中国科技大学交叉学科理论研究中心

Probing dark energy beyond ΛCDM with observations

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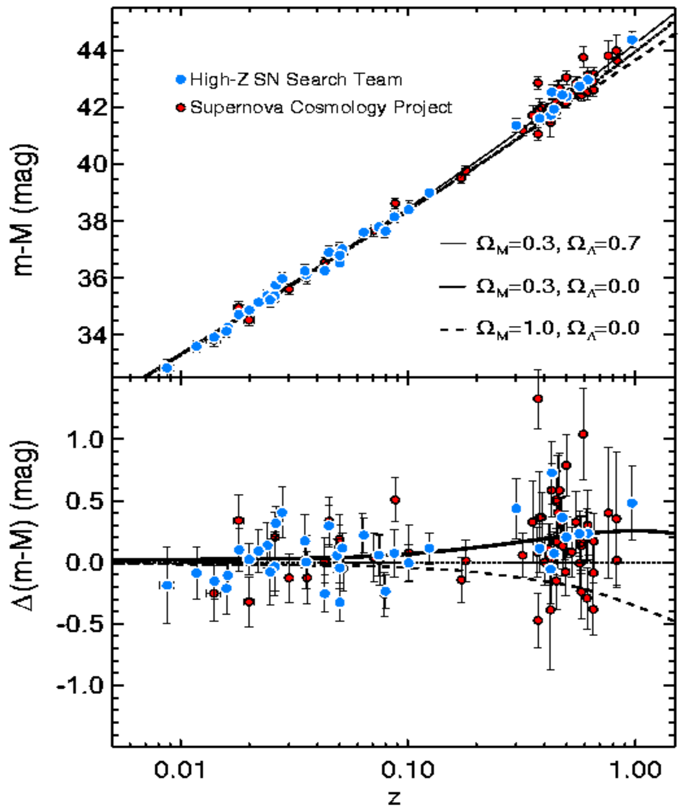
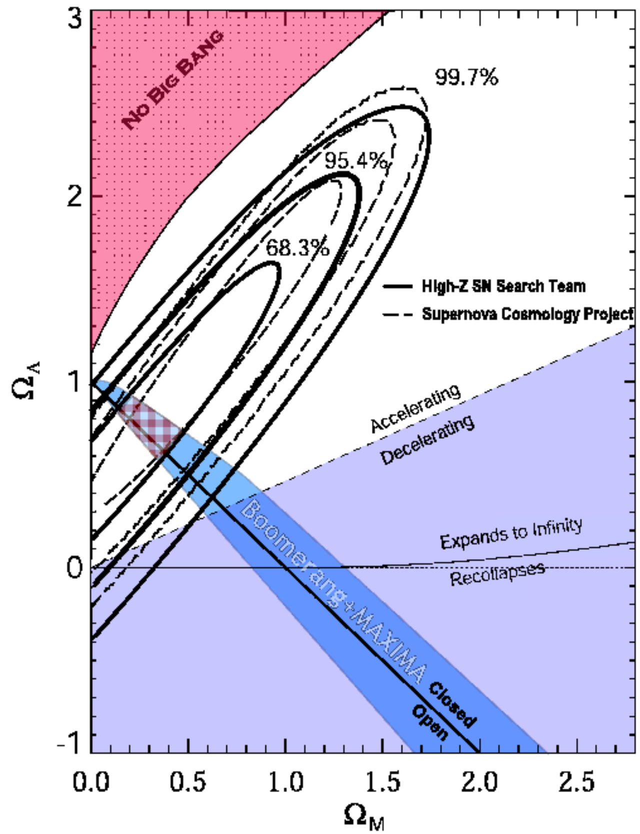
Outline

- Cosmic acceleration: most profound puzzle in contemporary physics
- Most important questions
- Current status: DE & MG
- MG: f(R) and its test (scale-dependent growth rate)
- IDE: problem & solution
- PPF for IDE
- Observational test for IDE

Discovery of cosmic acceleration





SUPERNOVA SEARCH TEAM collaboration, A.G. Riess et al., *Observational evidence from supernovae for an accelerating universe and a cosmological constant*, Astron. J. **116** (1998) 1009 [astro-ph/9805201] [INSPIRE];
SUPERNOVA COSMOLOGY PROJECT collaboration, S. Perlmutter et al., *Measurements of Omega and Lambda from 42 high redshift supernovae*, Astrophys. J. **517** (1999) 565 [astro-ph/9812133] [INSPIRE].

Why was the SN evidence for cosmic acceleration accepted so quickly by the community at large?

- Internal checks carried out by the two teams
- Substantial indirect evidence provided by CMB and LSS data
- Balloon-borne CMB experiments soon mapped the first acoustic peak and measured its angular location (Ω=1)
- Hubble constant measurement (H_0=71+/-6 km s^{-1} Mpc^{-1})

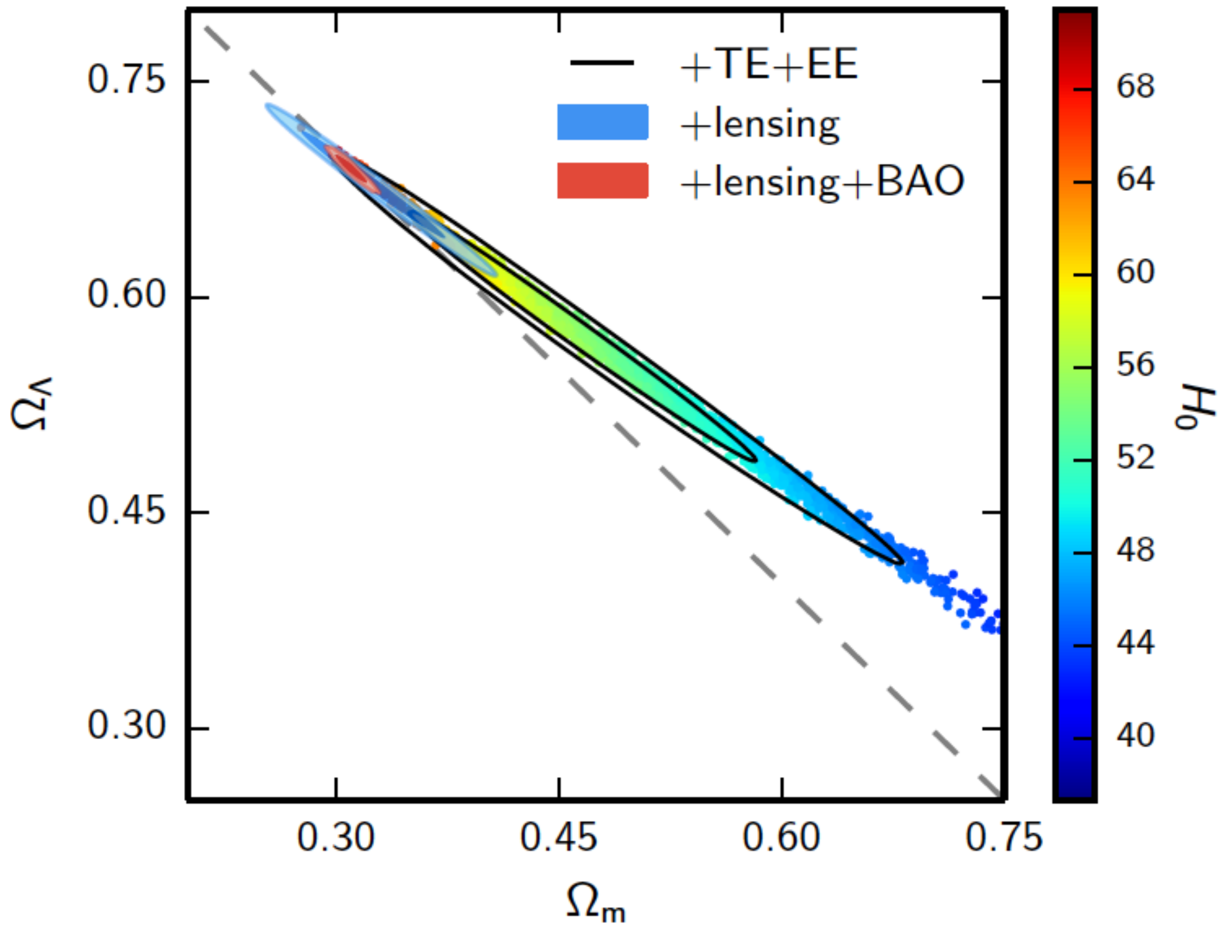
$t_0=(2/3)H_0^{-1}$=9.5 Gyr, too young to accommodate the 12–14 Gyr ages estimated for globular clusters (e.g., Chaboyer, 1998).

2011 Nobel Prize in Physics Awarded for Discovery of the Accelerating Expansion of the Universe

Saul Perlmutter   Brian P. Schmidt   Adam G. Riess

The universe is now dominated by some gravitationally repulsive source, dark energy, whose characteristic is negative pressure.

Planck 2015



$$\Omega_K = 0.000 \pm 0.005 \quad (95\%, \textit{Planck}\ \mathrm{TT+lowP+lensing+BAO})$$

existence of Ω_Λ : > 110 σ

Questions

- **MG or DE?**

  Does acceleration arise from a breakdown of GR on cosmological scales or from a new energy component that exerts repulsive gravity with in GR?

- **CC or Q?**

  If acceleration is caused by a new component, is its energy density constant in space and time, as expected for a fundamental vacuum energy, or does it show variations that indicate a dynamical field?

- **IDE?**

  Is there any direct non-gravitational coupling between dark energy and dark matter?

Why so difficult?

measure w(z)

$$\Omega_{\rm DE}\frac{\rho_{\rm DE}(z)}{\rho_{\rm DE}(z=0)} = \Omega_{\rm DE}\exp\left[3\int_0^z [1+w(z')]\frac{dz'}{1+z'}\right]$$

$$\frac{H^2(z)}{H_0^2} = \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_k(1+z)^2 + \Omega_\phi \frac{u_\phi(z)}{u_\phi(z=0)}$$

$$D_C(z) = \frac{c}{H_0}\int_0^z dz' \frac{H_0}{H(z')}$$

$$\ddot{G}_{\rm GR} + 2H(z)\dot{G}_{\rm GR} - \frac{3}{2}\Omega_m H_0^2(1+z)^3 G_{\rm GR} = 0$$

$$\delta(\mathbf{x},t) \equiv \frac{\rho_m(\mathbf{x},t) - \bar{\rho}_m(t)}{\bar{\rho}_m(t)} = \delta(\mathbf{x},t_i)\times\frac{G(t)}{G(t_i)}$$

- DE affects H(z) by an integral over w(z)
- DE affects distances and the growth factor by a further integration over functions of H(z)

Why so difficult?

search for MG (test GR)

- Search for MG by comparing the history of cosmic structure growth to the history of cosmic expansion
- Within GR, these two are linked by a consistency relation
- MG can change the predicted rate of structure growth, and it can make the growth rate dependent on scale or environment
- MG alters the combination of potentials for gravitational lensing and the dynamics of non-relativistic tracer (such as galaxies or stars) in different ways
- A general strategy to test MG is to precisely measure both expansion history and growth history to see whether they yield consistent results for H(z) or w(z)

$$f_{\mathrm{GR}}(z) \equiv \frac{d \ln G_{\mathrm{GR}}}{d \ln a} \approx [\Omega_m(z)]^{\gamma}$$

$$\gamma = 0.55 + 0.05[1 + w(z = 1)]$$

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Measuring growth index in a Universe with sterile neutrinos

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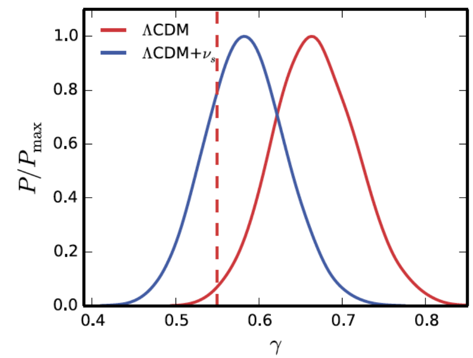
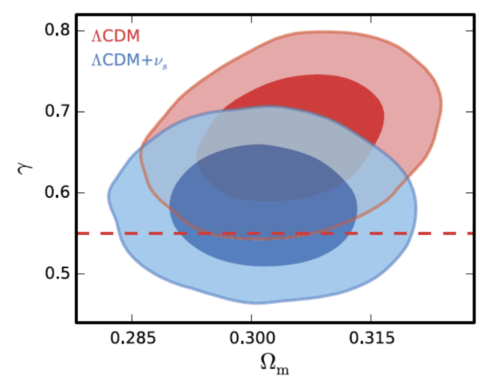
$$f(a)\sigma_8(a) = d\sigma_8(a)/d\ln a$$

- RSD constrain γ： testing GR
- Samushia et al. 2012 (BOSS-DR9)： $\gamma = 0.75 \pm 0.09$
- Beutler et al. 2013 (BOSS-DR11): $\gamma = 0.772^{+0.124}_{-0.097}$
- Discrepant from GR at 2-3σ level





$$\gamma = 0.584^{+0.047}_{-0.048}$$

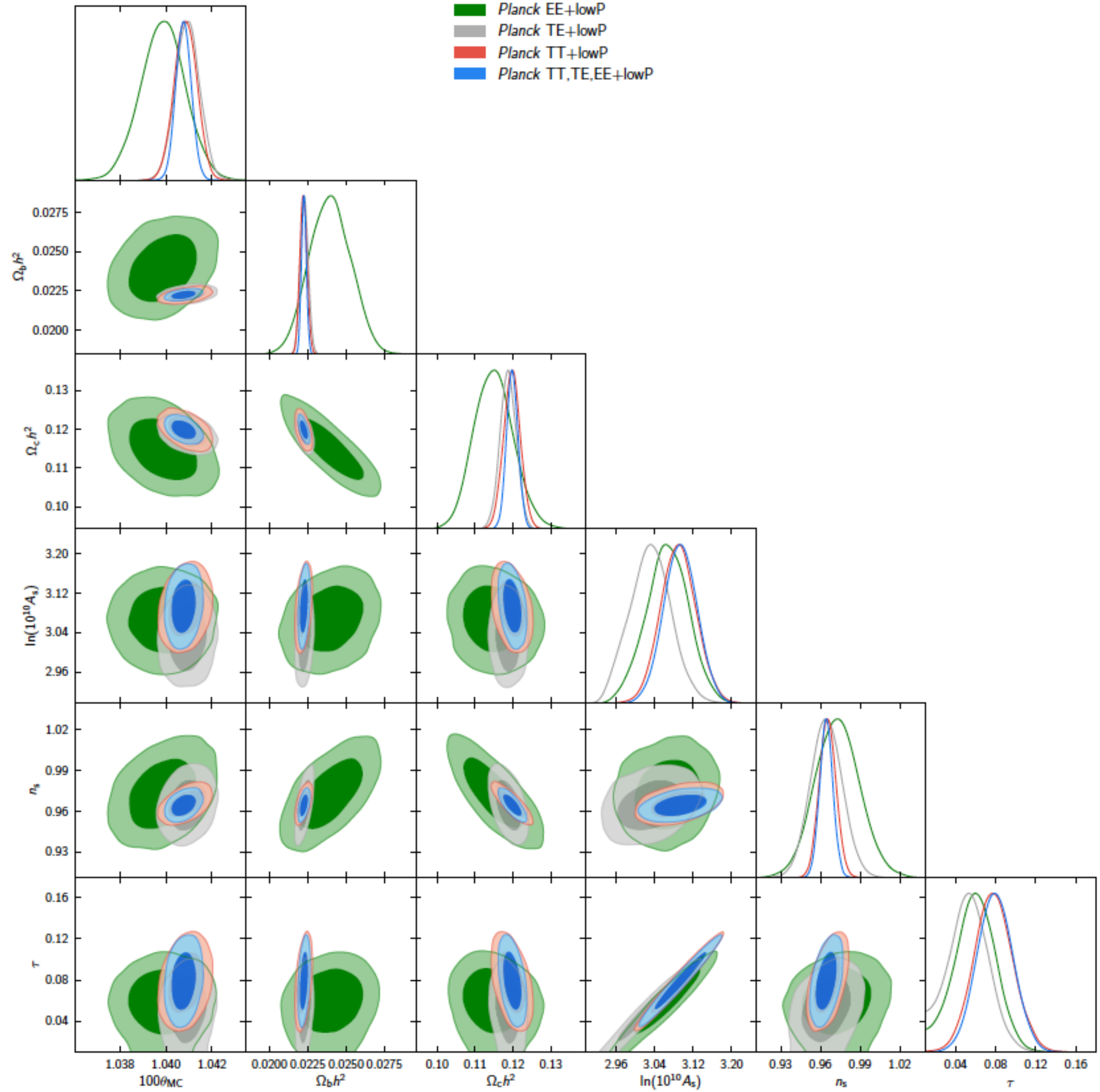
the consistency with GR is at the 0.6σ level

Observational probes

- Cosmic Microwave Background Anisotropies
- Type Ia Supernovae
- Baryon Acoustic Oscillations (BAO)
- Weak Gravitational Lensing
- Clusters of Galaxies
- Redshift-Space Distortions (RSD)
- Direct Determination of H_0

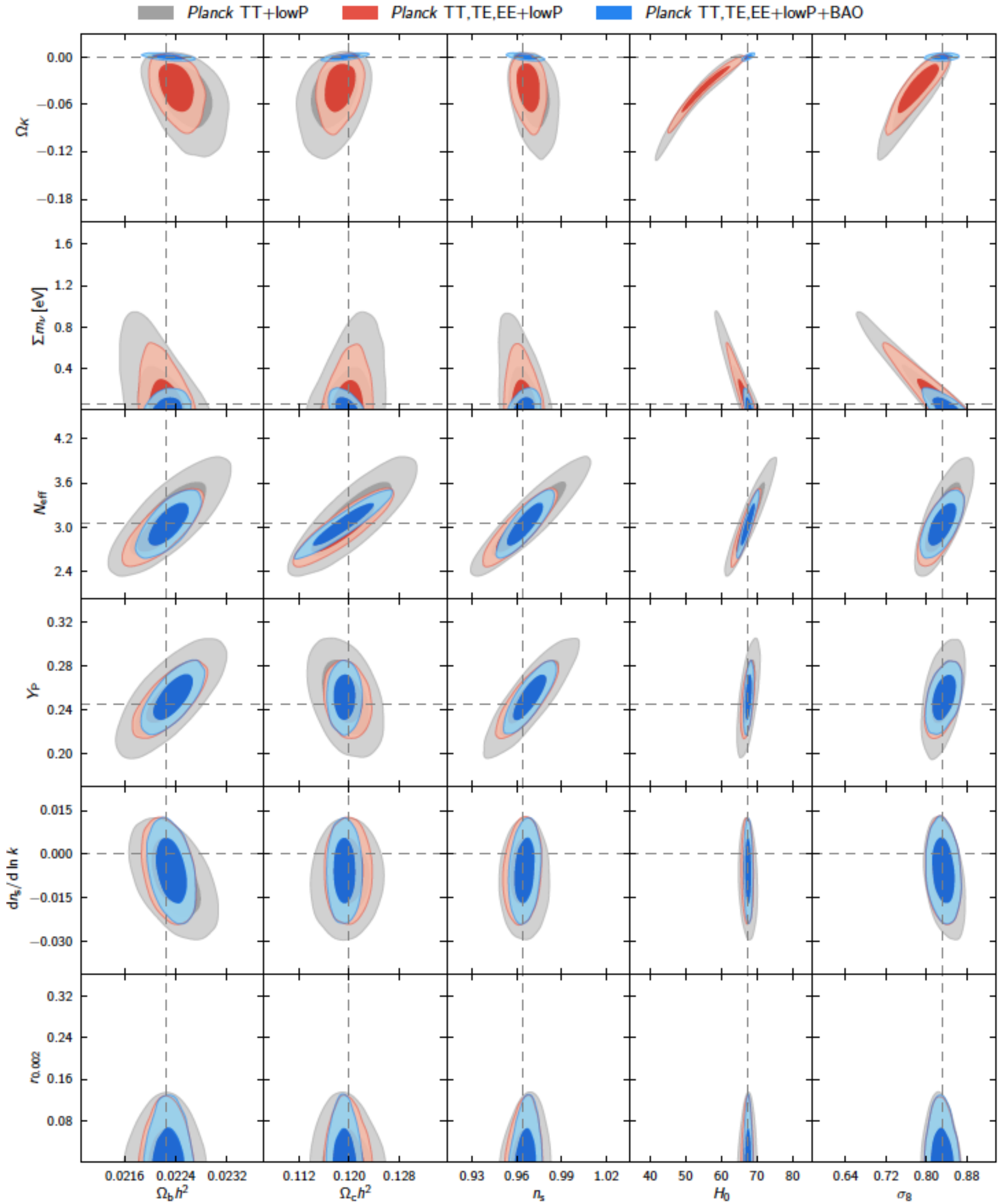
ΛCDM cosmology after Planck

- 6-parameter base ΛCDM model
- Tests of the base model
- Tests of the extensions to the base model
- Planck conclusion: strongly favor the base ΛCDM model



Planck EE+lowP
Planck TE+lowP
Planck TT+lowP
Planck TT,TE,EE+lowP
100θ_MC
Ω_b h^2
Ω_c h^2
ln(10^10 A_s)
n_s
τ

Parameter	TT+lowP 68 % limits	TT+lowP+lensing 68 % limits	TT+lowP+lensing+ext 68 % limits	TT,TE,EE+lowP 68 % limits	TT,TE,EE+lowP+lensing 68 % limits	TT,TE,EE+lowP+lensing+ext 68 % limits
$\Omega_b h^2$	0.02222 ± 0.00023	0.02226 ± 0.00023	0.02227 ± 0.00020	0.02225 ± 0.00016	0.02226 ± 0.00016	0.02230 ± 0.00014
$\Omega_c h^2$	0.1197 ± 0.0022	0.1186 ± 0.0020	0.1184 ± 0.0012	0.1198 ± 0.0015	0.1193 ± 0.0014	0.1188 ± 0.0010
$100\theta_{MC}$	1.04085 ± 0.00047	1.04103 ± 0.00046	1.04106 ± 0.00041	1.04077 ± 0.00032	1.04087 ± 0.00032	1.04093 ± 0.00030
τ	0.078 ± 0.019	0.066 ± 0.016	0.067 ± 0.013	0.079 ± 0.017	0.063 ± 0.014	0.066 ± 0.012
$\ln(10^{10} A_s)$	3.089 ± 0.036	3.062 ± 0.029	3.064 ± 0.024	3.094 ± 0.034	3.059 ± 0.025	3.064 ± 0.023
n_s	0.9655 ± 0.0062	0.9677 ± 0.0060	0.9681 ± 0.0044	0.9645 ± 0.0049	0.9653 ± 0.0048	0.9667 ± 0.0040
H_0	67.31 ± 0.96	67.81 ± 0.92	67.90 ± 0.55	67.27 ± 0.66	67.51 ± 0.64	67.74 ± 0.46
Ω_Λ	0.685 ± 0.013	0.692 ± 0.012	0.6935 ± 0.0072	0.6844 ± 0.0091	0.6879 ± 0.0087	0.6911 ± 0.0062
Ω_m	0.315 ± 0.013	0.308 ± 0.012	0.3065 ± 0.0072	0.3156 ± 0.0091	0.3121 ± 0.0087	0.3089 ± 0.0062
$\Omega_m h^2$	0.1426 ± 0.0020	0.1415 ± 0.0019	0.1413 ± 0.0011	0.1427 ± 0.0014	0.1422 ± 0.0013	0.14170 ± 0.00097
$\Omega_m h^3$	0.09597 ± 0.00045	0.09591 ± 0.00045	0.09593 ± 0.00045	0.09601 ± 0.00029	0.09596 ± 0.00030	0.09598 ± 0.00029
σ_8	0.829 ± 0.014	0.8149 ± 0.0093	0.8154 ± 0.0090	0.831 ± 0.013	0.8150 ± 0.0087	0.8159 ± 0.0086
$\sigma_8 \Omega_m^{0.5}$	0.466 ± 0.013	0.4521 ± 0.0088	0.4514 ± 0.0066	0.4668 ± 0.0098	0.4553 ± 0.0068	0.4535 ± 0.0059
$\sigma_8 \Omega_m^{0.25}$	0.621 ± 0.013	0.6069 ± 0.0076	0.6066 ± 0.0070	0.623 ± 0.011	0.6091 ± 0.0067	0.6083 ± 0.0066
z_{re}	$9.9^{+1.8}_{-1.6}$	$8.8^{+1.7}_{-1.4}$	$8.9^{+1.3}_{-1.2}$	$10.0^{+1.7}_{-1.5}$	$8.5^{+1.4}_{-1.2}$	$8.8^{+1.2}_{-1.1}$
$10^9 A_s$	$2.198^{+0.076}_{-0.085}$	2.139 ± 0.063	2.143 ± 0.051	2.207 ± 0.074	2.130 ± 0.053	2.142 ± 0.049
$10^9 A_s e^{-2\tau}$	1.880 ± 0.014	1.874 ± 0.013	1.873 ± 0.011	1.882 ± 0.012	1.878 ± 0.011	1.876 ± 0.011
Age/Gyr	13.813 ± 0.038	13.799 ± 0.038	13.796 ± 0.029	13.813 ± 0.026	13.807 ± 0.026	13.799 ± 0.021
z_*	1090.09 ± 0.42	1089.94 ± 0.42	1089.90 ± 0.30	1090.06 ± 0.30	1090.00 ± 0.29	1089.90 ± 0.23
r_*	144.61 ± 0.49	144.89 ± 0.44	144.93 ± 0.30	144.57 ± 0.32	144.71 ± 0.31	144.81 ± 0.24
$100\theta_*$	1.04105 ± 0.00046	1.04122 ± 0.00045	1.04126 ± 0.00041	1.04096 ± 0.00032	1.04106 ± 0.00031	1.04112 ± 0.00029
z_{drag}	1059.57 ± 0.46	1059.57 ± 0.47	1059.60 ± 0.44	1059.65 ± 0.31	1059.62 ± 0.31	1059.68 ± 0.29
r_{drag}	147.33 ± 0.49	147.60 ± 0.43	147.63 ± 0.32	147.27 ± 0.31	147.41 ± 0.30	147.50 ± 0.24
k_D	0.14050 ± 0.00052	0.14024 ± 0.00047	0.14022 ± 0.00042	0.14059 ± 0.00032	0.14044 ± 0.00032	0.14038 ± 0.00029
z_{eq}	3393 ± 49	3365 ± 44	3361 ± 27	3395 ± 33	3382 ± 32	3371 ± 23
k_{eq}	0.01035 ± 0.00015	0.01027 ± 0.00014	0.010258 ± 0.000083	0.01036 ± 0.00010	0.010322 ± 0.000096	0.010288 ± 0.000071
$100\theta_{s,eq}$	0.4502 ± 0.0047	0.4529 ± 0.0044	0.4533 ± 0.0026	0.4499 ± 0.0032	0.4512 ± 0.0031	0.4523 ± 0.0023


Planck TT+lowP
Planck TT,TE,EE+lowP
Planck TT,TE,EE+lowP+BAO
ΩK
Σmν [eV]
Neff
YP
dns/d ln k
r0.002
Ωbh2
Ωch2
ns
H0
σ8

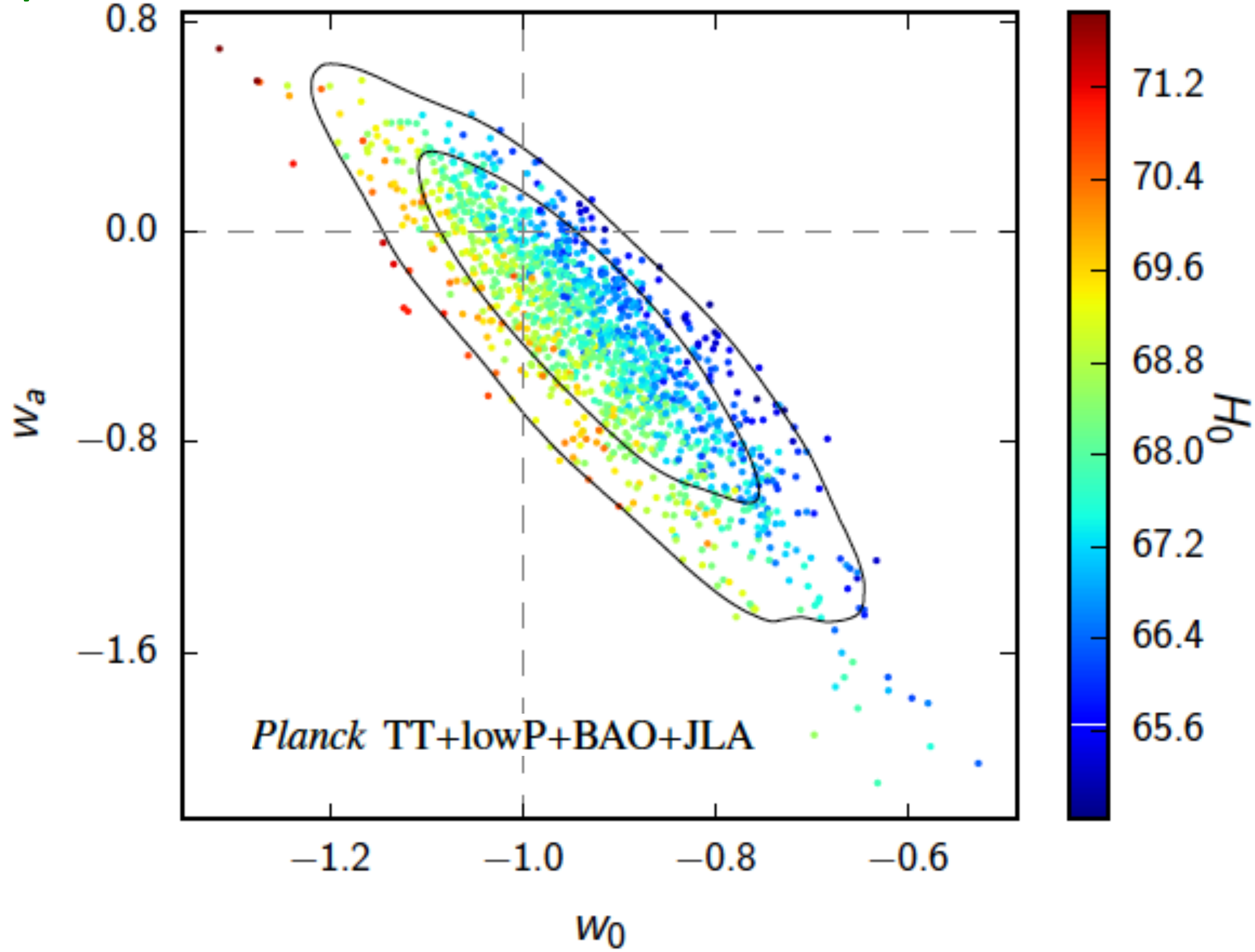
Constant w DE (wCDM)

95% CL

$$w = -1.023^{+0.091}_{-0.096} \quad \textit{Planck}\ \mathrm{TT{+}lowP{+}ext}\,;$$
$$w = -1.006^{+0.085}_{-0.091} \quad \textit{Planck}\ \mathrm{TT{+}lowP{+}lensing{+}ext}\,;$$
$$w = -1.019^{+0.075}_{-0.080} \quad \textit{Planck}\ \mathrm{TT,TE,EE{+}lowP{+}lensing{+}ext}$$

- Highly consistent with CC (though slightly phantom)
- Precision < 10%
- Physically unrealistic (w=const)
- One should probe evolution of w

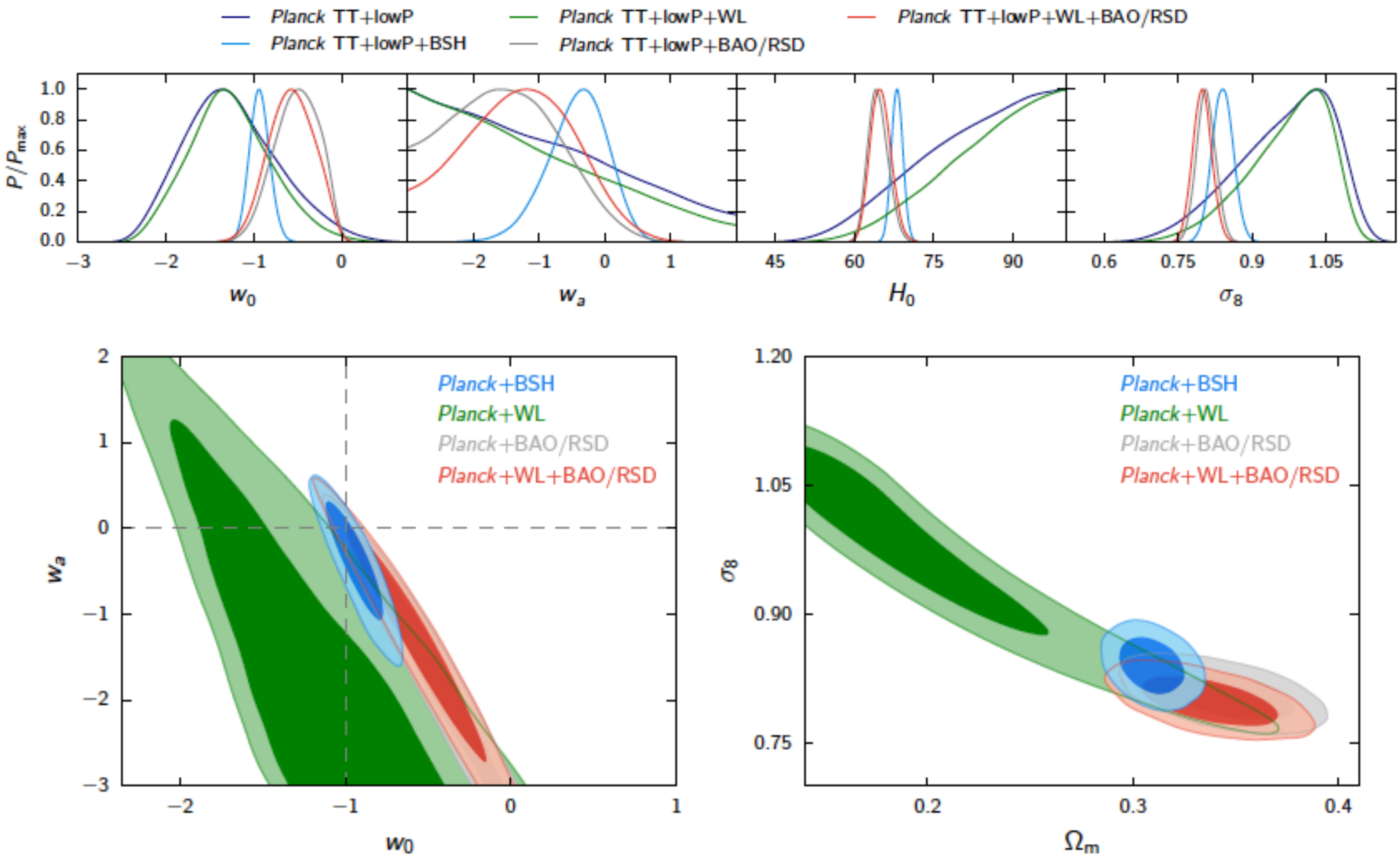
$w=w_0+w_a(1-a)$



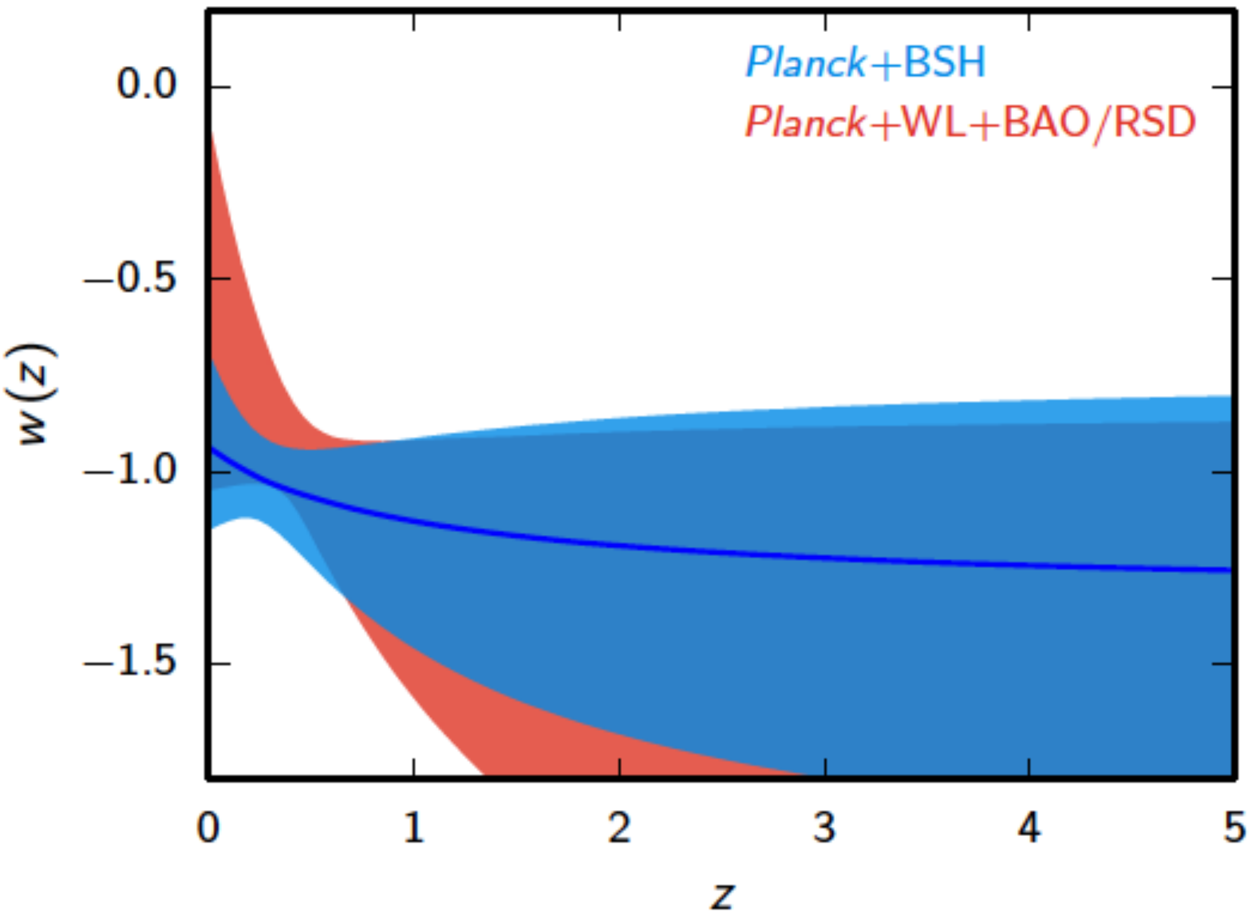
find no evidence for a departure from the base ΛCDM cosmology

$H_0 = (68.2 \pm 1.1)\ \mathrm{km\ s^{-1} Mpc^{-1}}$

higher values of H_0 would favour the phantom regime, $w < -1$

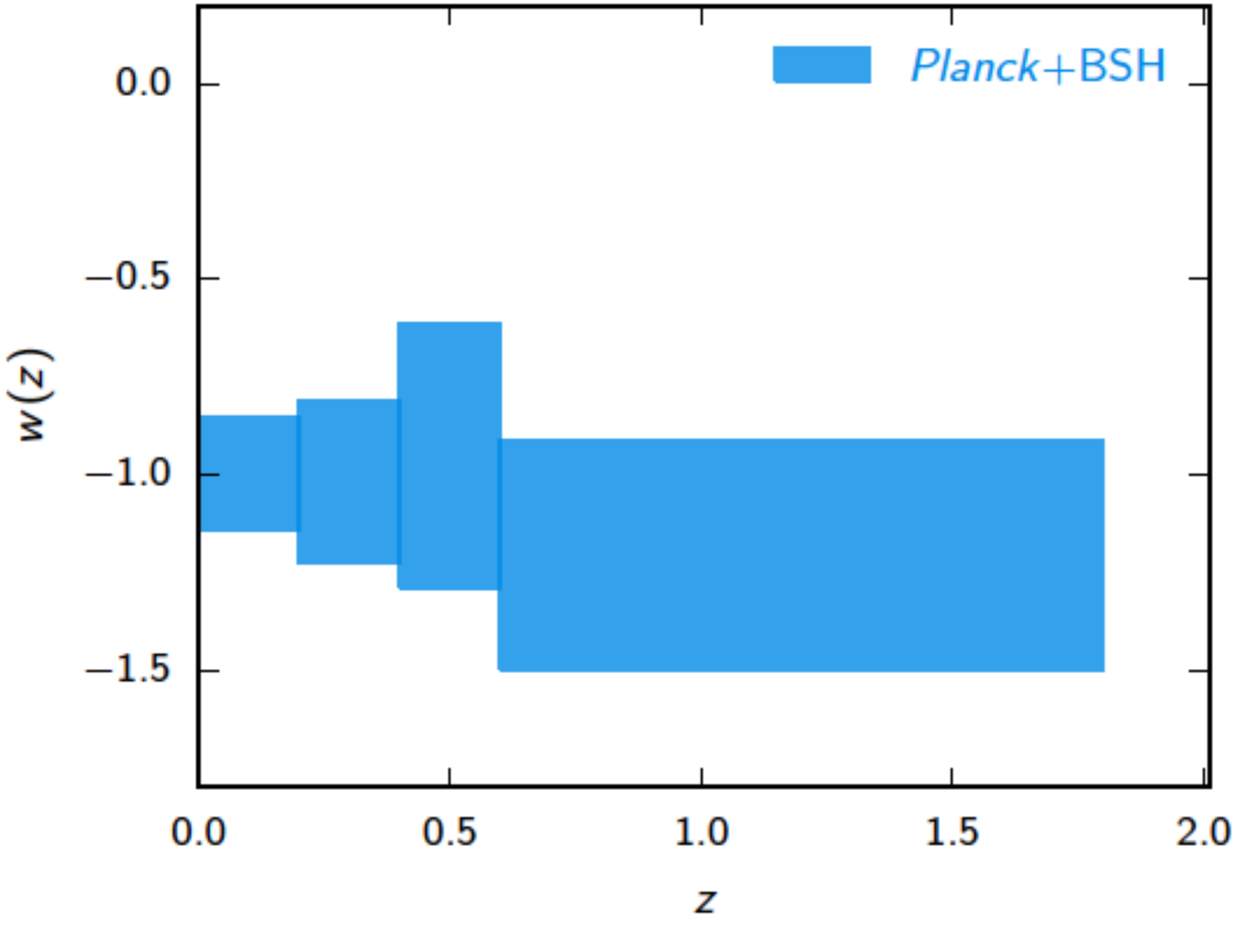


- Planck TT+lowP+BSH is most secure and gives the strongest constraints
- WL and RSD are weaker, since we consider a smooth dark energy model where perturbations are suppressed on small scales
- The use of WL is very conservative



Planck+BSH
Planck+WL+BAO/RSD
0.0
−0.5
−1.0
−1.5
w(z)
0
1
2
3
4
5
z

Principal Component Analysis (PCA)



Scalar-field DE

Huang, Z., Bond, J. R., & Kofman, L., Parameterizing and Measuring Dark Energy Trajectories from Late Inflatons. 2011, ApJ, 726, 64, arXiv:1007.5297

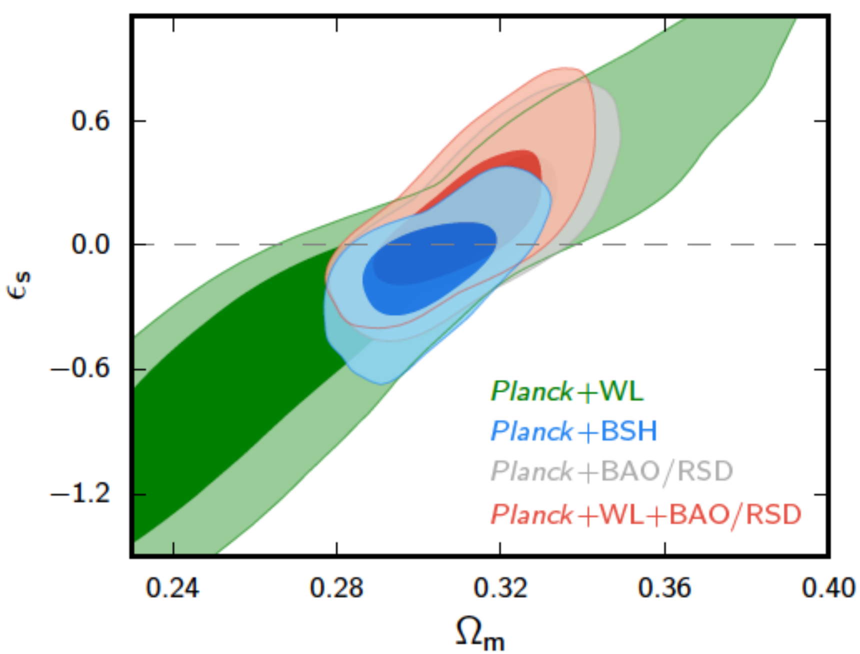
$$w = -1 + \frac{2}{3}\epsilon_s F^2\left(\frac{a}{a_{de}}\right)$$

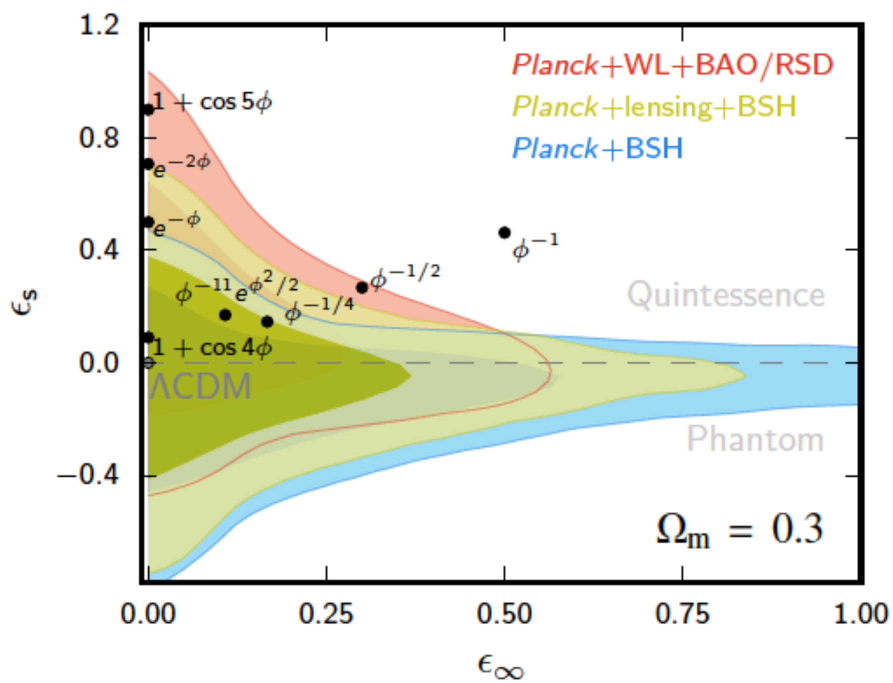
$$\epsilon_s \equiv \epsilon_V|_{a=a_{de}}$$

$$F(x) \equiv \frac{\sqrt{1+x^3}}{x^{3/2}} - \frac{\ln\left(x^{3/2}+\sqrt{1+x^3}\right)}{x^3}$$

$$\epsilon_V \equiv \left(\frac{d\ln V}{d\phi}\right)^2 M_P^2/2$$

$$\epsilon_\infty \equiv \lim_{a\to 0}\epsilon_V(a)\Omega_\phi(a)$$

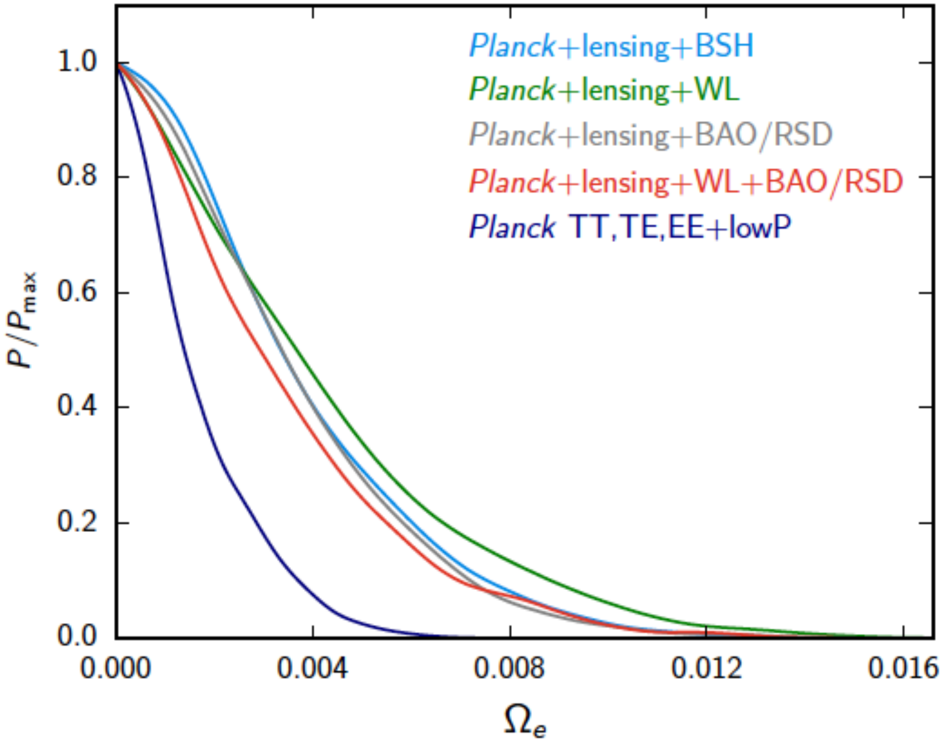




Early Dark Energy

Doran, M. & Robbers, G., Early Dark Energy Cosmologies. 2006, JCAP, 0606 (2006) 026

$$\Omega_{de}(a) = \frac{\Omega_{de}^0 - \Omega_e(1 - a^{-3w_0})}{\Omega_{de}^0 + \Omega_m^0 a^{3w_0}} + \Omega_e(1 - a^{-3w_0})$$



$\Omega_e < 0.0036$ *Planck* TT,TE,EE+lowP+BSH

Perturbation parameterizations

Modified gravity and the gravitational potentials

$$ds^2 = a^2\left[-(1+2\Psi)d\tau^2 + (1-2\Phi)dx^2\right]$$

1. $Q(a,\boldsymbol{k})$, which modifies the relativistic Poisson equation through extra DE clustering according to

$$-k^2\Phi \equiv 4\pi G a^2 Q(a,\boldsymbol{k})\rho\Delta, \tag{3}$$

   where Δ is the comoving density perturbation;
2. $\mu(a,\boldsymbol{k})$ (sometimes also called $Y(a,\boldsymbol{k})$), which modifies the equivalent equation for Ψ rather than Φ:

$$-k^2\Psi \equiv 4\pi G a^2 \mu(a,\boldsymbol{k})\rho\Delta; \tag{4}$$

3. $\Sigma(a,\boldsymbol{k})$, which modifies lensing (with the lensing/Weyl potential being $\Phi+\Psi$), such that

$$-k^2(\Phi+\Psi) \equiv 8\pi G a^2 \Sigma(a,\boldsymbol{k})\rho\Delta; \tag{5}$$

4. $\eta(a,\boldsymbol{k})$, which reflects the presence of a non-zero anisotropic stress, the difference between Φ and Ψ being equivalently written as a deviation of the ratio[2]

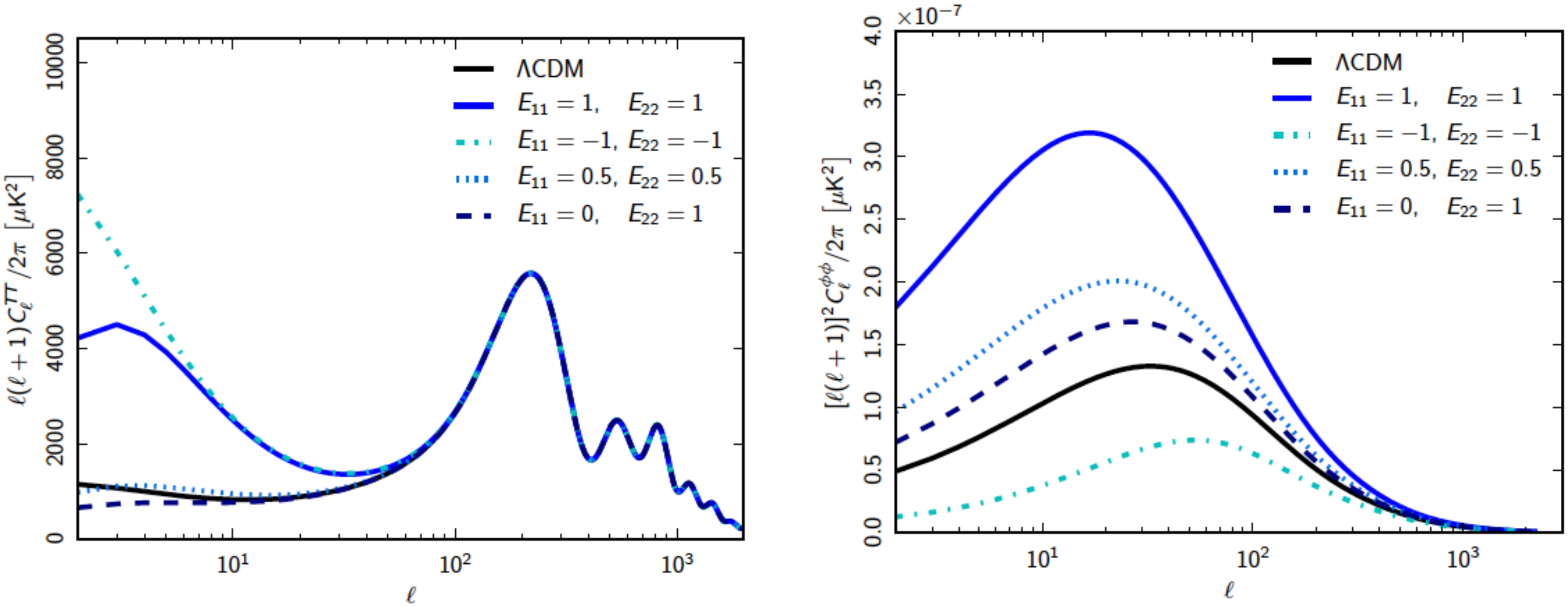
$$\eta(a,\boldsymbol{k}) \equiv \Phi/\Psi. \tag{6}$$

$$\rho\Delta = \rho_{\rm m}\Delta_{\rm m} + \rho_{\rm r}\Delta_{\rm r}$$

$$\mu(a,k) = 1 + f_1(a)\frac{1 + c_1(\lambda H/k)^2}{1 + (\lambda H/k)^2};$$

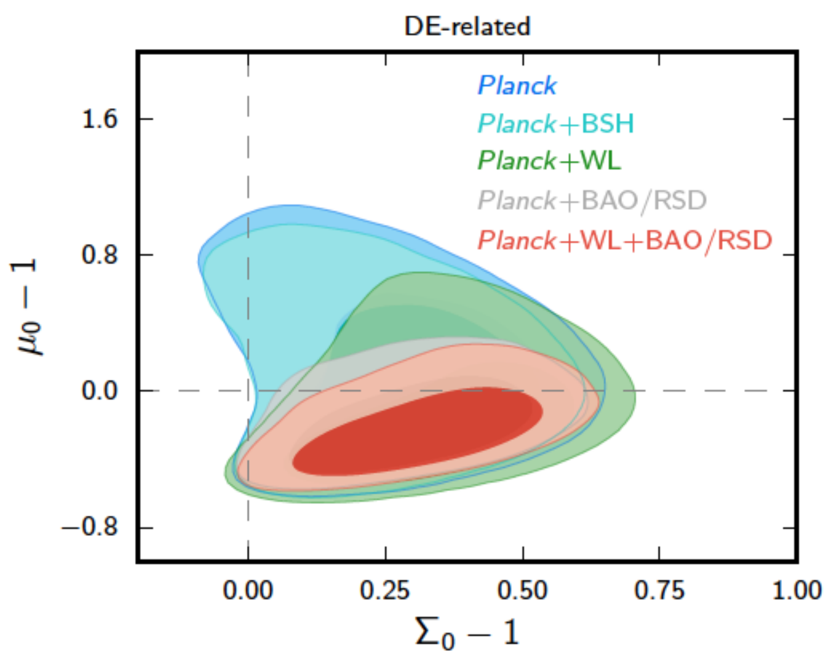
$$\eta(a,k) = 1 + f_2(a)\frac{1 + c_2(\lambda H/k)^2}{1 + (\lambda H/k)^2}.$$

1. coefficients related to the DE density, $f_i(a) = E_{ii}\Omega_{DE}(a)$,
2. time-related evolution, $f_i(a) = E_{i1} + E_{i2}(1 - a)$.



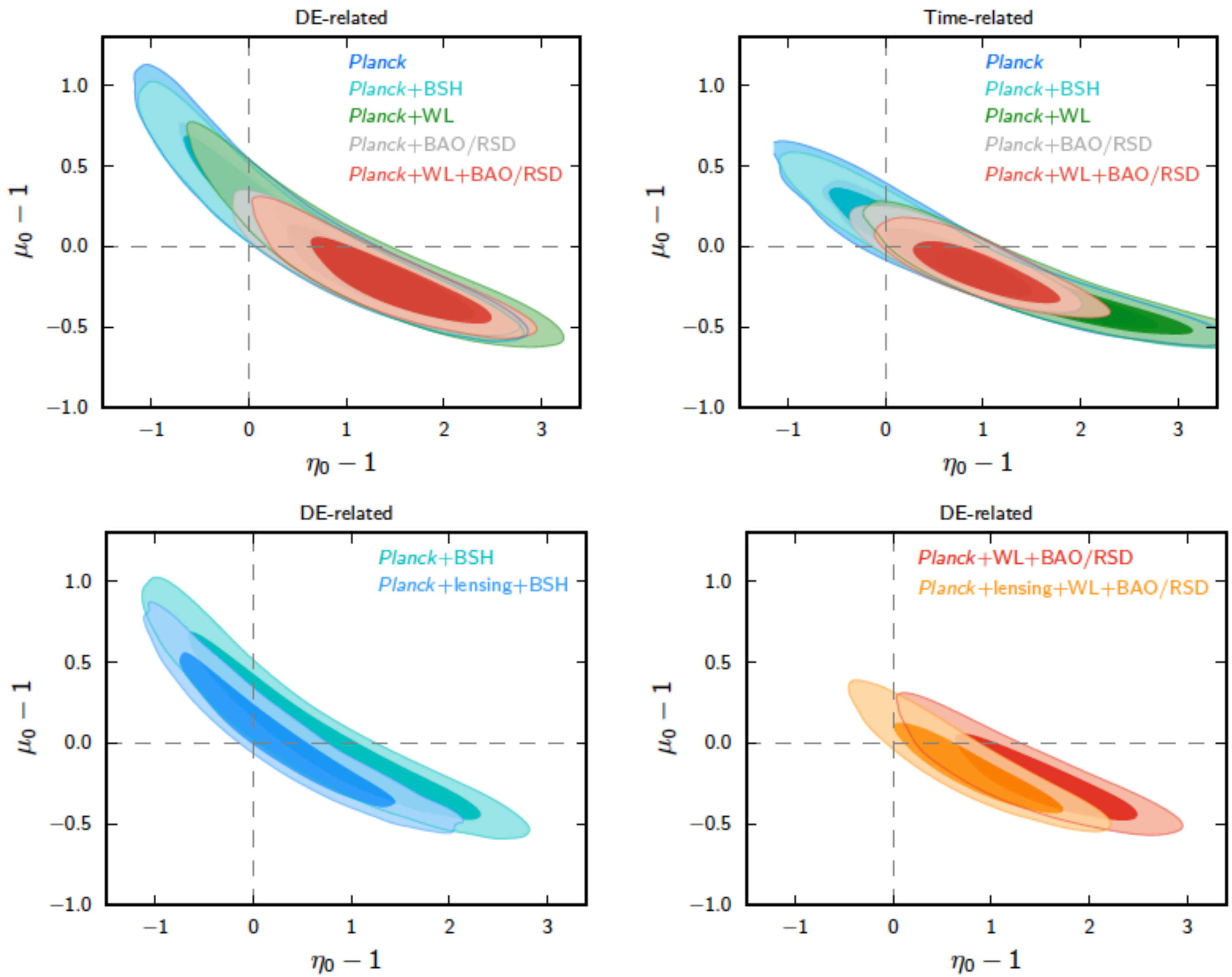
even when the temperature spectrum is very close to LCDM (as for $E_{11} = E_{22} = 0.5$) the lensing potential is still different with respect to LCDM

Parameter	*Planck* TT+lowP	*Planck* TT+lowP +BSH	*Planck* TT+lowP +WL	*Planck* TT+lowP +BAO/RSD	*Planck* TT+lowP +WL+BAO/RSD	*Planck* TT,TE,EE+lowP +BSH
E_{11}	$0.099^{+0.34}_{-0.73}$	$0.06^{+0.32}_{-0.69}$	$-0.20^{+0.19}_{-0.47}$	$-0.24^{+0.19}_{-0.33}$	$-0.30^{+0.18}_{-0.30}$	$0.08^{+0.33}_{-0.69}$
E_{22}	0.99 ± 1.3	1.03 ± 1.3	$1.92^{+1.4}_{-0.96}$	1.77 ± 0.88	2.07 ± 0.85	0.9 ± 1.2
$\mu_0 - 1$	$0.07^{+0.24}_{-0.51}$	$0.04^{+0.22}_{-0.48}$	$-0.14^{+0.13}_{-0.34}$	$-0.17^{+0.14}_{-0.23}$	$-0.21^{+0.12}_{-0.21}$	$0.06^{+0.23}_{-0.48}$
$\eta_0 - 1$	0.70 ± 0.94	0.72 ± 0.90	$1.36^{+1.0}_{-0.69}$	1.23 ± 0.62	1.45 ± 0.60	0.60 ± 0.86
$\Sigma_0 - 1$	0.28 ± 0.15	0.27 ± 0.14	$0.34^{+0.17}_{-0.14}$	0.29 ± 0.13	0.31 ± 0.13	0.23 ± 0.13
τ	0.065 ± 0.021	0.063 ± 0.020	$0.061^{+0.020}_{-0.022}$	0.062 ± 0.019	0.057 ± 0.019	0.060 ± 0.019
H_0 (km/s/Mpc) .	68.5 ± 1.1	68.17 ± 0.58	69.2 ± 1.1	68.26 ± 0.69	68.55 ± 0.66	67.90 ± 0.48



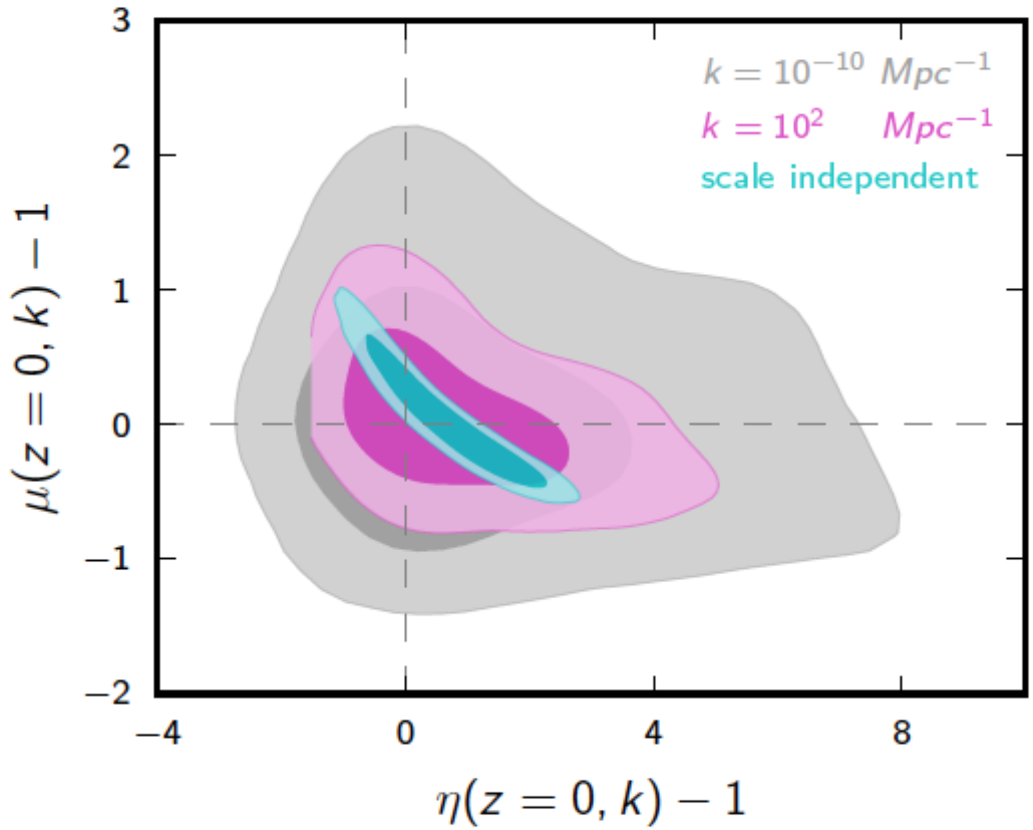
$$\Sigma = (\mu/2)(1 + \eta)$$

- Scale-independent case: c_1=c_2=1
- Adding BSH does not significantly increase the constraining power
- Adding the RSD data tightens the constraints significantly
- Tensions with LCDM



- Tension with LCDM
- Tension is reduced when CMB lensing is added

Scale dependence



Planck TT+lowP+BSH

Allowing for scale dependence, tension with LCDM is washed out

Universal couplings: $f(R)$ cosmologies

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}(R + f(R)) + \int d^4x \mathcal{L}_{\rm M}(\chi_i, g_{\mu\nu})$$

Planck 2015: **EFTcamb**

MGCAMB

Zhao, G.-B., Pogosian, L., Silvestri, A., & Zylberberg, J., Searching for modified growth patterns with tomographic surveys. 2009, Phys.Rev., D79, 083513, arXiv:0809.3791

Hojjati, A., Pogosian, L., & Zhao, G.-B., Testing gravity with CAMB and CosmoMC. 2011, JCAP, 1108, 005, arXiv:1106.4543

$$k^2\Psi = -\mu(k,a)4\pi G a^2\{\rho\Delta + 3(\rho + P)\sigma\},$$

$$k^2[\Phi - \gamma(k,a)\Psi] = \mu(k,a)12\pi G a^2(\rho + P)\sigma$$

$$\mu(k,a) = \frac{1}{1 - 1.4\cdot 10^{-8}|\lambda_1|^2 a^3}\frac{1 + \frac{4}{3}\lambda_1^2 k^2 a^4}{1 + \lambda_1^2 k^2 a^4},$$

$$\gamma(k,a) = \frac{1 + \frac{2}{3}\lambda_1^2 k^2 a^4}{1 + \frac{4}{3}\lambda_1^2 k^2 a^4},$$

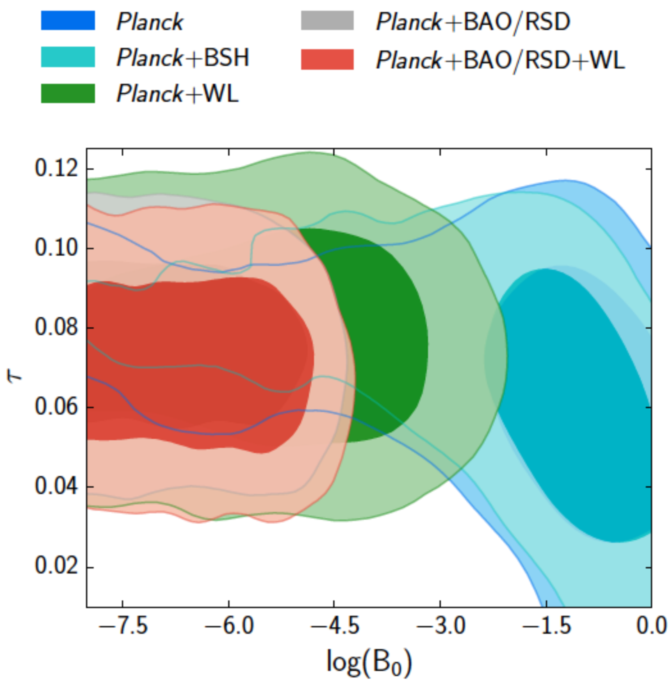
E. Bertschinger, P. Zukin, Phys. Rev. D 78 (2008) 024015, arXiv:0801.2431 [astro-ph].

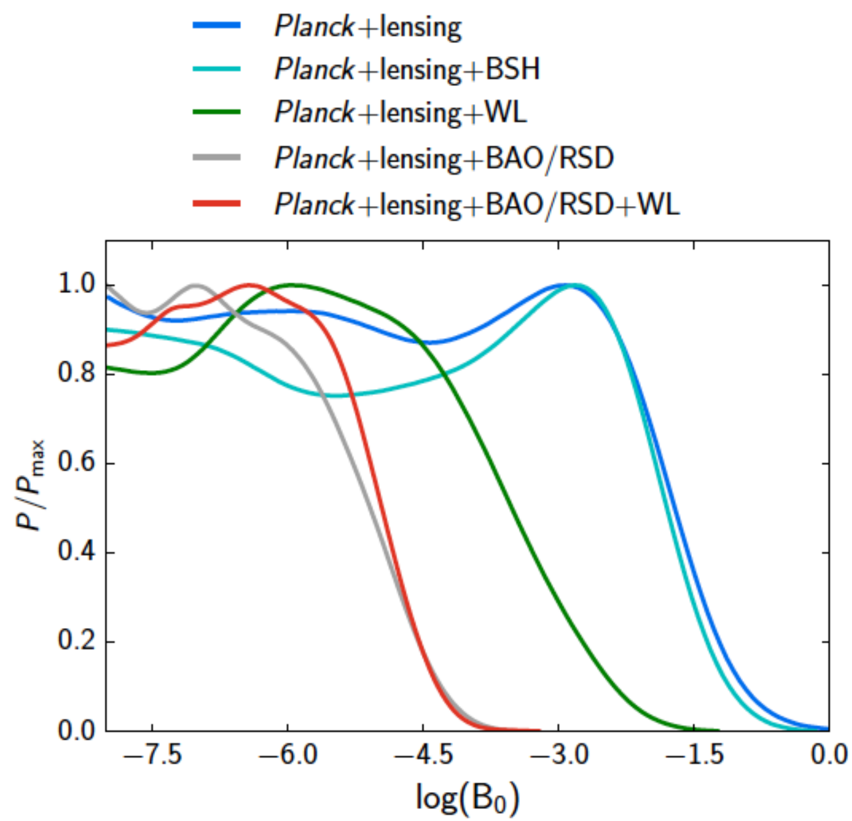
T. Giannantonio, M. Martinelli, A. Silvestri, A. Melchiorri, J. Cosmol. Astropart. Phys. 1004 (2010) 030, arXiv:0909.2045 [astro-ph.CO].

$$\lambda_1^2 = B_0 c^2/(2H_0^2) \qquad B \equiv \frac{f_{RR}}{1 + f_R}\frac{dR}{d\ln a}\left(\frac{d\ln H}{d\ln a}\right)^{-1}$$

Y.S. Song, W. Hu, I. Sawicki, Phys. Rev. D 75 (2007) 044004, arXiv:astro-ph/0610532.

$f(R)$ models	*Planck* TT+lowP	*Planck* TT+lowP +BSH	*Planck* TT+lowP +WL	*Planck* TT+lowP +BAO/RSD	*Planck* TT+lowP +WL+BAO/RSD
B_0	< 0.79 (95 % CL)	< 0.69 (95 % CL)	< 0.10 (95 % CL)	$< 0.90 \times 10^{-4}$ (95 % CL)	$< 0.86 \times 10^{-4}$ (95 % CL)
B_0 (+lensing)	< 0.12 (95 % CL)	< 0.07 (95 % CL)	< 0.04 (95 % CL)	$< 0.97 \times 10^{-4}$ (95 % CL)	$< 0.79 \times 10^{-4}$(95 % CL)





- A degeneracy between optical depth τ and f(R) parameter B_0 for Planck TT+lowP+BSH
- Adding any structure formation probe, such as WL, RSD, or CMB lensing, breaks the degeneracy

Physics Letters B 744 (2015) 213–217

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Probing $f(R)$ cosmology with sterile neutrinos via measurements of scale-dependent growth rate of structure

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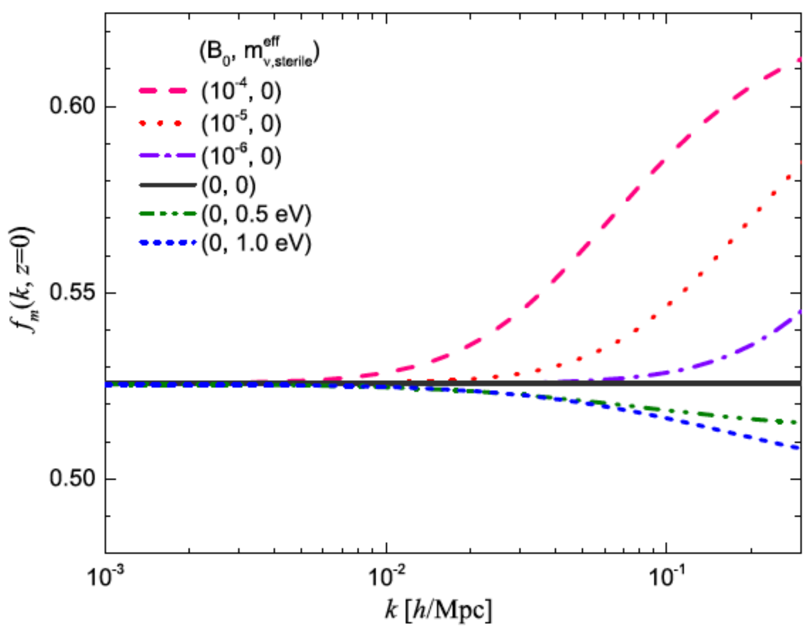


- f(R) cosmology: scale dependent growth rate – enhance small scale matter perturbations
- For $B_0=10^{-4}$case, deviation is 14% at 0.3 h Mpc^{-1}, larger than RSD precision (7%) – induce substantial systematics for the fit
- N-body simulation: for f(R), RSD only for k<0.6 h Mpc^{-1} at z=0
- Sterile neutrino: suppress perturbations – a degeneracy between B_0 and neutrino mass

PV:

Ref. [25] tried to measure the scale-dependent growth rate by using the observations of peculiar velocities of galaxies from 6dF Galaxy Survey velocity sample in combination with a newly-compiled sample of low-redshift type Ia supernovae. The measurement obtained the scale-dependent growth rates in five k bins at

A. Johnson, C. Blake, J. Koda, Y.Z. Ma, M. Colless, M. Crocce, T.M. Davis, H. Jones, et al., Mon. Not. R. Astron. Soc. 444 (2014) 3926, arXiv:1404.3799 [astro-ph.CO].

$z = 0$: $f_m\sigma_8(k) = 0.79 \pm 0.21$, 0.30 ± 0.14, 0.32 ± 0.19, 0.64 ± 0.17, and 0.48 ± 0.22, for $k \in [0.005, 0.02]$, $[0.02, 0.05]$, $[0.05, 0.08]$, $[0.08, 0.12]$, and $[0.12, 0.15]$ $h\,\mathrm{Mpc}^{-1}$

RSD:

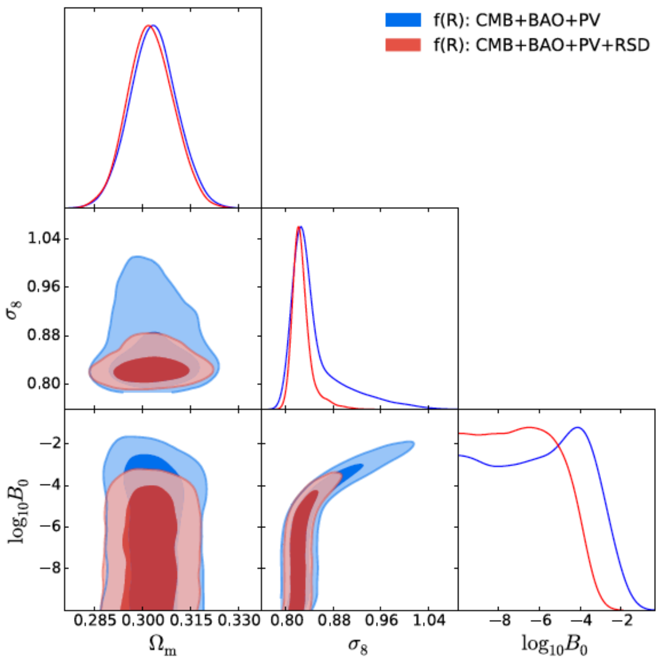
growth rate measurements $f_m\sigma_8(z) = 0.35 \pm 0.06$ at $z = 0.25$ and $f_m\sigma_8(z) = 0.46 \pm 0.04$ at $z = 0.37$ from SDSS DR7 [19]. These two data points actually belong to the RSD measurement but are obtained from the power spectrum with $k \in [0.005, 0.033]$ $h\,\mathrm{Mpc}^{-1}$.

CMB+BAO:

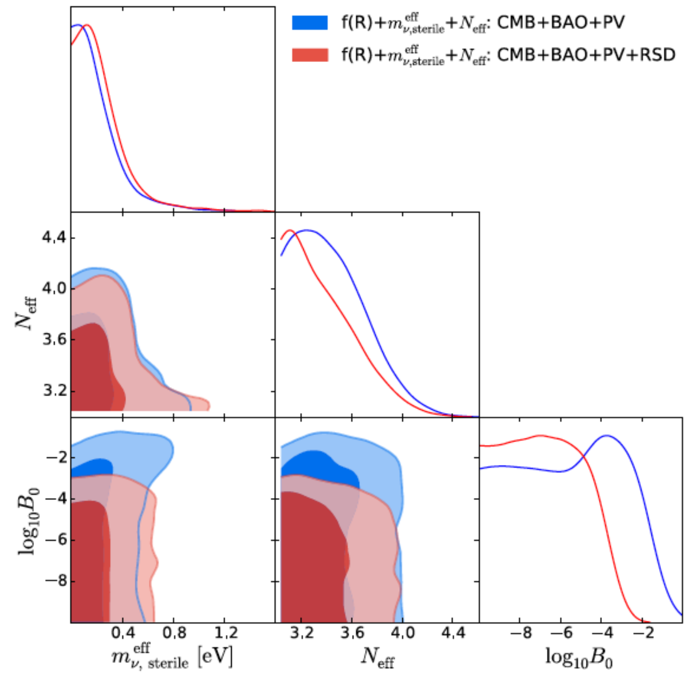
Planck TT + WP + Planck lensing

To constrain other cosmological parameters and break degeneracies between them, we also employ the observational data from cosmic microwave background (CMB) and baryon acoustic oscillation (BAO) observations. For the CMB data, we use the temperature power spectrum data C_ℓ^{TT} [54] and lensing data $C_\ell^{\phi\phi}$ [55] from Planck[3] in combination with the WMAP 9-yr polarization (TE and EE) power spectrum data [56]. For the BAO data, we use the measurements from 6dFGS ($z = 0.1$) [57], SDSS DR7 ($z = 0.35$) [58], WiggleZ ($z = 0.44, 0.60,$ and 0.73) [59], and BOSS DR11 ($z = 0.32$ and 0.57) [60]. Note that the late-time integrated Sachs–Wolfe (ISW) effect and the lensing signal from the CMB measurements can also give constraint on B_0, since B_0 affects the evolutions of the metric perturbation potentials Φ and Ψ from Eqs. (2)–(4), and the ISW effect and CMB lensing potential are directly related to $\dot\Phi + \dot\Psi$ and $\Phi + \Psi$, respectively.

Parameter	$f(R)$		$\Lambda CDM + m^{eff}_{\nu,sterile} + N_{eff}$		$f(R) + m^{eff}_{\nu,sterile} + N_{eff}$	
	CMB+BAO+PV	+RSD	CMB+BAO+PV	+RSD	CMB+BAO+PV	+RSD
$m^{eff}_{\nu,sterile}$	...	...	$< 0.43(0.18)$	$< 0.56(0.23)$	$< 0.61(0.22)$	$< 0.62(0.24)$
N_{eff}	...	...	$< 3.96(3.47)$	$< 3.92(3.43)$	$< 3.95(3.45)$	$< 3.90(3.40)$
$\log_{10} B_0$	$< -2.7(-6.1)$	$< -4.1(-7.0)$	...	...	$< -1.8(-5.6)$	$< -3.8(-6.8)$
Ω_m	0.3035 ± 0.0071	$0.3027^{+0.0069}_{-0.0075}$	0.3047 ± 0.0080	$0.3047^{+0.0078}_{-0.0079}$	$0.3040^{+0.0080}_{-0.0081}$	$0.3047^{+0.0078}_{-0.0084}$
σ_8	$0.850^{+0.006}_{-0.049}$	$0.827^{+0.008}_{-0.018}$	$0.809^{+0.029}_{-0.022}$	$0.799^{+0.031}_{-0.023}$	$0.851^{+0.032}_{-0.079}$	$0.807^{+0.031}_{-0.030}$
H_0	68.1 ± 0.6	68.2 ± 0.6	$69.8^{+1.1}_{-1.8}$	$69.6^{+1.0}_{-1.7}$	$69.8^{+1.1}_{-1.8}$	$69.5^{+1.0}_{-1.7}$
$-\ln \mathcal{L}_{max}$	4912.405	4913.993	4912.085	4913.852	4912.361	4914.293



$\log_{10} B_0 < -4.1$



$\log_{10} B_0 < -3.8$

For the massive sterile neutrino parameters, we get $m^{eff}_{\nu,sterile} < 0.62$ eV and $N_{eff} < 3.90$ (2σ). As a comparison, we also obtain $m^{eff}_{\nu,sterile} < 0.56$ eV and $N_{eff} < 3.92$ (2σ) in the standard ΛCDM model.

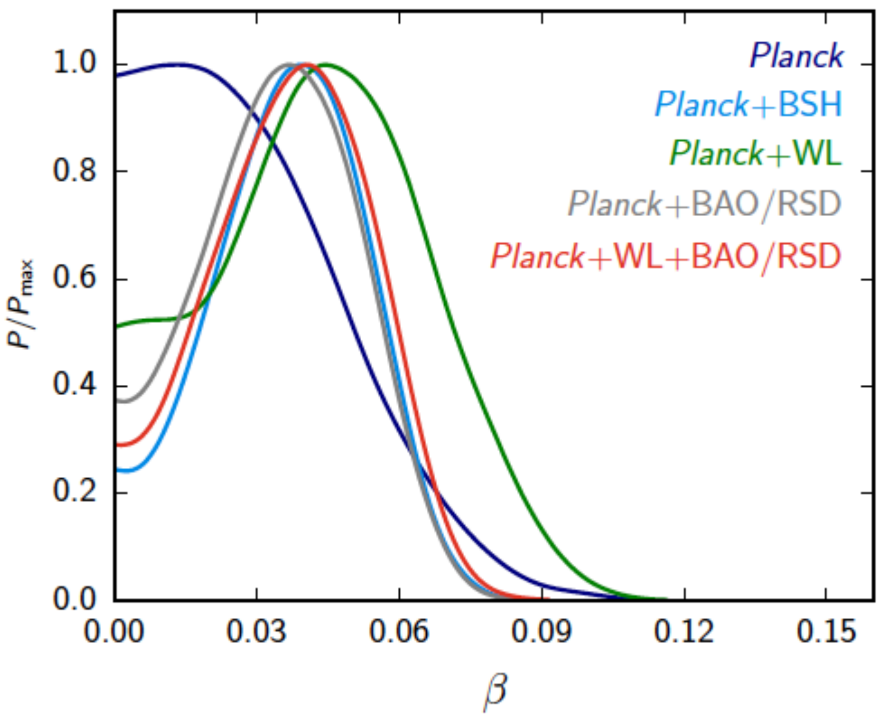
Non-universal couplings: coupled Dark Energy

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi) - m(\phi)\bar{\psi}\psi + \mathcal{L}_{\rm kin}[\psi]$$

$$m(\phi) = m_0 \exp^{-\beta\phi}$$

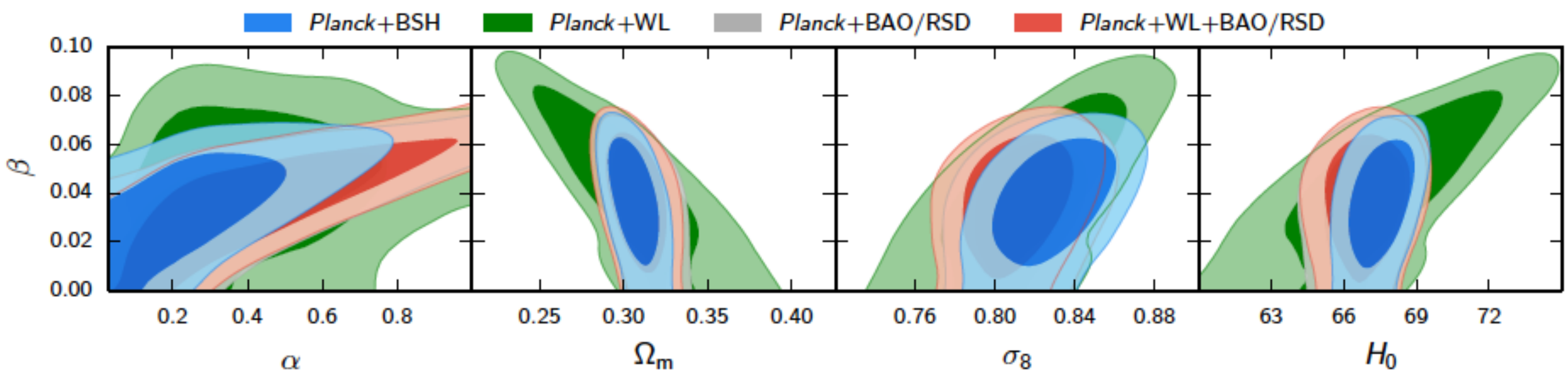
$$\rho'_\phi = -3\mathcal{H}\rho_\phi(1+w_\phi) + \beta\rho_c\phi' ,$$
$$\rho'_c = -3\mathcal{H}\rho_c - \beta\rho_c\phi' .$$

$$\phi'' + 2\mathcal{H}\phi' + a^2\frac{dV}{d\phi} = a^2\beta\rho_c$$
$$V = V_0\phi^{-\alpha}$$



- $\beta > 0$ means transfer of energy from DM to DE
- Data prefer $\beta > 0$
- Planck + BSH is consistent with Planck + WL + RSD
- Planck + BSH: 2.5 σ
- Planck + WL + RSD: 2.3 σ

CDE models	*Planck* TT+lowP	*Planck* TT+lowP +BSH	*Planck* TT+lowP +WL	*Planck* TT+lowP +BAO/RSD	*Planck* TT+lowP +WL+BAO/RSD
β	< 0.066 (95 %)	$0.037^{+0.018}_{-0.015}$	$0.043^{+0.026}_{-0.022}$	$0.034^{+0.019}_{-0.016}$	$0.037^{+0.020}_{-0.016}$
α	$0.43^{+0.15}_{-0.33}$	$0.29^{+0.077}_{-0.26}$	$0.44^{+0.18}_{-0.29}$	$0.40^{+0.15}_{-0.29}$	$0.45^{+0.17}_{-0.33}$
H_0 (km/s/Mpc)	$65.4^{+3.2}_{-2.6}$	$67.47^{+0.88}_{-0.79}$	67.6 ± 2.8	66.7 ± 1.1	66.9 ± 1.1
. . . .	TT,TE,EE+lowP	TT,TE,EE+lowP +BSH	TT,TE,EE+lowP +WL	TT,TE,EE+lowP +BAO/RSD	TT,TE,EE+lowP +WL+BAO/RSD
β	< 0.062 (95 %)	$0.036^{+0.016}_{-0.013}$	$0.036^{+0.019}_{-0.026}$	$0.034^{+0.018}_{-0.015}$	$0.038^{+0.018}_{-0.014}$
α	$0.42^{+0.14}_{-0.32}$	< 0.58 (95 %)	$0.42^{+0.13}_{-0.33}$	$0.37^{+0.13}_{-0.28}$	$0.42^{+0.16}_{-0.31}$



- Degeneracy between β and α
- 2D plots: compatible with ΛCDM
- Due to degeneracy, marginalization over α takes more contributions from higher β
- $\beta > 0$ means transfer of energy from DM to DE, so a larger β prefers larger H_0 and higher σ_8
- Addition of polarization tightens bounds: $\beta > 0$ at 2.8 σ for Planck+BSH, and at 2.7 σ for Planck+WL+RSD

Interacting Dark Energy

- Motivation: deviation from ΛCDM – IΛCDM
- Problem: early-time large-scale instability for IwCDM
- Solution: new framework – PPF for IDE
- Observational constraints
- IDE after Planck

PHYSICAL REVIEW D **90,** 063005 (2014)

Parametrized post-Friedmann framework for interacting dark energy

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(Received 25 April 2014; published 16 September 2014)

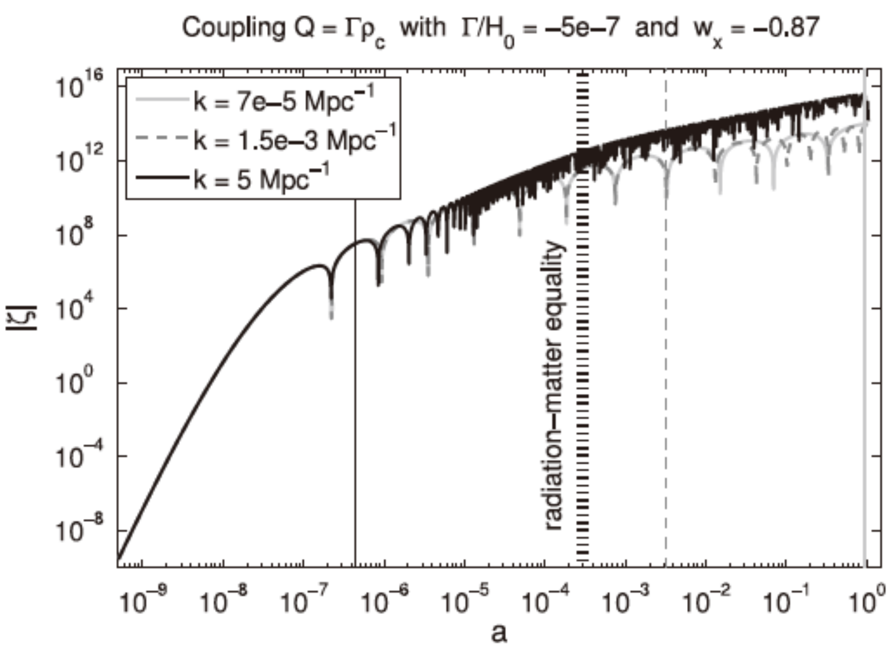
$$\rho'_{\rm de} = -3\mathcal{H}(1+w)\rho_{\rm de} + aQ_{\rm de},$$

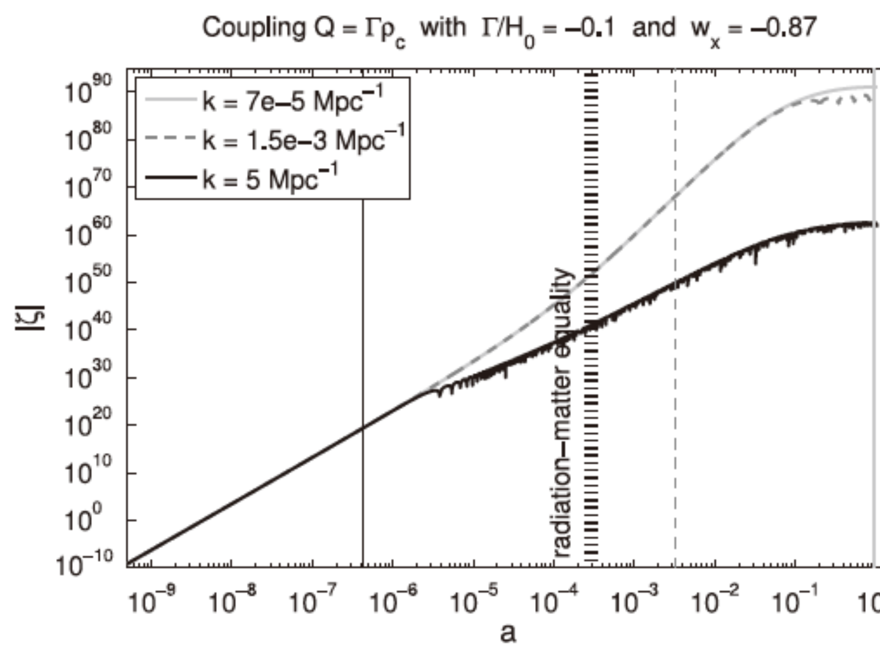
$$\nabla_\nu T^{\mu\nu}_I = Q^\mu_I, \qquad \sum_I Q^\mu_I = 0$$

$$\rho'_c = -3\mathcal{H}\rho_c + aQ_c, \qquad Q_{\rm de} = -Q_c = Q$$

$$Q^I_\mu = a(-Q_I(1+AY) - \delta Q_I Y, [f_I + Q_I(v-B)]Y_i)$$

相互作用形式	稳定性
$Q \propto \rho_{de}$	$w > -1, \Gamma > 0$ 稳定，其它情形发散
$Q \propto \rho_c$	$w < -1$ 稳定，其它情形发散





J. Valiviita, E. Majerotto, and R. Maartens, J. Cosmol. Astropart. Phys. 07 (2008) 020.

($k = 7\times10^{-5}$ Mpc^{-1}) stays super-Hubble all the way up to today. The intermediate scale ($k = 1.5\times10^{-3}$ Mpc^{-1}) enters the horizon during matter domination, and the smallest scale ($k = 5$ Mpc^{-1}) enters deep in the radiation era.

- $\delta\rho_{\rm de}$ — 暗能量密度扰动 （满足能量守恒定律）
- $v_{\rm de}$ — 暗能量速度 （满足动量守恒定律）
- $\pi_{\rm de}$ — 暗能量各向异性应力 （一般可忽略）
- $\delta p_{\rm de}$ — 暗能量压强扰动

$$\delta\rho_I' + 3(\delta\rho_I + \delta p_I) + (\rho_I + p_I)(k_H v_I + 3H_L') = \frac{1}{H}(\delta Q_I - AQ_I),$$

$$\frac{[a^4(\rho_I + p_I)(v_I - B)]'}{a^4 k_H} - \delta p_I + \frac{2}{3}c_K p_I \Pi_I - (\rho_I + p_I)A$$
$$= \frac{a}{k}[Q_I(v - B) + f_I].$$

$$\delta p_{\rm de} = c_{s,{\rm de}}^2 \delta\rho_{\rm de} + (c_{s,{\rm de}}^2 - c_{a,{\rm de}}^2)[3(1+w)\rho_{\rm de} - \frac{Q}{H}]\frac{v_{\rm de} - B}{k_H}$$

- Perturbation blow-up reveals our ignorance about nature of DE
- DE blows up at w=-1 crossing: (nonadiabatic) fluid treatment is NOT OK!
- IDE blows up: (nonadiabatic) fluid treatment is NOT OK!
- We need new approach to treat DE perturbations

PPF for IDE

$$\rho_{\rm de}\Delta_{\rm de} = -3(\rho_{\rm de}+p_{\rm de})\frac{V_{\rm de}-V_T}{k_H} - \frac{k^2 c_K}{4\pi G a^2}\Gamma,$$

$$\frac{V_{\rm de}-V_T}{k_H} = \frac{-H^2}{4\pi G(\rho_{\rm de}+p_{\rm de})F} \times \left[S - \Gamma' - \Gamma + f_\zeta \frac{4\pi G(\rho_T+p_T)}{H^2}\frac{V_T}{k_H}\right]$$

$$(1+c_\Gamma^2 k_H^2)[\Gamma' + \Gamma + c_\Gamma^2 k_H^2 \Gamma] = S$$

$$F = 1 + 3\frac{4\pi G a^2}{k^2 c_K}(\rho_T + p_T)$$

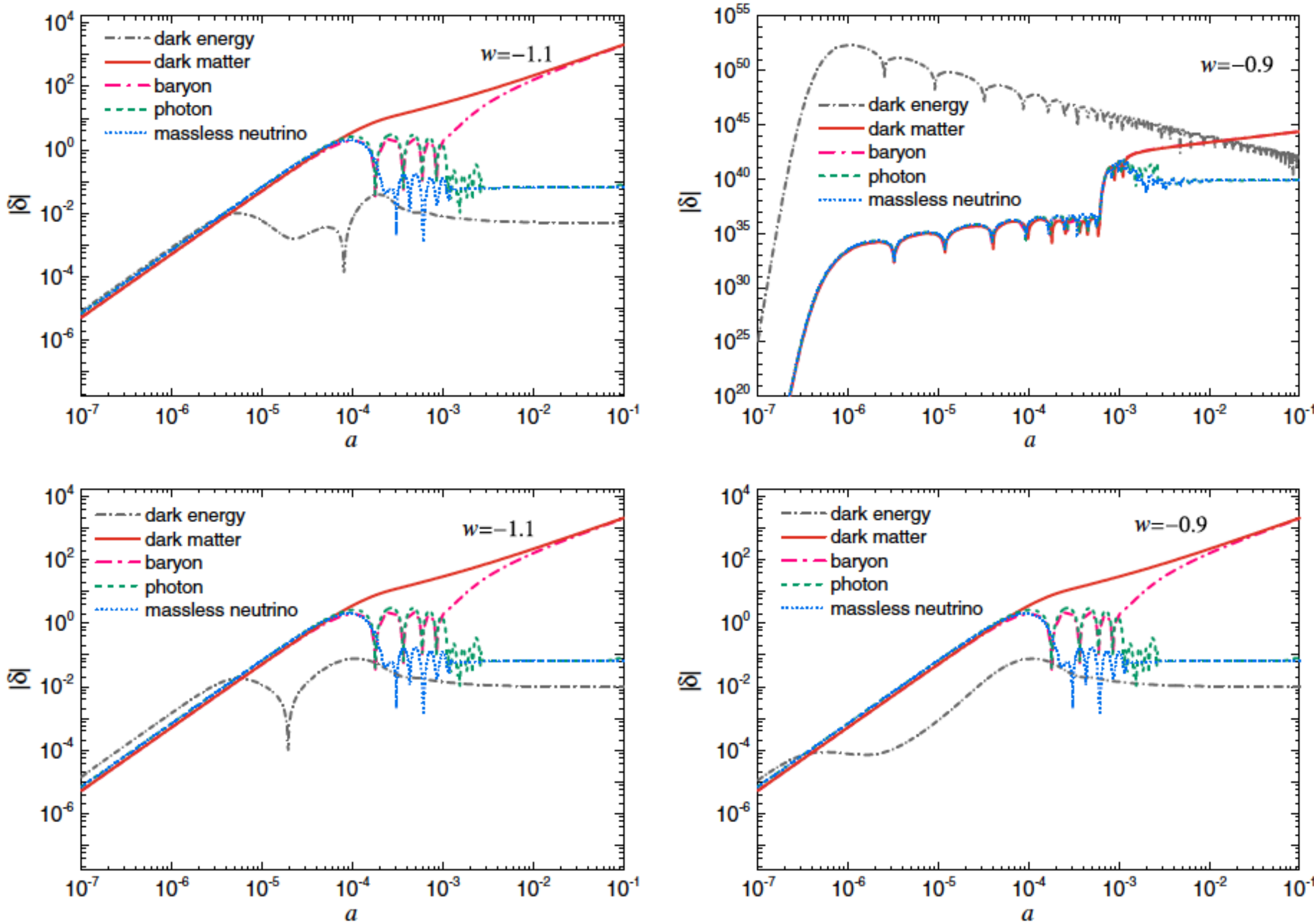
$$S = \frac{4\pi G}{k_H^2 H^2}\left\{[(\rho_{\rm de}+p_{\rm de}) - f_\zeta(\rho_T+p_T)]k_H V_T + \frac{3a}{k c_K}[Q_c(V-V_T)+f_c] + \frac{1}{H c_K}(\Delta Q_c - \xi Q_c)\right\}$$

$$\lim_{k_H \ll 1}\frac{4\pi G}{H^2}(\rho_{\rm de}+p_{\rm de})\frac{V_{\rm de}-V_T}{k_H} = -\frac{1}{3}c_K f_\zeta(a) k_H V_T$$

$$\lim_{k_H \gg 1}\Phi = \frac{4\pi G}{c_K k_H^2 H^2}\Delta_T \rho_T$$

$$\Phi + \Gamma = \frac{4\pi G}{c_K k_H^2 H^2}\Delta_T \rho_T$$

$$\Phi = \zeta + V_T/k_H$$

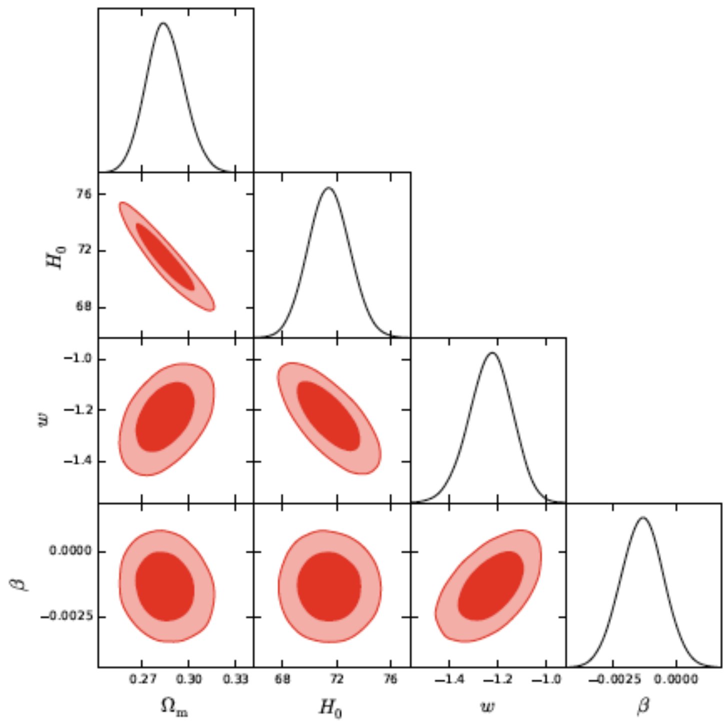


$$Q^{\mu} = 3\beta H \rho_c u_c^{\mu}$$

- β > 0: DM → DE
- β < 0: DE → DM
- Example: k = 0.1 Mpc^{-1} and β = -10^{-17}

TABLE I. Fit results for the IDE model with $Q^\mu = 3\beta H \rho_c u_c^\mu$.

Parameter	Best fit	68% limits
$\Omega_b h^2$	0.02227	0.02218 ± 0.00028
$\Omega_c h^2$	0.12199	0.1224 ± 0.0022
H_0	71.16	71.5 ± 1.5
τ	0.0955	$0.090^{+0.012}_{-0.014}$
w	−1.2050	$-1.228^{+0.093}_{-0.084}$
β	−0.00137	-0.0013 ± 0.0008
n_s	0.9630	$0.9609^{+0.0066}_{-0.0065}$
$\ln(10^{10} A_s)$	3.096	$3.086^{+0.024}_{-0.027}$
Ω_Λ	0.7139	$0.7152^{+0.0128}_{-0.0116}$
Ω_m	0.2861	$0.2848^{+0.0116}_{-0.0128}$
Age/Gyr	13.826	13.831 ± 0.064



$Q^\mu = 3\beta H \rho_c u_c^\mu$: Planck+BAO+SN+$H_0$

注意：新方法适用当前所有相互作用暗能量模型！

- Whole parameter space can be explored
- Prefer $\beta < 0$: DE → DM
- $\beta = O(10^{-3})$, precision 60%
- CMB itself can constrain β well
- RSD do not improve significantly

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Exploring the full parameter space for an interacting dark energy model with recent observations including redshift-space distortions: Application of the parametrized post-Friedmann approach

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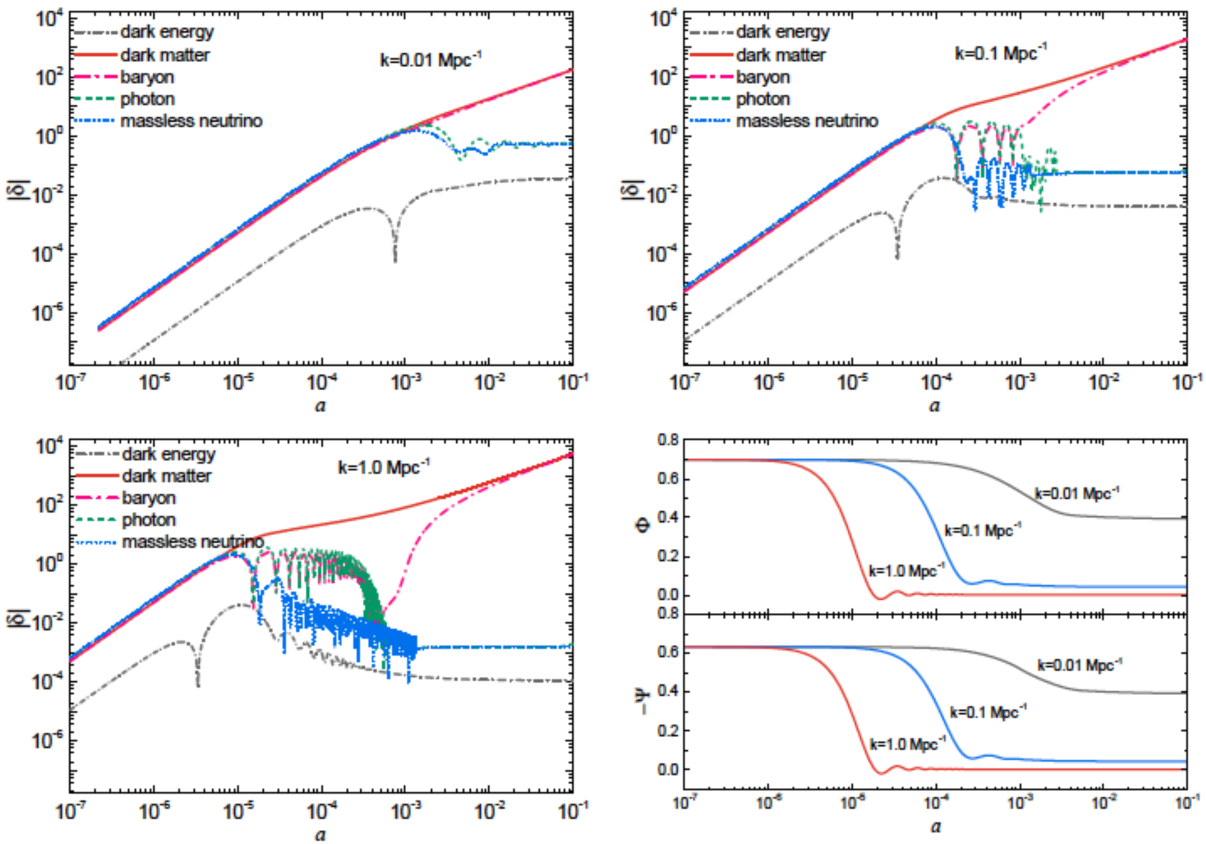
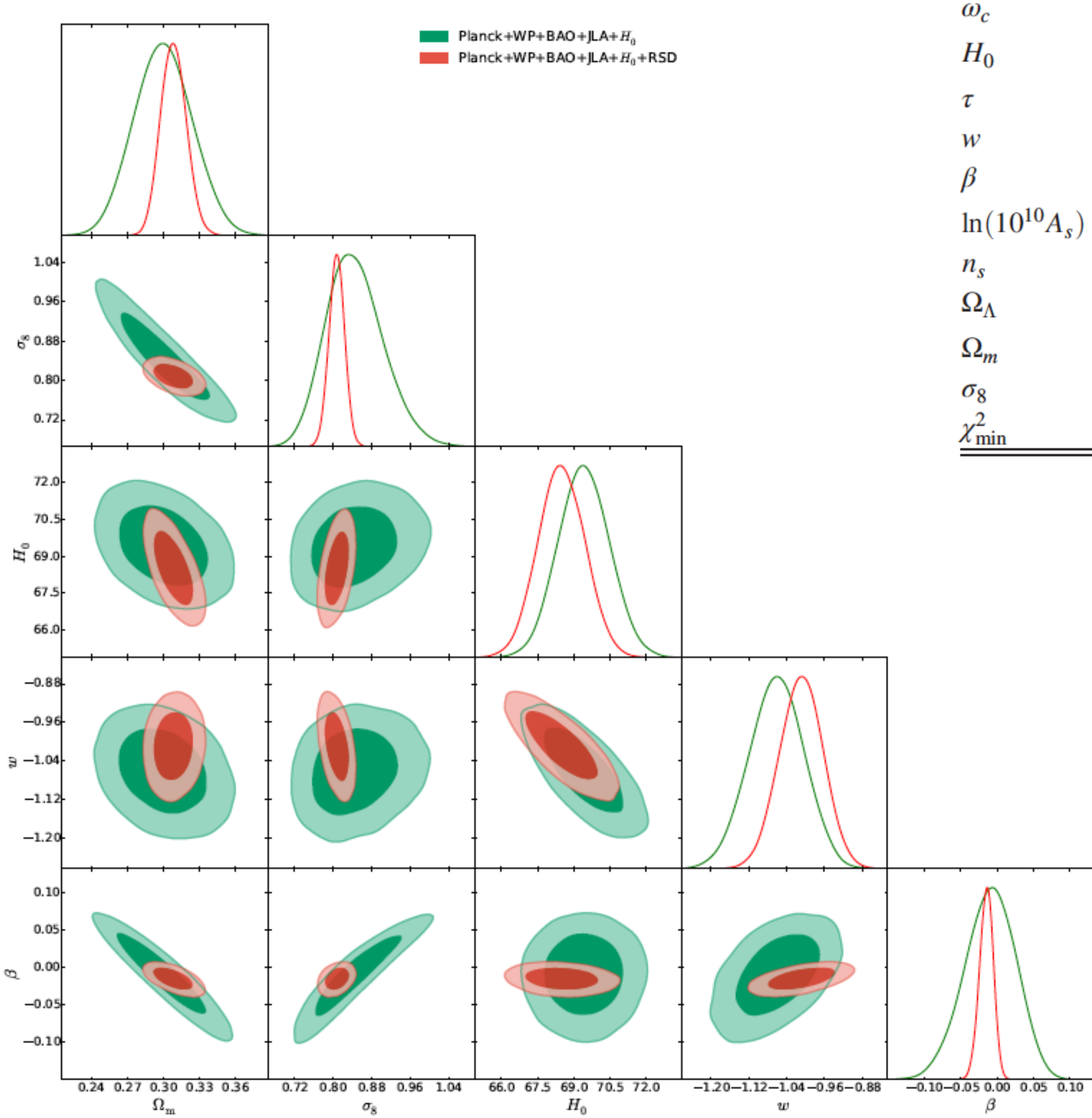
(Received 27 September 2014; published 16 December 2014)



$Q^{\mu} = 3\beta H \rho_{\rm de} u_c^{\mu}$ $\qquad w = -1.05, \beta = -0.01$



Parameters	Planck + WP+ BAO + JLA + H_0	+RSD
ω_b	$0.02209^{+0.00025}_{-0.00026}$	$0.02220^{+0.00025}_{-0.00024}$
ω_c	0.123 ± 0.011	$0.1226^{+0.0039}_{-0.0038}$
H_0	69.4 ± 1.0	$68.5^{+1.0}_{-0.9}$
τ	$0.089^{+0.012}_{-0.014}$	$0.087^{+0.012}_{-0.013}$
w	-1.061 ± 0.056	-1.009 ± 0.045
β	$-0.010^{+0.037}_{-0.033}$	$-0.0148^{+0.0100}_{-0.0089}$
$\ln(10^{10}A_s)$	$3.088^{+0.024}_{-0.026}$	$3.079^{+0.023}_{-0.026}$
n_s	0.9601 ± 0.0057	0.9638 ± 0.0057
Ω_Λ	0.700 ± 0.024	$0.691^{+0.011}_{-0.010}$
Ω_m	0.300 ± 0.024	$0.309^{+0.010}_{-0.011}$
σ_8	$0.846^{+0.051}_{-0.065}$	0.808 ± 0.016
χ^2_{min}	10508.090	10519.498

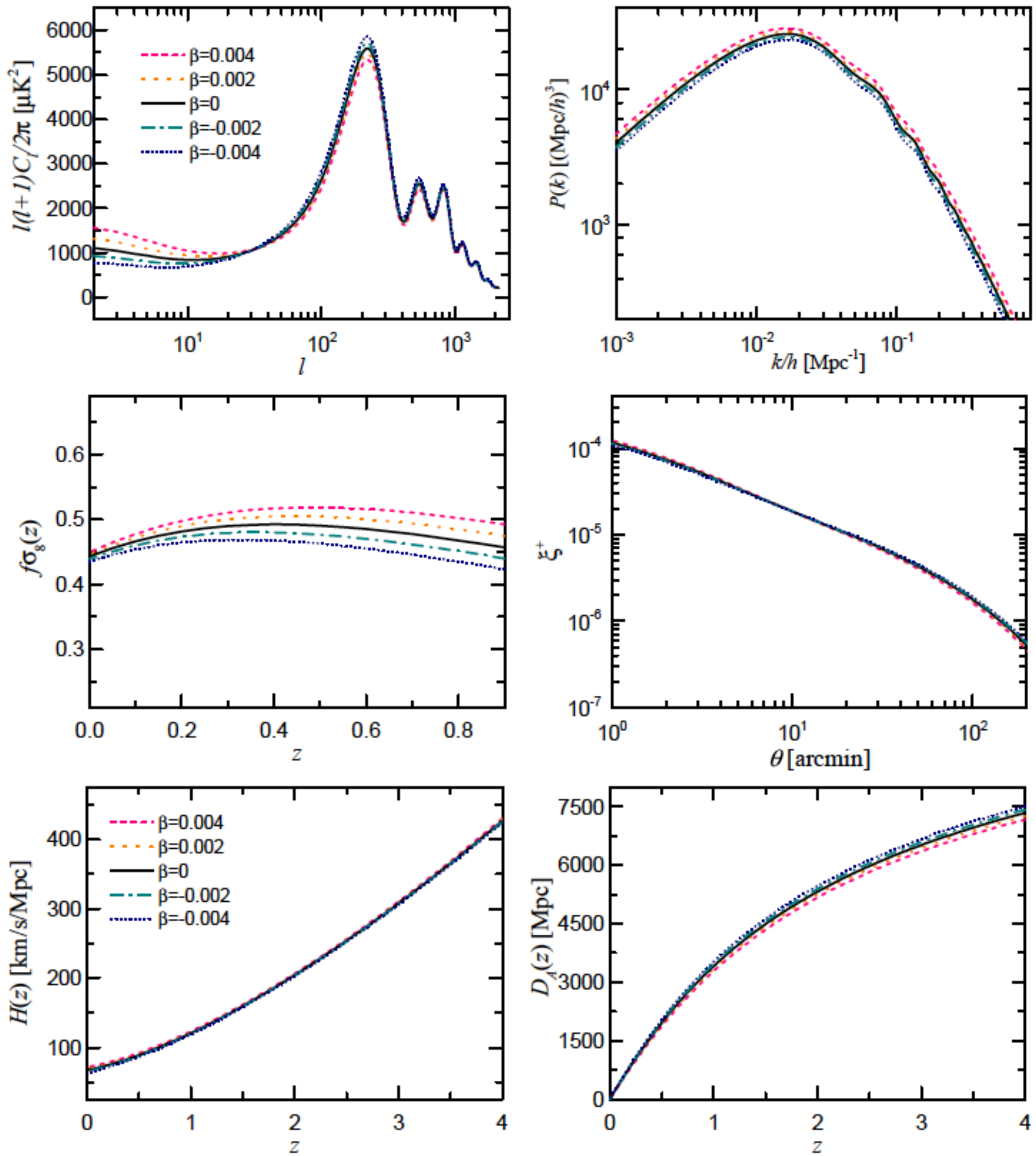
- Previous works assume w>-1 and β>0, so the fit result is positive β～O(10^{-3}), which is wrong!
- RSD break degeneracy and improve fit significantly
- RSD favor a negative β～O(10^{-2})
- w is around -1: **IΛCDM** model

Test IΛCDM

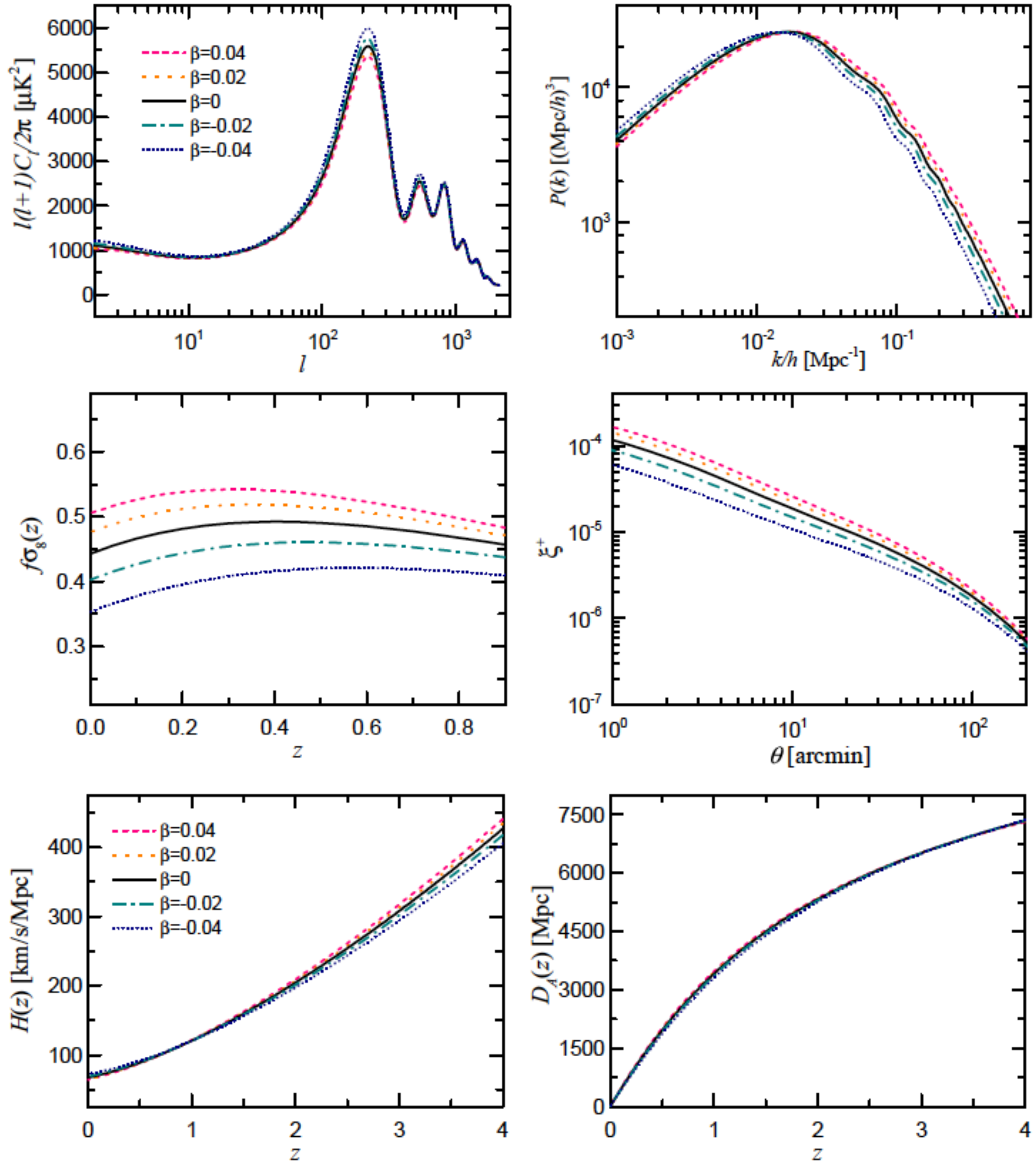
- **One-parameter extension of ΛCDM: wCDM & IΛCDM**
- **PPF is applicable to all IDE models**
- **DE perturbations in IΛCDM (missed in the literature)**
- **Combination of background and perturbation data**
- **Current status**

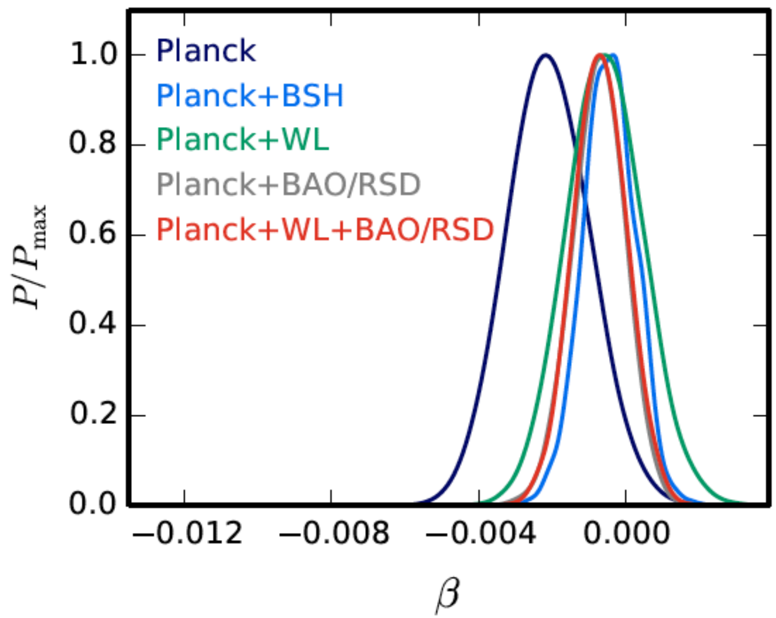
Y. H. Li & X. Zhang, in preparation

$Q \propto \rho_c$

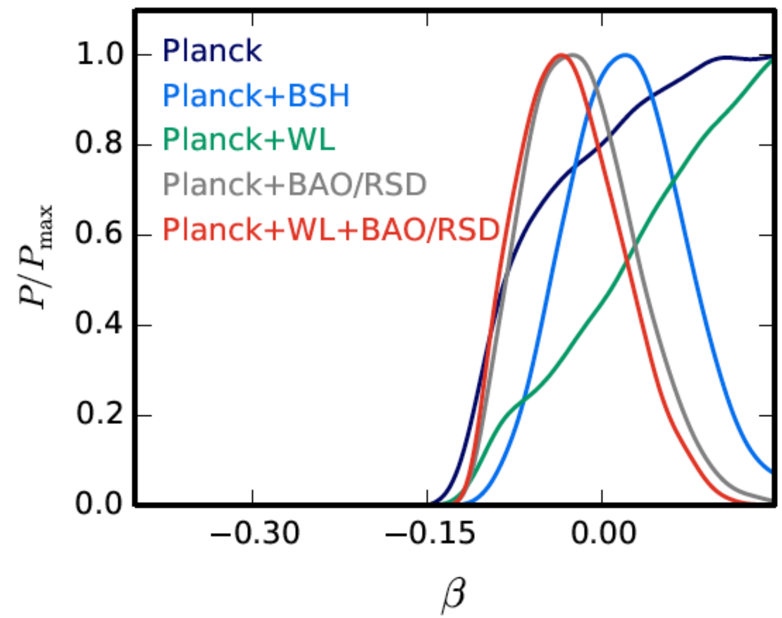


$Q \propto \rho_{de}$





$Q \propto \rho_c$

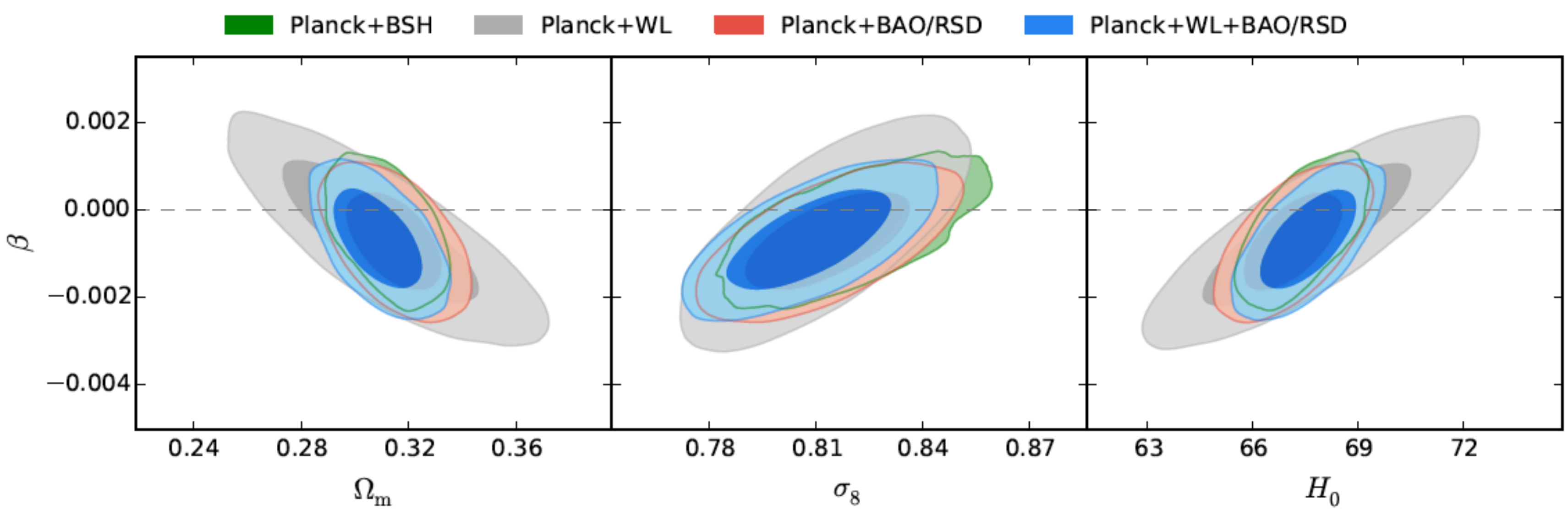


$Q \propto \rho_{\rm de}$

- Model I: Planck itself can constrain β well; BSH is extremely consistent with RSD; $\beta = O(10^{-4})$
- Model II: Planck cannot provide tight constraint; BSH is basically consistent with RSD; $\beta = O(10^{-2})$
- More RSD data may help

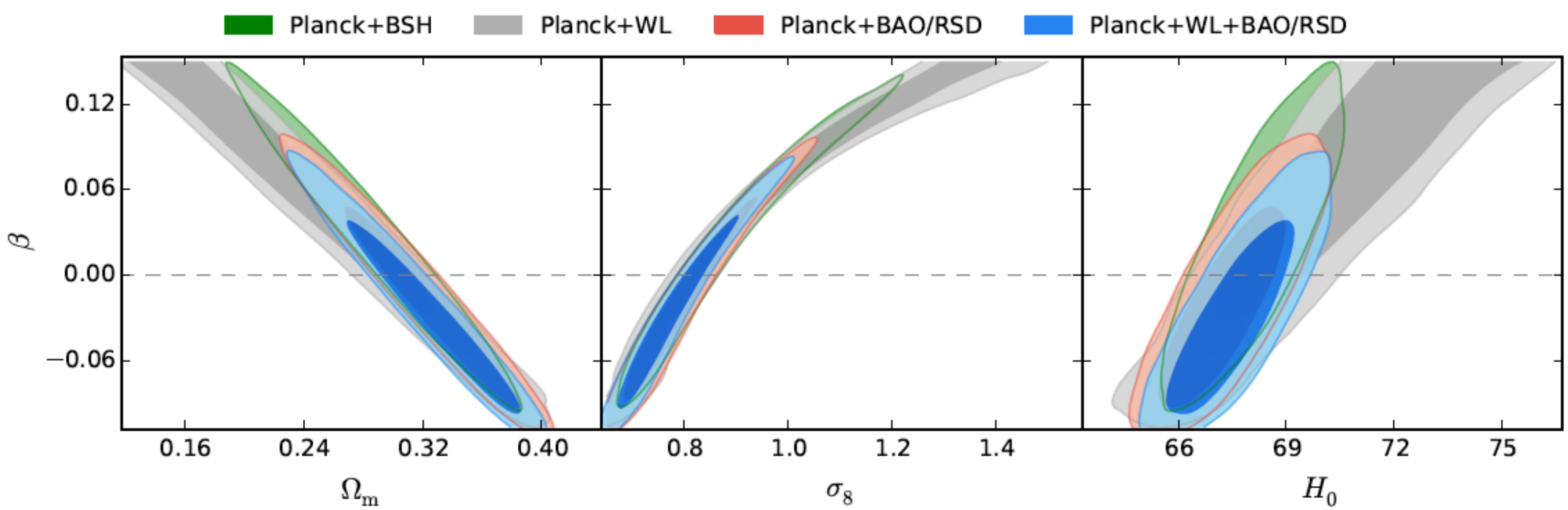
$Q \propto \rho_c$

Parameters	Planck	Planck+BSH	Planck+WL	Planck+BAO/RSD	Planck+WL+BAO/RSD
Ω_m	$0.371^{+0.035}_{-0.039}$	$0.3114^{+0.0086}_{-0.0083}$	$0.31^{+0.023}_{-0.025}$	0.315 ± 0.011	$0.309^{+0.010}_{-0.011}$
β	-0.0021 ± 0.0011	$-0.00044^{+0.00069}_{-0.00066}$	-0.0006 ± 0.0010	$-0.00073^{+0.00075}_{-0.00069}$	$-0.00069^{+0.00072}_{-0.00071}$
σ_8	$0.809^{+0.015}_{-0.017}$	$0.82^{+0.014}_{-0.015}$	0.812 ± 0.016	0.813 ± 0.014	0.808 ± 0.014
H_0	$63.4^{+2.2}_{-2.5}$	$67.45^{+0.69}_{-0.77}$	67.6 ± 1.9	67.13 ± 0.87	67.58 ± 0.86
τ	$0.089^{+0.013}_{-0.015}$	$0.094^{+0.013}_{-0.015}$	0.09 ± 0.013	0.091 ± 0.013	$0.089^{+0.012}_{-0.014}$
n_s	0.9576 ± 0.0073	$0.9642^{+0.0063}_{-0.0062}$	0.9658 ± 0.0067	$0.9647^{+0.0062}_{-0.0063}$	$0.9666^{+0.0061}_{-0.0062}$



$$Q \propto \rho_{\rm de}$$

Parameters	Planck	Planck+BSH	Planck+WL	Planck+BAO/RSD	Planck+WL+BAO/RSD
Ω_m	$0.281^{+0.066}_{-0.094}$	$0.293^{+0.043}_{-0.038}$	$0.242^{+0.047}_{-0.088}$	$0.323^{+0.046}_{-0.032}$	$0.324^{+0.045}_{-0.031}$
β	$0.036^{+0.114}_{-0.039}$	$0.02^{+0.048}_{-0.053}$	$0.058^{+0.092}_{-0.026}$	$-0.017^{+0.039}_{-0.055}$	$-0.026^{+0.036}_{-0.053}$
σ_8	$0.95^{+0.12}_{-0.25}$	$0.879^{+0.072}_{-0.132}$	$1.01^{+0.16}_{-0.28}$	$0.804^{+0.048}_{-0.103}$	$0.782^{+0.044}_{-0.094}$
H_0	$68.3^{+2.8}_{-2.6}$	68.1 ± 1.0	$70.3^{+2.8}_{-2.1}$	$67.4^{+1.0}_{-1.3}$	$67.6^{+1.0}_{-1.2}$
τ	$0.089^{+0.013}_{-0.014}$	$0.09^{+0.012}_{-0.014}$	$0.089^{+0.012}_{-0.015}$	$0.09^{+0.012}_{-0.015}$	$0.088^{+0.012}_{-0.014}$
n_s	$0.9593^{+0.0071}_{-0.0073}$	0.9607 ± 0.0059	0.9657 ± 0.0068	0.9628 ± 0.006	0.965 ± 0.0059



Summary

- Acceleration is confirmed, but its physics is still in the dark
- Most important mission is to distinguish between DE & MG
- MG can mimic DE (Λ) in the background (expansion history), but modifies perturbations (growth history)
- Base ΛCDM fits almost all data well
- Tests of DE & MG: towards ΛCDM
- MG (5th force): f(R) – universal coupling & CQ – non-universal coupling
- IDE modifies both background and perturbation
- PPF for IDE established: whole parameter space can be explored
- Testing IΛCDM: towards ΛCDM
- Future high-precision data are important: long way to go

Thanks!