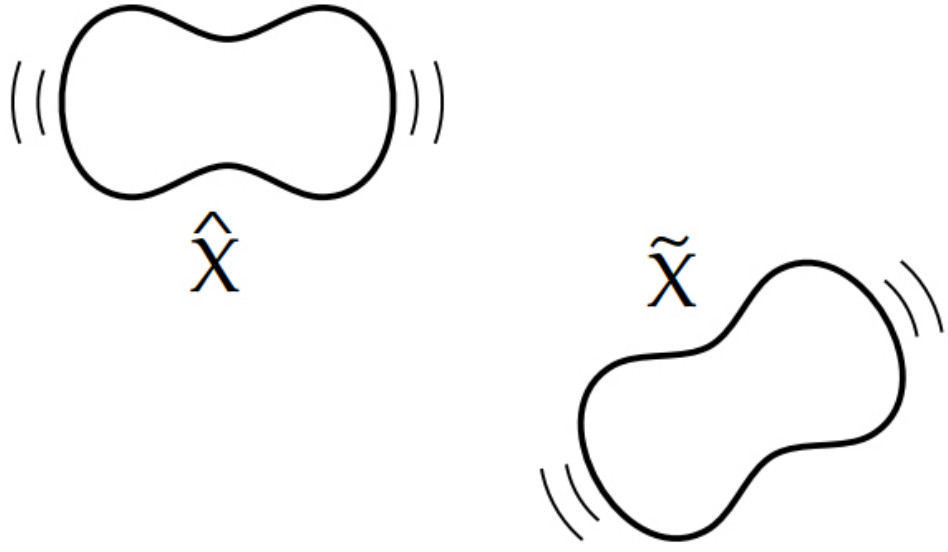
Consider a block-diagonal set of matrices

$$X^i = \begin{pmatrix} \hat{X}^i & 0 \\ 0 & \tilde{X}^i \end{pmatrix} \tag{3}$$

the bosonic matrix theory action is separable

$$S[X^i] = \hat{S}[\hat{X}^i] + \tilde{S}[\tilde{X}^i] \tag{4}$$

$\rightarrow$ Two objects (Bound systems of D0-branes).

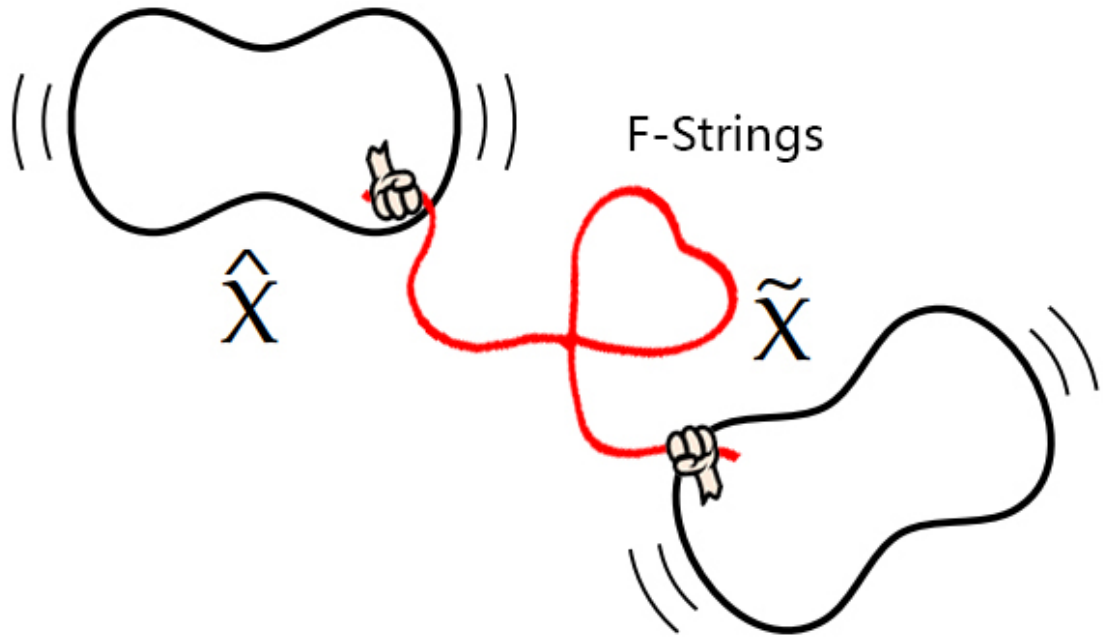


Two objects described by block-diagonal blocks

Meanings of the Matrices

- Diagonal blocks $\rightarrow$ bound states.
- Off-diagonal blocks $\rightarrow$ interactions.

$$X^i = \begin{pmatrix} \hat{X}^i & Y \\ Y^\dagger & \tilde{X}^i \end{pmatrix} \tag{5}$$



Interactions between bound states

Summary

- Matrix theory is
  - Matrix-valued quantum mechanics.
  - Membrane theory.
  - D0 branes.
  - Describe light-cone compactified M-theory.
- D0-branes
  - Diagonal blocks $\rightarrow$ bound states.
  - Off-diagonal blocks $\rightarrow$ interactions.

Next

- Try to recover M-theory's objects from Matrix-theory.

Supergravitons

Consider $N = 1$ (and $P^+ = 1/R$):

$$H = \frac{1}{2R}\dot{X}^2. \tag{6}$$

Fermionic sector:

$$\{\theta_\alpha, \theta_\beta\} = \delta_{\alpha\beta}, \quad \alpha = 1, .., 16. \tag{7}$$

Rewritten as:

$$\theta_i^\pm = \frac{1}{\sqrt{2}}(\theta_i \pm \mathrm{i}\theta_{i+8}), \quad 1 \leq i \leq 8, \tag{8}$$

and we have 8 creation & annihilation operators

$$\{\theta_i^+, \theta_j^-\} = \delta_{ij}, \tag{9}$$

$$\{\theta_i^+, \theta_j^+\} = \{\theta_i^-, \theta_j^-\} = 0. \tag{10}$$

Supergravitons

The Fock space is[7]

$$(\theta_8^+)^{s_8}...(\theta_1^+)^{s_1}|0\rangle \tag{11}$$

where $s_i = 0, 1$, and the dimension is

$$2^8 = 256 = 128_B \oplus 128_F. \tag{12}$$

which agrees with the number of states in supergravity.

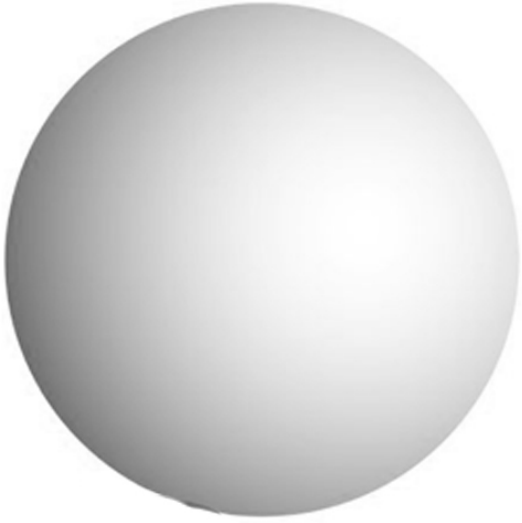
$$g_{\mu\nu} : 44, \tag{13}$$

$$A_{\mu\nu\rho} : 84, \tag{14}$$

$$\Psi_\mu : 128. \tag{15}$$

[7] We assume the existence of a unique ground state

Extended objects



M2-brane in M-theory

LARGE N

Collection of D0-branes

- M2/5 branes are discretized into D0-branes in matrix theory.
- If they exists.

Transverse 5-branes issue

M-Theory	+	−	1	2	3	4	5	6	7	8	9	In Matrix Model
L2-Branes	●	●	●									YES
T2-Branes	●		●	●								YES
L5-Branes	●	●	●	●	●	●						YES
T5-Branes	●		●	●	●	●	●					???

M2/5 -branes in M-theory and in Matrix model

- Light-cone coordinates: $X^{\pm} = (X^0 \pm X^{10})/\sqrt{2}$
- Transverse 5-branes seem to be devoid in Matrix model?

$$\{Q, Q\} \sim H + Z_{L2} + Z_{T2} + Z_{L5} + \cancel{Z_{T5}}$$

- Compact T5-branes have zero charge $\rightarrow$ may be OK.

Other type of matrix model (BMN)

Ordinary matrix model is still difficult:

- Flat directions in the potential.
- No tunable parameters.

BMN model: Another matrix model built on pp-wave:

$$\begin{aligned} \mathrm{d}s^2 = &- 2\mathrm{d}x^+\mathrm{d}x^- + \sum_{A=1}^{9} \mathrm{d}x^A \mathrm{d}x^A \\ &- \left(\sum_{i=1}^{3} \frac{\mu^2}{9} x^i x^i + \sum_{a=4}^{9} \frac{\mu^2}{36} x^a x^a \right) \mathrm{d}x^+ \mathrm{d}x^+ . \end{aligned} \tag{16}$$

with $F_{123+} = \mu$.

BMN Model[8]

Matrix theory on pp-wave. The lagrangian is

$$L = T(X^i, X^a, \psi) - V. \tag{17}$$

where the potential is

$$\begin{aligned} V \sim & \mathrm{Tr}[(\frac{\mu}{3R}X^i + i\epsilon^{ijk}X^jX^k)^2 + \frac{1}{2}\left(i[X^a, X^b]\right)^2 \\ & + \left(i[X^a, X^i]\right)^2 + \left(\frac{\mu}{6R}\right)^2 (X^a)^2]. \end{aligned} \tag{18}$$

V=0:

- $X^a = 0$.
- $X^i = \frac{\mu}{3R}J^i$, where $[J^i, J^k] = i\epsilon^{ijk}J^k$ is SU(2) algebras.

Vacuums: Arbitrary N-dim SU(2) representation.

[8] Berenstein, Maldacena, Nastase, 2002

Conclusion

- Two ways of thinking about Matrix theory
  - A quantized theory of supermembrane.
  - Discrete light-cone quantization of M-theory.
- Reproduce M-theory objects.
- Other aspects include:
  - Two body interactions.
  - Other backgrounds.
  - non-commutate geometry.
  - ...

Still,

- Transverse five-brane issue.
- Background dependent.
- ...