High dimensional seesaw for tiny neutrino masses
中微子微小质量的高量纲跷跷板机制

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based on
PLB694, PLB698, JHEP1106

1. Introduction
2. Higher-dimensional mass operators
3. One example: cascade seesaw
4. Conclusion

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1. Introduction

1.1 Experimental facts on neutrinos

• Precision data –
3 active, almost massless neutrinos interact as assigned in SM

• Oscillation data –
neutrinos have mass, with differences

$$\Delta m_{21}^2 \sim 7.6 \times 10^{-5}\ \mathrm{eV}^2,\ |\Delta m_{31}^2| \sim 2.4 \times 10^{-3}\ \mathrm{eV}^2$$

and leptons mix in CC weak interactions

$$\sin^2\theta_{12} \sim 0.32,\ \sin^2\theta_{23} \sim 0.50,\ \theta_{13} \neq 0.$$

• β decays, cosmological arguments, ... –
neutrinos have sub-eV mass; at least 2 out of 3 are massive

Why are neutrinos so light?

1.2 Weinberg operator: SM as EFT

• Within SM

SM contains only LH neutrinos – Only Majorana ν mass could be possible

But gauge symmetries of SM do *not* allow such a mass –

$\Rightarrow$ $m_\nu \neq 0$ calls for phys beyond SM

Trivial extension:
adding RH ν to form massive Dirac ν as we do for charged fermions –

must tolerate tiny Yukawa coupling less than 10^{-11}

• SM as EFT

We can only see SM particles at low energies below certain scale Λ.
Weinberg (1980): SM gau. symmetries allow a dim-5, L-violating operator

$$\frac{\lambda}{\Lambda}\mathcal{O}_5 + \text{h.c.}, \quad \mathcal{O}_5 = \left(\overline{F_L{}^C}\epsilon H\right)\left(H^T\epsilon F_L\right)$$

H : Higgs doublet
F_L : LH lepton doublet

$$\Rightarrow \quad \frac{1}{2}\frac{\lambda \text{v}^2}{\Lambda}\overline{\nu_L^C}\nu_L + \text{h.c. via } \langle H^0\rangle = \frac{\text{v}}{\sqrt{2}}$$

1.3 Conventional seesaws

Weinberg operator $\mathcal{O}_5$ for m_ν is unique.

Ma (1998): From a purely group theor. analysis for $SU(2)_L \otimes U(1)_Y$:

$$F_L = (\mathbf{2}, -1),\ H = (\mathbf{2}, +1)$$

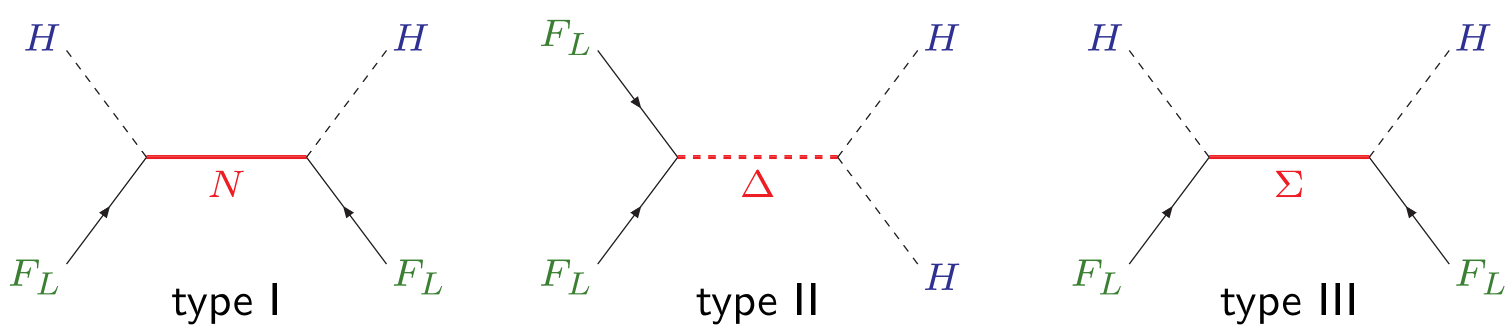
there are 3 and only 3 apparently different realizations of $\mathcal{O}_5$ at tree level.

They may hint at 3 different origins from an underlying theory –

(1) $F_L \otimes H$ as a (fermion) singlet: type I seesaw by adding fermion singlets

(2) $F_L \otimes F_L$ as a (scalar) triplet: type II seesaw by adding scalar triplets

(3) $F_L \otimes H$ as a (fermion) triplet: type III seesaw by adding fermion triplets



Back to $\mathcal{O}_5$:

for example, for $\mathrm{v} \sim 250\ \mathrm{GeV},\ \lambda \sim 1,\ m_\nu \sim 0.1\ \mathrm{eV}$ requires

$$\Lambda \sim 10^{15}\ \mathrm{GeV}$$

Dilemma:

m_ν tends to demand extremely large Λ, while

accessibility to new phys responsible for m_ν relies on a not-too-high Λ

What to do with this tension?

What kind of underlying phys for m_ν would possibly be detectable?

1.4 Beyond conventional seesaws

Goal: lower Λ

Basic approaches

- m_ν induced radiatively:

one loop (Zee '80),
two loops (Zee '85, Babu '88),
three loops (Krauss et al '03), ...

Usually amounts to higher-dim operators with additional small factors

Global symmetries usually required to forbid lower-loop contri.

- m_ν induced at tree level from higher-dim operators

Fields in higher-dim reps required to forbid lower-dim operators

Global symmetries not necessary

Summary

- For realistic pheno, m_ν should be induced from higher-dim operators
- What are higher-dim neutrino mass operators?
- How to realize them in underlying theories?

2. Higher-dimensional mass operators

2.1 SM as low energy effective theory PLB694

Relevant fields are all $SU(2)_L$ doublets with hypercharge in parentheses:

$$F_L\ (-1),\ \ H\ (+1).$$

Mass operator unique at every higher dimension:

$$\mathcal{O}_{2n+5} = \left(\overline{F_L{}^C}\epsilon H\right)\left(H^T\epsilon F_L\right)\left(H^\dagger H\right)^n$$

inducing a neutrino mass upon $\langle H^0\rangle = \mathrm{v}/\sqrt{2}$:

$$\mathcal{O}_{2n+5}:\ \ m_\nu \sim \frac{\lambda \mathrm{v}^2}{\Lambda}\left(\frac{\mathrm{v}^2}{\Lambda^2}\right)^n$$

- accommodates tiny m_ν for an appropriate n without requiring tiny λ or huge Λ
- operator with smallest possible n dominates

Key: Young tableau, **2** of $SU(2)$ is pseudoreal.

2.2 Two-Higgs doublet model (2HDM) as low energy effective theory PLB698

- 2 Higgs doublets arise naturally in extended models, e.g., SUSY models
- phenomenologically viable: $\rho = 1$, FCNC suppressed in type I and II models

Relevant fields are all $SU(2)_L$ doublets with hypercharge in parentheses:

$$F_L\ (-1),\ H_1\ (+1),\ H_2\ (-1).$$

3 operators at dim-5, e.g.,

$$T_5 = \overline{F_L{}^C}\epsilon H_1\ H_2{}^\dagger F_L.$$

12 operators at dim-7, e.g.,

$$T_7^1 = \overline{F_L{}^C}\epsilon H_1\ H_1{}^T\epsilon F_L\ H_2{}^\dagger H_2.$$

33 operators at dim-9, e.g.,

$$X_9^4 = \overline{F_L{}^C}\epsilon H_1\ H_2{}^\dagger F_L\ H_1{}^\dagger H_2\ H_2{}^\dagger H_1.$$

More possible operators at higher dims.

They induce m_ν upon $\langle H^0_{1,2}\rangle = \mathrm{v}_{1,2}/\sqrt{2}$.

• The phys responsible for mass operators may also induce lepton-number violating interactions ($\Delta L = 2$) with gauge bosons.

Such operators started at dim-7. A few examples:

$$J^2 = \Big(\big(D_\mu \overline{F_L{}^C} \big) \epsilon H_1 \Big) \Big(\big(D^\mu F_L \big) \epsilon H_1 \Big) \quad \text{14 operators in total}$$

$$M(B) = \Big(\overline{F_L{}^C} \epsilon H_1 \Big) \sigma^{\mu\nu} \Big(F_L \epsilon \tilde{H}_2 \Big) B_{\mu\nu} \quad \text{10 operators in total}$$

Phenomenology:

Upon SSB, there are the terms

$$J^2_{xy} = \frac{1}{2} g_2^2 \mathrm{v}_1^2 \overline{\ell^C_{Lx}} \ell_{Ly} W^{+\mu} W^+_\mu + \cdots \quad \text{neutrinoless 'double beta' decays}$$

$$M(B)_{xy} = \frac{1}{4} \mathrm{v}_1 \mathrm{v}_2^* \big(\sin\theta_W Z_{\mu\nu} - \cos\theta_W A_{\mu\nu} \big) \overline{\nu^C_{Lx}} \sigma^{\mu\nu} \nu_{Ly}$$

transition dipole interactions for Majorana ν

Systematic analysis of all related processes are possible with effective operators.

3. One example: cascade seesaw

Goal: lowest-dim operators with a given set of fields that give m_ν upon SSB

Require fields in high-dim rep to push operators to higher dim

L must be broken to get seesaw mass

3.1 Basic considerations

- Gauge fields irrelevant; restrict to new fermion and scalar fields

- With new scalars alone, F_L must be in a state of $I = 1, Y = -2$: $\overline{F_L^C} F_L$

$\Rightarrow$ type II seesaw

- With new fermions alone, they must couple F_L to H and thus $I = 0, 1$; $Y = 0, \pm 2$.

$\Rightarrow$ $Y = 0$: type I and III seesaws

$Y = \pm 2$: mix leptons, but not change # of massless/massive modes

Conclusion - need both new fermions and scalars to go beyond conventional seesaws

Assume one new scalar Φ and one new fermion Σ

Assume without losing generality $Y_\Phi \geq 0,\ Y_\Sigma \geq 0$.

Restrictions on their quantum numbers

(1) $(I_\Sigma, Y_\Sigma) \neq (0,0),\ (1,0);\ (I_\Phi, Y_\Phi) \neq (1,2)$, to avoid conventional seesaws.

$(I_\Sigma, Y_\Sigma) \neq (0,2)$, to avoid trivial mixing.

(2) $|I_\Sigma - I_\Phi| = \frac{1}{2}$ to couple (Φ, Σ, F_L).

(3) $I_\Sigma,\ Y_\Sigma/2$ are integral (half-integral) while $I_\Phi,\ Y_\Phi/2$ are half-integral (integral), with $Y_\Sigma/2 \leq I_\Sigma,\ Y_\Phi/2 \leq I_\Phi$ to incorporate neutral members.

(4) Neutrality in Y allows:

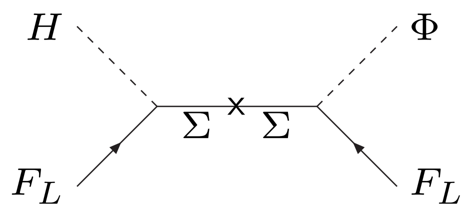
(4a) $Y_\Sigma + Y_\Phi = 1$ for $(F_L \Sigma \Phi)$

(4b) $Y_\Sigma - Y_\Phi = 1$ for $(F_L \Sigma \Phi^\dagger)$

(4c) $Y_\Sigma - Y_\Phi = -1$ for $(F_L \Sigma^C \Phi)$

(5) L must be broken.

3.2 Simplest case: both H and Φ couple to (F_L, Σ)



- Couple (H, Σ, F_L): 4 options for I_Σ, Y_Σ –

2 correspond to type I and III seesaws, 1 to trivial mixing,

1 remaining: $(I_\Sigma, Y_\Sigma) = (1, 2)$ with $(F_L \Sigma H^\dagger)$.

- Then $I_\Phi = 1/2,\ 3/2$: 4 options for (I_Φ, Y_Φ) –

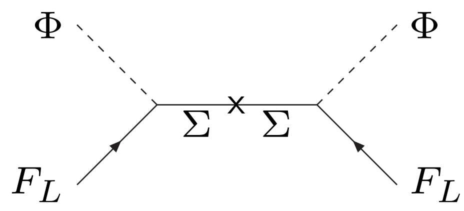
$Y_\Phi = 1$: not possible to break L

$Y_\Phi = 0$: not possible for Yukawa coupling

1 remaining: $(I_\Phi, Y_\Phi) = (3/2, 3)$ with $(F_L \Sigma^C \Phi)$.

This is the model in arXiv: 0905.2710 – dim-7 neutrino mass operator

3.3 Next simplest/more symmetric case: only Φ couples to (F_L, Σ)



- $(I_\Sigma, Y_\Sigma) = (0,0), (1,0), (0,2), (1,2)$ are excluded to avoid coupling (Σ, F_L, H)

- Options for coupling (Σ, F_L, Φ) –

(4b+4c) not possible; (4a+4b) not break L

remaining (4a+4c): $Y_\Sigma = 0, Y_\Phi = 1$

$\Rightarrow$ I_Σ integral and I_Φ half-integral

$\Rightarrow$ $I_\Sigma \geq 2$ to avoid type III, and then $I_\Phi \geq 3/2$

minimal choice: $(I_\Sigma, Y_\Sigma) = (2,0), (I_\Phi, Y_\Phi) = (3/2, 1)$

Simpler than the model in arXiv:0911.1374 – dim-9 neutrino mass operator

m_ν is induced from Yukawa coupling and $\langle\Phi\rangle \neq 0$

Seesaw operates with heavy Σ and small $\langle\Phi\rangle$

Issue: naturally small $\langle\Phi\rangle$

$$V \supset -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2$$
$$+\mu_\Phi^2 \Phi^\dagger \Phi - \left[\kappa\left(\Phi \tilde{H} H \tilde{H}\right)_0 + \text{h.c.}\right] \qquad \tilde{H} \equiv \epsilon H^*$$

- $\kappa = 0$: small $\langle\Phi\rangle$ by fine-tuning parameters $\Rightarrow$ unnatural
  and occurrence of Majoron $\Rightarrow$ not acceptable
- κ term explicitly breaks $L \Rightarrow \kappa$ naturally small:

$$\langle H^0\rangle \approx \sqrt{\frac{\mu_H^2}{2\lambda_H}}, \quad \langle\Phi^0\rangle \approx \kappa^* \frac{\langle H^0\rangle^3}{\mu_\Phi^2}, \; m_\Phi^2 \approx \mu_\Phi^2$$

Other terms in V: small corrections to VEV.

m_ν from dim-9 operator $\mathcal{O}_9$

$$-\mathcal{L}_{\text{Yuk+mass}} = m_\Sigma \overline{\Sigma}\Sigma + \left[y_{ij}\overline{F_{Li}} H f_{Rj} + x_j\left(\overline{F_{Lj}^C}\Phi\Sigma\right)_0 + z_j\left(\overline{\Sigma}\Phi F_{Lj}\right)_0 + \text{h.c.}\right]$$
$$\Rightarrow (m_\nu)_{jk} \sim (x_j z_k + x_k z_j)\frac{\langle\Phi^0\rangle^2}{m_\Sigma} \sim (x_j z_k + x_k z_j)\langle H^0\rangle \kappa^{*2}\frac{\langle H^0\rangle^5}{m_\Sigma m_\Phi^4}$$

3.4 Generalization of next-simplest case to higher dimensions

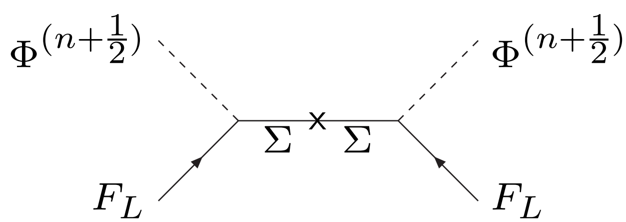
New fields:

Single fermion field Σ of $(I,Y)=(n+1,0)$ with integral $n\geq 1$

A sequence of scalar fields $\Phi^{(m+\frac{1}{2})}$ of $(I,Y)=(m+1/2,1)$ with $1\leq m\leq n$

Consequences:

- Only $\Phi^{(n+\frac{1}{2})}$ can Yukawa couple to (Σ,F_L) due to $SU(2)_L$ invariance:



- Only $\Phi^{(\frac{3}{2})}$ can directly develop a naturally small VEV,

while $\Phi^{(n+\frac{1}{2})}$ develops a naturally small VEV by cascading seesaw – see below

- m_ν is induced from a dim-$(5+4n)$ operator.
- No global sym is imposed to realize the above.

Small $\langle\Phi^{(n+\frac{1}{2})}\rangle$ by cascading seesaw

largest contri. to $\langle\Phi^{(n+\frac{1}{2})}\rangle \leftrightarrow$ lowest-dim operator for $m_\nu \leftrightarrow$ leading contri. to m_ν

- With $n+1$ multiplets of scalars that share the same Y but have $I = 1/2, 3/2, \cdots, n+1/2$, there are many terms in V

But most of them only provide couplings, mixing and mass splitting suppressed by VEV's – not of our concern

- L must be explicitly broken.

Since L is necessarily violated by $\kappa(\Phi^{(3/2)}\tilde{H}H\tilde{H})_0$ to induce $\langle\Phi^{(3/2)}\rangle \neq 0$, only need consider L-conserving terms for all others, that transmit VEV from $\langle\Phi^{(3/2)}\rangle \neq 0$ towards $\langle\Phi^{(n+1/2)}\rangle \neq 0$.

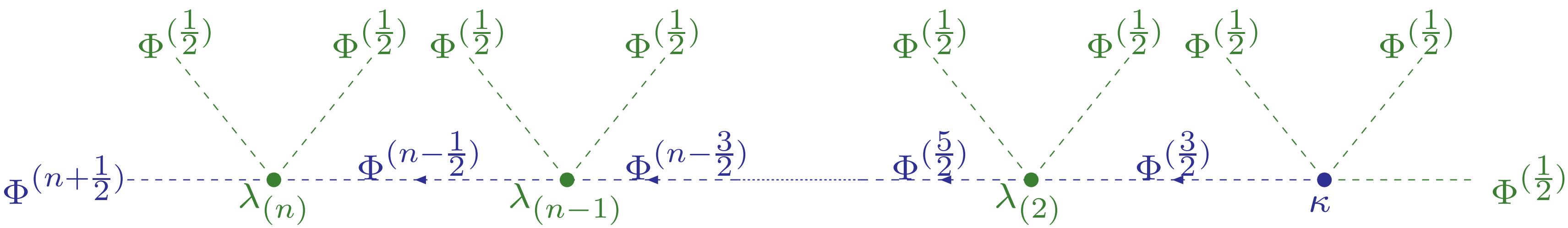
$$V^{(n+\frac{1}{2})} \supset -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 \qquad \Phi^{(\frac{1}{2})} \equiv H,\ \lambda_{(1)} \equiv \kappa$$
$$+\sum_{k=1}^{n} \mu_{(k)}^2 \Phi^{(k+\frac{1}{2})\dagger}\Phi^{(k+\frac{1}{2})} - \sum_{k=1}^{n}\left[\lambda_{(k)}\left(\Phi^{(k+\frac{1}{2})}\tilde{\Phi}^{(k-\frac{1}{2})}H\tilde{H}\right)_0 + \text{h.c.}\right],$$

Leading term of VEV's:

$$\langle \Phi_0^{(k+\frac{1}{2})} \rangle = \frac{1}{2\sqrt{2k+1}} \frac{|\langle H_0 \rangle|^2}{\mu_{(k)}^2} \lambda_{(k)}^* \langle \Phi_0^{(k-\frac{1}{2})} \rangle, \quad n \geq k \geq 1,$$

$$\Rightarrow \quad \langle \Phi_0^{(n+\frac{1}{2})} \rangle = \langle H_0 \rangle |\langle H_0 \rangle|^{2n} \prod_{k=1}^{n} \frac{1}{2\sqrt{2k+1}} \frac{\lambda_{(k)}^*}{\mu_{(k)}^2}.$$

In diagrams



The first cascade is suppressed by L-violation.

Remaining cascades are suppressed by heavy scalar masses.

Seesaw neutrino mass

$$
\begin{aligned}
m^{\nu}_{jk} &= (x_j z_k + x_k z_j)\frac{1}{m_\Sigma}(-1)^n \frac{1}{2(2n+3)}\Big(\langle \Phi_0^{(n+\frac{1}{2})}\rangle\Big)^2 \\
&= (-1)^n (x_j z_k + x_k z_j)\frac{1}{m_\Sigma}\frac{1}{2(2n+3)}\langle H_0\rangle^2 \big|\langle H_0\rangle\big|^{4n}\Big(\prod_{k=1}^{n}\frac{1}{2\sqrt{2k+1}}\frac{\lambda^*_{(k)}}{\mu^2_{(k)}}\Big)^2
\end{aligned}
$$

- suppressed by $(4n+1)$ powers of heavy scales
  and one power of small L-violating coupling

$\Leftrightarrow$ mass operator of dimension $(4n+5)$

- One neutrino massless at tree level. Other two massive in either hierarchy.
- Order of magnitude estimate:

ignore Clebsch-Gordan coeffi.

$x \sim z$, $m_\Sigma \sim \mu_{(k)} \sim M$, $\lambda_{(k)} \sim \lambda$ (for $k > 1$)

$\Rightarrow m \sim x^2 \lambda^{2(n-1)} \kappa^2 \langle H_0\rangle^{2+4n} M^{-1-4n}$

For example, with $x \sim 10^{-2}$, $\lambda \sim 10^{-1}$, $\kappa \sim 10^{-3}$, $\langle H_0\rangle \sim 174\ \mathrm{GeV}$

$m \sim 0.1\ \mathrm{eV}$ requires $M \sim 490\ \mathrm{GeV}$ at $n = 1$, $M \sim 190\ \mathrm{GeV}$ at $n = 2$

4. Conclusion

• Conventional seesaws for neutrino mass imply that underlying physics is generally hard to access.

• There are two basic approaches to effectively lower new physics scale.

• By employing one new fermion multiplet and a sequence of new scalars which develop VEV's via cascading seesaw.

$\Rightarrow$ Neutrino mass operators first appear at dim-$(5+4n)$.

Tiny neutrino mass is induced without requiring too small couplings or a too high scale.

Underlying physics can possibly be explored at high-energy colliders and in low-energy precision measurements – work in progress.

谢谢！