Precision Study of Gluon Saturation: Experimental Analysis versus Theoretical Approach

Bo-Wen Xiao

School of Science and Engineering, CUHK-Shenzhen

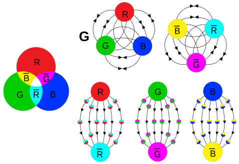
G. A. Chirilli, Bo-Wen Xiao, Feng Yuan, Phys. Rev. Lett. **108**, 122301 (2012).
Y. Shi, L. Wang, S.Y. Wei, Bo-Wen Xiao, Phys. Rev. Lett. **128**, 202302 (2022).

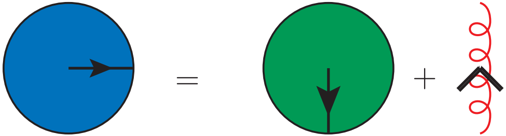
QCD and Gauge Symmetry

Trinity of Color:

$$\text{QCD Lagrangian :} \qquad \mathcal{L} = i\bar{\psi}\gamma^{\mu}\mathcal{D}_{\mu}\psi - m\bar{\psi}\psi - \frac{1}{4}\left[\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu}\right]$$

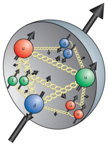
with $\mathcal{D}_{\mu} \equiv \partial_{\mu} + igA^{a}_{\mu}t^{a}$ and $F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} - gf_{abc}A^{b}_{\mu}A^{c}_{\nu}$.

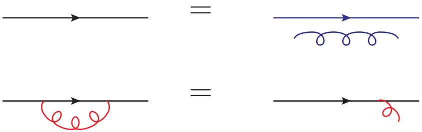




- Gauge symmetry is w.r.t. the local phase of charge particles.
- The same physics (phase) can be described using different orientations of the arrows (phases of the quark wavefunction) with a compensating gauge field.
- Non-Abelian gauge field theory. Invariant under SU(3) gauge transformation.

Kinoshita-Lee-Nauenberg Theorem





KLN theorem: In a theory with massless fields, transition rates are free of the infrared divergence (soft and collinear) if the summation over initial and final degenerate states is carried out.

- The KLN theorem: infrared divergences appear because some of states are physically "degenerate", but we treat them as different.
- A state with a quark accompanied by a collinear gluon is degenerate with a state with a single quark.
- A state with a soft gluon is almost degenerate with a state with no gluon (virtual).

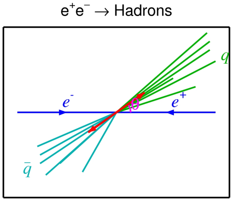
Infrared Safety

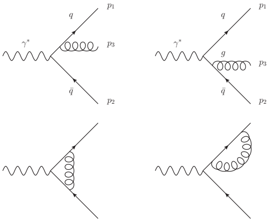
- Two kinds of IR divergences: collinear and soft divergences.
- For a suitable defined inclusive observable (e.g., $\sigma_{e^+e^-\to\text{hadrons}}$), there is a cancellation between the soft and collinear singularities occurring in the real and virtual contributions. Kinoshita-Lee-Nauenberg theorem
- Any new observables must have a definition which does not distinguish between

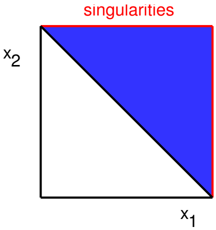
$$\text{parton} \leftrightarrow \text{parton} + \text{soft gluon}$$
$$\text{parton} \leftrightarrow \text{two collinear partons}$$

- Observables that respect the above constraint are called infrared safe observables. Infrared safety is a requirement that the observable is calculable in pQCD.

e^+e^- annihilation







- Born diagram () gives $\sigma_0 = \alpha_{em}\sqrt{s}N_c \sum_q e_q^2 \left(\frac{4\pi\mu^2}{s}\right)^{\epsilon} \frac{\Gamma[2-\epsilon]}{\Gamma[2-2\epsilon]}$
- NLO: real contribution (3 body final state). $x_i \equiv \frac{2E_i}{Q}$ with $Q = \sqrt{s}$

$$\frac{d\sigma_3}{dx_1 dx_2} = C_F \frac{\alpha_s}{2\pi}\sigma_0 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Dimensional Regularization

Dimensional regularization:

- Analytically continue in the number of dimensions from $d = 4$ to $d = 4 - 2\epsilon$.
- Convert the soft and collinear divergence into poles in ϵ.
- To keep g_s dimensionless, substitue $g_s \to g_s \mu^\epsilon$ with renormalization scale μ.

At the end of the day, one finds

$$\begin{aligned} \sigma_r &= \sigma_0 \frac{\alpha_s(\mu)}{2\pi} C_F \left(\frac{Q^2}{4\pi\mu^2} \right)^{-\epsilon} \frac{\Gamma[1-\epsilon]}{\Gamma[1-2\epsilon]} \left[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \frac{2\pi^2}{3} \right] \\ \sigma_v &= \sigma_0 \frac{\alpha_s(\mu)}{2\pi} C_F \left(\frac{Q^2}{4\pi\mu^2} \right)^{-\epsilon} \frac{\Gamma[1-\epsilon]}{\Gamma[1-2\epsilon]} \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{2\pi^2}{3} \right] \end{aligned}$$

and the sum $\lim_{\epsilon \to 0} \sigma = \sigma_0 \left(1 + \frac{3}{4} C_F \frac{\alpha_s(\mu)}{\pi} + \mathcal{O}(\alpha_s^2) \right)$.

- (Almost) Complete Cancellation between real and virtual.
- For more exclusive observables, the cancellation is not always complete.

Factorization and NLO Calculation

- Factorization is about separation of short distant physics (perturbatively calculable hard factor) from large distant physics (Non perturbative).

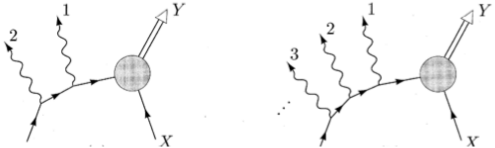
$$\sigma \sim xf(x) \otimes \mathcal{H} \otimes D_h(z) \otimes \mathcal{F}(k_\perp)$$

- NLO (1-loop) calculation always contains various kinds of divergences.
  - Some divergences can be absorbed into the corresponding evolution equations.
  - Renormalization: cutting off infinities and hiding the ignorance.
  - The rest of divergences should be canceled.
- Hard factor

$$\mathcal{H} = \mathcal{H}^{(0)}_{\mathrm{LO}} + \frac{\alpha_s}{2\pi}\mathcal{H}^{(1)}_{\mathrm{NLO}} + \cdots$$

should always be finite and free of divergence of any kind.

Large Logarithms



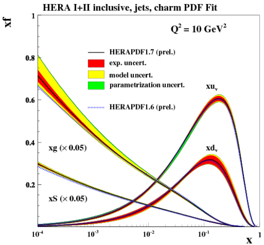
- NLO vs NLL   Naive α_s expansion sometimes is not sufficient!

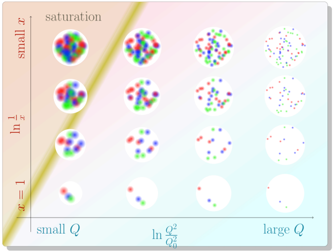
	LO	NLO	NNLO	$\cdots$
LL	1	$\alpha_s L$	$(\alpha_s L)^2$	$\cdots$
NLL		α_s	$\alpha_s\,(\alpha_s L)$	$\cdots$
$\cdots$			$\cdots$	$\cdots$

- Evolution $\rightarrow$ Resummation of large logs.
  LO evolution resums LL; NLO $\Rightarrow$ NLL.

Saturation Physics (Color Glass Condensate)

色玻璃凝聚理论: 描述高能极限下强子内部高密度胶子的涌现性质

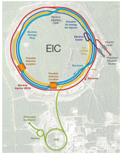


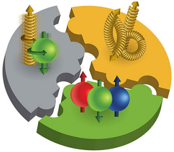
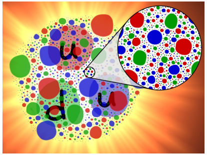


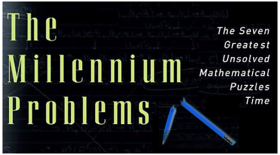
- Gluon density grows rapidly as x gets small. BFKL evolution!
- Resummation of the $\alpha_s \ln \frac{1}{x}$ $\Rightarrow$ BFKL equation. Hard at NLO! (20 years)
- Many gluons with fixed size packed in a confined hadron, gluons overlap and recombine $\Rightarrow$ Non-linear QCD dynamics (BK/JIMWLK) $\Rightarrow$ ultra-dense gluonic matter
- Saturation = Multiple Scattering (MV model) + Small-x (high energy) evolution

Ultimate Questions and Challenges in QCD

To understand our physical world, we have to understand QCD!





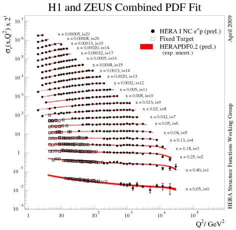
Three pillars of EIC Physics:

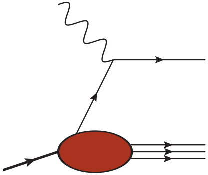
- How does the spin of proton arise? (Spin puzzle)
- What are the emergent properties of dense gluon system?
- How does proton mass arise? Mass gap: million dollar question.

EICs: keys to unlocking these mysteries! Many opportunities will be in front of us!

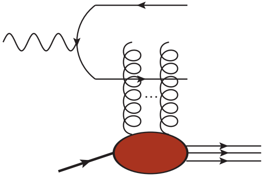
Dual Descriptions of Deep Inelastic Scattering

深度非弹性散射的双重描述:





Bjorken frame



Dipole frame

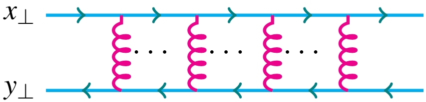
- Bjorken: partonic picture is manifest. Saturation shows up as limit of number density.
- Dipole: the partonic picture is no longer manifest. Saturation appears as the unitarity limit for scattering. Convenient to resum the multiple gluon interactions.

$$F_2(x, Q^2) = \sum_f e_f^2 \frac{Q^2}{4\pi^2\alpha_{\rm em}} S_\perp \int_0^1 \mathrm{d}z \int \mathrm{d}^2 r_\perp \left|\psi\left(z, r_\perp, Q\right)\right|^2 \left[1 - S^{(2)}\left(Q_s r_\perp\right)\right]$$

Wilson Lines in Color Glass Condensate Formalism

The Wilson loop (color singlet dipole) in McLerran-Venugopalan (MV) model

$$\frac{1}{N_c}\left\langle \mathrm{Tr}U(x_\perp)U^\dagger(y_\perp)\right\rangle = e^{-\frac{Q_s^2(x_\perp-y_\perp)^2}{4}}$$

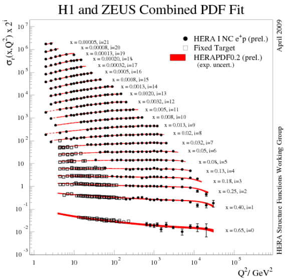


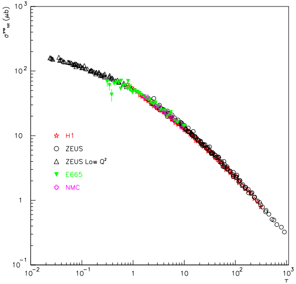
- Dipole amplitude $S^{(2)}$ then produces the quark k_T spectrum via Fourier transform

$$\mathcal{F}(k_\perp) \equiv \frac{dN}{d^2k_\perp} = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp-y_\perp)} \frac{1}{N_c}\left\langle \mathrm{Tr}U(x_\perp)U^\dagger(y_\perp)\right\rangle.$$

Geometrical Scaling in DIS

深度非弹性散射总截面[Golec-Biernat, Stasto, Kwiecinski; 01, Munier, Peschanski, 03]





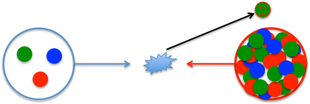
- All data ($x \leq 0.01$, $Q^2 \leq 450GeV^2$) is function of a single variable $\tau = Q^2/Q_s^2$.
- Define $Q_s^2(x) = (x_0/x)^\lambda \mathrm{GeV}^2$ with $x_0 = 3.04 \times 10^{-3}$ and $\lambda = 0.288$.

Forward hadron production in *pA* collisions

利用相对稀疏的质子（氘核）作为探针来探测重核中的稠密胶子的性质

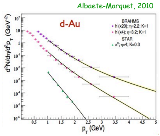
[Dumitru, Jalilian-Marian, 02] Dilute-dense factorization at forward rapidity

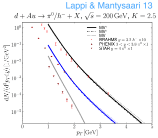
$$\frac{d\sigma_{\rm LO}^{pA\to hX}}{d^2p_\perp dy_h}=\int_\tau^1\frac{dz}{z^2}\left[x_1q_f(x_1,\mu)\mathcal{F}_{x_2}(k_\perp)D_{h/q}(z,\mu)+x_1g(x_1,\mu)\tilde{\mathcal{F}}_{x_2}(k_\perp)D_{h/g}(z,\mu)\right].$$



projectile: $x_1 \sim \frac{p_\perp}{\sqrt{s}}e^{+y}\sim 1$ valence

target: $x_2 \sim \frac{p_\perp}{\sqrt{s}}e^{-y}\ll 1$ gluon



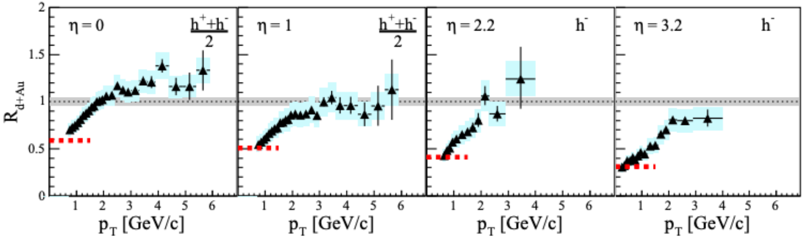


- Proton: Collinear PDFs and FFs (Strongly depends on μ^2).; Nucleus: Small-*x* gluon!
- Early attempts: [Dumitru, Hayashigaki, Jalilian-Marian, 06; Altinoluk, Kovner 11] [Altinoluk, Armesto, Beuf, Kovner, Lublinsky, 14]
- Full NLO: [Chirilli, BX and Yuan, 12]

d+Au collisions at RHIC

相对论重离子对撞机上的单强子产生: 氘+金核碰撞/质子+质子碰撞

$$R_{d+Au} = \frac{1}{\langle N_{\rm coll} \rangle} \frac{d^2N_{d+Au}/d^2p_T d\eta}{d^2N_{pp}/d^2p_T d\eta}$$



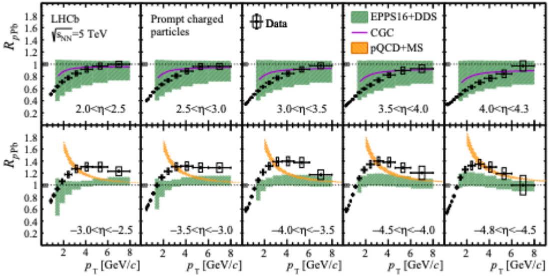
BRAHMS

- Cronin effect at middle rapidity
- Rapidity evolution of the nuclear modification factors R_{d+Au}
- Promising evidence for gluon saturation effects

New LHCb Results

[R. Aaet al. (LHCb Collaboration), Phys. Rev. Lett. 128 (2022) 142004]

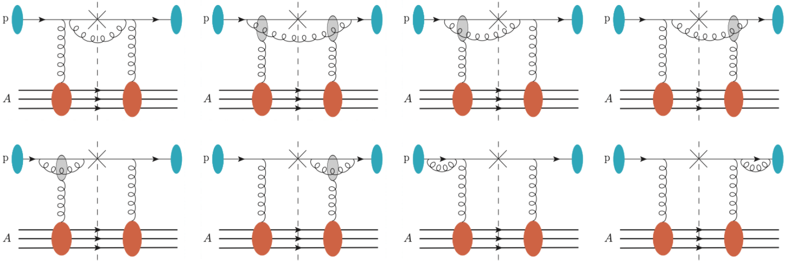
$$R_{pPb} = \frac{1}{\langle N_{\rm coll} \rangle} \frac{d^2N_{p+Pb}/d^2p_T d\eta}{d^2N_{pp}/d^2p_T d\eta}$$



- Rapidity evolution of the nuclear modification factors R_{pPb} similar to RHIC

NLO diagrams in the $q \to q$ channel

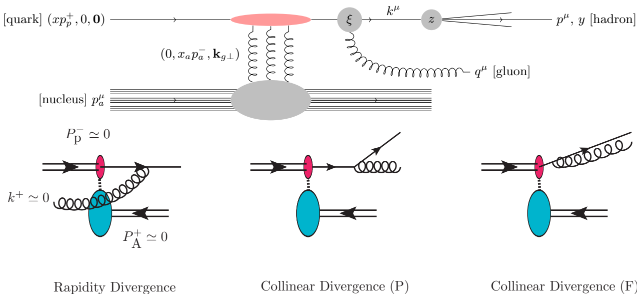
G. A. Chirilli, Bo-Wen Xiao, Feng Yuan, Phys. Rev. Lett. **108**, 122301 (2012).



- Take into account real (top) and virtual (bottom) diagrams together!
- Non-linear multiple interactions inside the grey blobs!
- Integrate over gluon phase space $\Rightarrow$Divergences!.

Factorization for single inclusive hadron productions

Factorization for the $p + A \to H + X$ process [Chirilli, BX and Yuan, 12]



- Include all real and virtual graphs in all channels $q \to q$, $q \to g$, $g \to q(\bar{q})$ and $g \to g$.
- 1. collinear to target nucleus; rapidity divergence $\Rightarrow$ BK evolution for UGD $\mathcal{F}(k_\perp)$.
- 2. collinear to the initial quark; $\Rightarrow$ DGLAP evolution for PDFs
- 3. collinear to the final quark. $\Rightarrow$ DGLAP evolution for FFs.

Numerical implementation of the NLO result

Single inclusive hadron production up to NLO

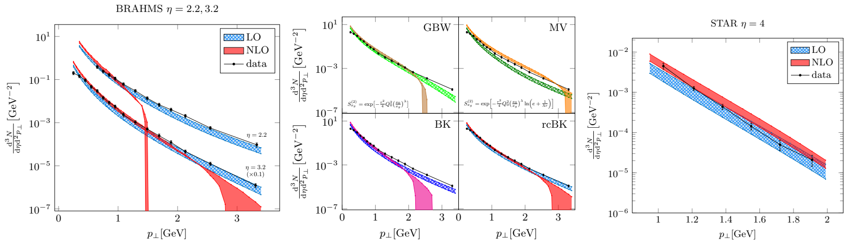
$$d\sigma = \int xf_a(x) \otimes D_a(z) \otimes \mathcal{F}_a^{x_g}(k_\perp) \otimes \mathcal{H}^{(0)} + \frac{\alpha_s}{2\pi} \int xf_a(x) \otimes D_b(z) \otimes \mathcal{F}_{(N)ab}^{x_g} \otimes \mathcal{H}_{ab}^{(1)}.$$

Consistent implementation should include all the NLO α_s corrections.

- NLO parton distributions. (MSTW or CTEQ)
- NLO fragmentation function. (DSS or others.)
- Use NLO hard factors. [Chirilli, BX and Yuan, 12]
- Use the one-loop approximation for the running coupling
- rcBK evolution equation for the dipole gluon distribution [Balitsky, Chirilli, 08; Kovchegov, Weigert, 07]. Full NLO BK evolution not available.
- Saturation physics at One Loop Order (SOLO). [Stasto, Xiao, Zaslavsky, 13]

Numerical implementation of the NLO result

Saturation physics at One Loop Order (SOLO). [Stasto, Xiao, Zaslavsky, 13]



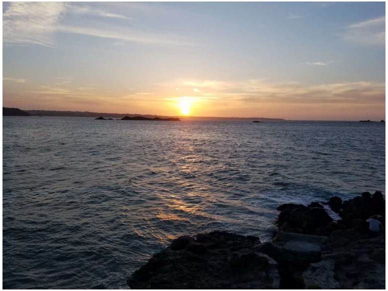
- Reduced factorization scale dependence!
- Catastrophe: Negative NLO cross-sections at high p_T.
- Fixed order calculation in field theories is not guaranteed to be positive.
- Rapidity sub with kinematic constraints. [Watanabe, Xiao, Yuan, Zaslavsky, 15]

Extending the applicability of CGC calculation

- Goal: find a solution within our current factorization (exactly resum $\alpha_s \ln 1/x_g$) to extend the applicability of CGC. Other scheme choices certainly is possible.
- A lot of logs arise in pQCD loop-calculations: DGLAP, small-x, threshold, Sudakov.
- Breakdown of α_s expansion occurs due to the appearance of logs in certain PS.
- Demonstrate onset of saturation and visualize smooth transition to dilute regime.
- Add'l consideration: numerically challenging due to limited computing resources.
- Towards a more complete framework. [Altinoluk, Armesto, Beuf, Kovner, Lublinsky, 14; Kang, Vitev, Xing, 14; Ducloue, Lappi and Zhu, 16, 17; Iancu, Mueller, Triantafyllopoulos, 16; Liu, Ma, Chao, 19; Kang, Liu, 19; Kang, Liu, Liu, 20;]

Some thoughts

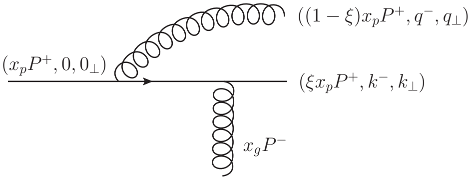
Spiritual Retreat under the Auspices of the (Color) Trinity

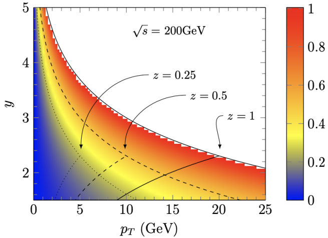


- Sabbatical leave is important!
- Learn new technique and put it aside sometimes.
- Keep trying and take as long as it takes!

Gluon Radiation at the Threshold

Near threshold: radiated gluon has to be soft! $\tau = \frac{p_\perp e^y}{\sqrt{s}}$ density ($\tau = x_p \xi z \leq 1$)





- Gluon momentum: $q^+ = (1-\xi)p_q^+ \to 0$
- Introduce an additional semi-hard scale Λ^2.

Threshold Logarithms

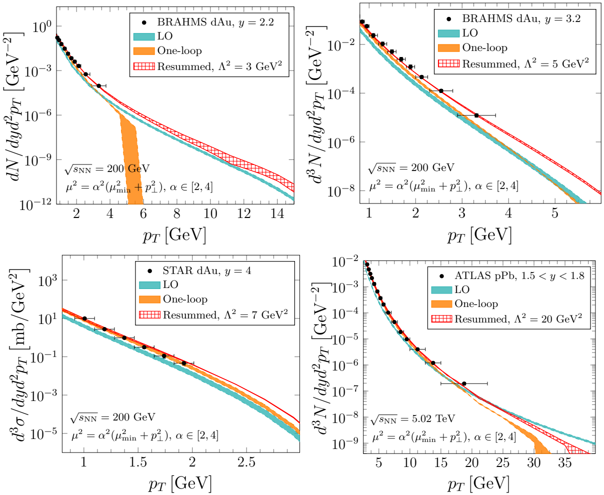
Y. Shi, L. Wang, S.Y. Wei, Bo-Wen Xiao, ▸ Phys. Rev. Lett. **128**, 202302 (2022).

- Numerical integration (8-d in total) is notoriously hard in $r_\perp$ space. Go to $k_\perp$ space.
- In the coordinate space, we can identify two types of logarithms

$$\text{single log: } \ln\frac{k_\perp^2}{\mu_r^2}\to\ln\frac{k_\perp^2}{\Lambda^2}\,,\quad \ln\frac{\mu^2}{\mu_r^2}\to\ln\frac{\mu^2}{\Lambda^2};\quad \text{double log: } \ln^2\frac{k_\perp^2}{\mu_r^2}\to\ln^2\frac{k_\perp^2}{\Lambda^2}\,,$$

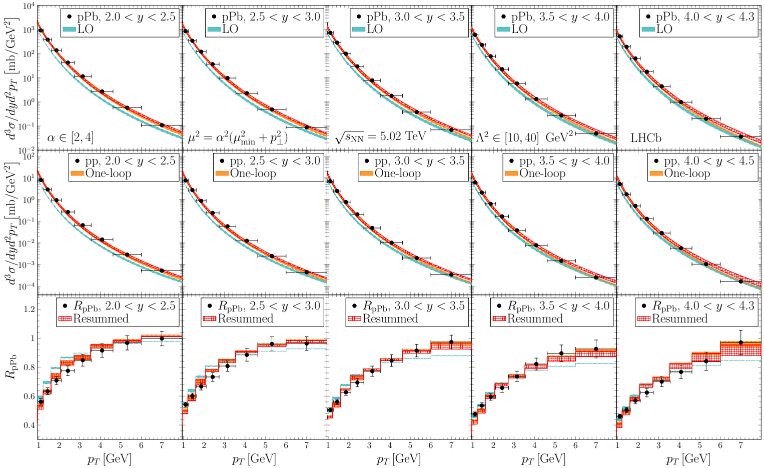
  with $\mu_r\equiv c_0/r_\perp$ with $c_0=2e^{-\gamma_E}$.
- Introduce a semi-hard auxiliary scale $\Lambda^2\sim\mu_r^2\gg\Lambda_{QCD}^2$. Identify dominant $r_\perp$!
- Dependences on μ^2, Λ^2 cancel order by order. Choose “natural" values at fixed order.
- For running coupling, $\Lambda^2=\Lambda_{QCD}^2\left[\frac{(1-\xi)k_\perp^2}{\Lambda_{QCD}^2}\right]^{C_R/[C_R+\beta_1]}$. Akin to CSS & Catani *et al*.

Numerical Results for pA spectra



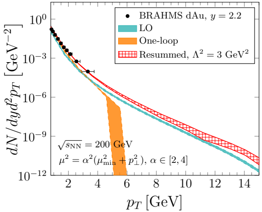
- $\mu^2 = \alpha^2(\mu^2_{\min} + p_T^2)$ & $\alpha \in [2, 4]$;
- RHIC: $\Lambda^2 \sim Q_s^2$; LHC, larger Λ^2.
- $\mu \sim Q \geq 2k_\perp$ ($\alpha > 2$) at high p_T. $2 \to 2$ hard scattering.
- Estimate higher order correction by varying μ and Λ.
- Threshold enhancement for σ.
- Nice agreement with data across many orders of magnitudes for different energies and p_T ranges!

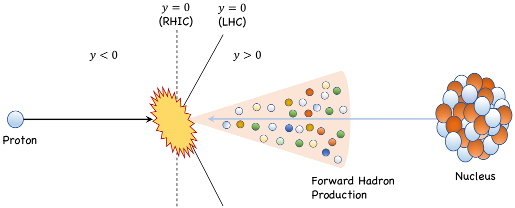
Comparison with the new LHCb data

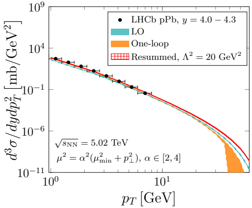


- LHCb data: 2108.13115
- Data Link DIS2021
- $\mu \sim (2 \sim 4)p_T$ with proper choice of Λ
- Threshold effect is not important at low p_T for LHCb data. Saturation effects are still dominant.
- Predictions are improved from LO to NLO.

Summary







- Ten-Year Odyssey in NLO hadron productions in pA collisions in CGC.
- Towards the precision test of saturation physics (CGC) at RHIC and LHC.
- Next Goal:Global analysis for CGC combining data from pA and DIS.
- Exciting time of NLO CGC phenomenology with the upcoming EIC.

Threshold resummation in the CGC formalism

Threshold logarithms: Sudakov soft gluon part and Collinear (plus-distribution) part.

- Soft single and double logs ($\ln k_\perp^2/\Lambda^2$, $\ln^2 k_\perp^2/\Lambda^2$) are resummed via Sudakov factor. Performing Fouier transformations

$$\int \frac{d^2 r_\perp}{(2\pi)^2} S(r_\perp) \ln \frac{\mu^2}{\mu_r^2} e^{-ik_\perp \cdot r_\perp} = -\int \frac{d^2 l_\perp}{\pi l_\perp^2} \left[F(k_\perp + l_\perp) - J_0(\frac{c_0}{\mu} l_\perp) F(k_\perp) \right]$$

$$= \quad -\frac{1}{\pi} \int \frac{d^2 l_\perp}{(l_\perp - k_\perp)^2} \left[F(l_\perp) - \frac{\Lambda^2}{\Lambda^2 + (l_\perp - k_\perp)^2} F(k_\perp) \right] + F(k_\perp) \ln \frac{\mu^2}{\Lambda^2}.$$

- Two equivalent methods to resum the collinear part ($P_{ab}(\xi) \ln \Lambda^2/\mu^2$):
  1. Reverse DGLAP evolution; 2. RGE method (threshold limit $\xi \to 1$).
- Introduce forward threshold quark jet function $\Delta^q(\Lambda^2, \mu^2, \omega)$, which satisfies

$$\frac{\mathrm{d}\Delta^q(\omega)}{\mathrm{d}\ln \mu^2} = -\frac{\mathrm{d}\Delta^q(\omega)}{\mathrm{d}\ln \Lambda^2} = -\frac{\alpha_s C_F}{\pi} \left[\ln \omega + \frac{3}{4} \right] \Delta^q(\omega) + \frac{\alpha_s C_F}{\pi} \int_0^\omega \mathrm{d}\omega' \frac{\Delta^q(\omega) - \Delta^q(\omega')}{\omega - \omega'}$$

- Consistent with the threshold resummation in SCET[Becher, Neubert, 06]!