New Loop-Regularization and Intrinsic Mass Scales in QFTs

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2004. 09. 06 ITP, Beijing, CAS (CAS)

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- **Y.L.Wu, Mod. Phys. Lett. A19 (2004) 2191, hep-th/0311082**

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Introduction

Symmetry and QFTs are the Basic Foundation in Physics

- **All known basic forces of nature, i.e., gravitational, electromagnetic, weak and strong forces, are governed by the symmetries:** $\mathbf{U(1)_Y \times SU(2)_L \times SU(3)_c \times SO(1,3) \times GL(4,R)}$

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- **Quantum field theories (QFTs) have successfully been applied to treat the underlying theories of elementary particles and also deal with effective theories for composite particles at low energies as well as critical phenomena (or phase transitions) in statistical mechanics and condensed matter physics.**

- **An important issue for making QFTs to be physically meaningful is the elimination of ultraviolet (UV) divergences without spoiling symmetries of the original theory.**

Real World Prefers the Symmetry Breaking Phase

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- **In Strong QCD,** $\mathbf{U(3)_L \times U(3)_R \to U(3)_V \to U(1)_V}$

Question 1: How to Avoid the Infinity Problem in QFTs

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$$\mathbf{1 = \frac{GN_c}{8\pi^2}\left(\Lambda^2 - m_t^2 \ln\frac{\Lambda^2}{m_t^2}\right)} \quad \textbf{Cut-off Reg.}$$

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Infinity Free $\leftrightarrow$ Intrinsic Mass Scales

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Dynamically Spontaneous Symmetry Breaking (DSSB)

Gap Equation $\leftrightarrow$ Minimal Condition

Symmetry-Preserving Loop Regularization (SPLR)

Why Quantum Field Theory

Weinberg's folk theorem:

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"Any quantum theory that at sufficiently low energy and large distances looks Lorentz invariant and satisfies the cluster decomposition principle will also at sufficiently low energy look like a quantum field theory."

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Wilson-Kadanoff and Gell-Mann-Low RG Flow

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"Any quantum theory that at sufficiently low energy and large distances looks Lorentz invariant and satisfies the cluster decomposition principle will also at sufficiently low energy look like a quantum field theory."

Wilson-Kadanoff and Gell-Mann-Low RG Flow

"Physical phenomena at any interesting renormalization energy scale can be described by integrating out the physics at higher energy scales."

Implications

- **There must exist in any case a characteristic energy scale (CES) M_c .**
- **One can always make an QFT description at a sufficiently low energy scale in comparison with the CES M_c.**

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- **The explicit regularization method is governed by a physically meaningful CES M_c and a physically interesting SES μ_s.**

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- **One can always make an QFT description at a sufficiently low energy scale in comparison with the CES M_c.**
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- **The explicit regularization method is governed by a physically meaningful CES M_c and a physically interesting SES μ_s.**
- **There should be no doubt of existing a new symmetry-preserving and infinity-free regularization scheme.**

Irreducible Loop Integrals (ILIs)

Introduction of the concept of irreducible loop integrals (ILIs): **"Any n-fold ILIs that are evaluated from n-loop overlapping Feynman integrals of loop momenta k_i ($i = 1, \cdots, n$) are defined as the loop integrals in which there are no longer the overlapping factors $(k_i - k_j + p_{ij})^2$ $(i \neq j)$ that appear in the original overlapping Feynman integrals. "**

1-fold ILIs for one loop integrals

$$\mathbf{I}_{-2\alpha} = \int \frac{\mathbf{d^4k}}{(2\pi)^4} \frac{\mathbf{1}}{(\mathbf{k^2} - \mathcal{M}^2)^{2+\alpha}} , \quad \mathbf{I}_{-2\alpha\ \mu\nu} = \int \frac{\mathbf{d^4k}}{(2\pi)^4} \frac{\mathbf{k}_\mu \mathbf{k}_\nu}{(\mathbf{k^2} - \mathcal{M}^2)^{3+\alpha}} ,$$

$$\mathbf{I}_{-2\alpha\ \mu\nu\rho\sigma} = \int \frac{\mathbf{d^4k}}{(2\pi)^4} \frac{\mathbf{k}_\mu \mathbf{k}_\nu \mathbf{k}_\rho \mathbf{k}_\sigma}{(\mathbf{k^2} - \mathcal{M}^2)^{4+\alpha}} , \quad \alpha = -1, 0, 1, \cdots$$

$\mathbf{I_2}$, $\mathbf{I}_{2\mu\nu\cdots} \rightarrow$ **Quadratically div.;** $\mathbf{I_0}$, $\mathbf{I}_{0\mu\nu\cdots} \rightarrow$ **Logarithmically div.**

Symmetry-Preserving Consistency Conditions

For Regularized ILIs

$$\mathbf{I^R_{2\mu\nu} = \frac{1}{2} g_{\mu\nu}\ I^R_2}, \quad \mathbf{I^R_{2\mu\nu\rho\sigma} = \frac{1}{8} g_{\{\mu\nu}g_{\rho\sigma\}}\ I^R_2},$$

$$\mathbf{I^R_{0\mu\nu} = \frac{1}{4} g_{\mu\nu}\ I^R_0}, \quad \mathbf{I^R_{0\mu\nu\rho\sigma} = \frac{1}{24} g_{\{\mu\nu}g_{\rho\sigma\}}\ I^R_0}$$

$$\mathbf{g_{\{\mu\nu}g_{\rho\sigma\}} \equiv g_{\mu\nu}g_{\rho\sigma} + g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\rho\nu}}$$

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Divergence Cancellation

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Divergence Cancellation

"The regularized quadratic divergences cancel each other for Underlying gauge theories due to Gauge Invariance."

Regulator-Free Scheme

"Consistency conditions lead to a regulator-free scheme for underlying QFTs."

General Theorem:

"The convergent integrations can safely be carried out, only the divergent integrations destroy the gauge invariance".

Demonstration

$$\mathbf{I_{2\mu\nu} = a\ g_{\mu\nu} I_2}, \qquad \mathbf{a = I_{2\mu\nu} g^{\mu\nu}/(4 I_2) = 1/4}$$

$$\mathbf{I_{2\ 00} = a\ g_{00} I_2}, \qquad \mathbf{a = 1/2}$$

General Regularization Prescription

Replacing in the ILIs the loop integrating variable k^2 and the loop integrating measure $\int d^4k$ by the regularized ones $[k^2]_l$ and $\int [d^4k]_l$

$$\mathbf{k^2 \rightarrow [k^2]_l \equiv k^2 + M_l^2} \ ,$$

$$\int \mathbf{d^4k} \rightarrow \int \mathbf{[d^4k]_l \equiv \lim_{N,M_l^2} \sum_{l=0}^{N} c_l^N \int d^4k}$$

with

$$\lim_{\mathbf{N,M_l^2}} \sum_{\mathbf{l=0}}^{\mathbf{N}} \mathbf{c_l^N \ (M_l^2)^n = 0 \quad c_0^N = 1 \quad (n = 0, 1, \cdots)}$$

Explicit and Simple Solution

$$\mathbf{M_l^2 = \mu_s^2 + l M_R^2, \quad c_l^N = (-1)^l \frac{N!}{(N-l)!\ l!}}$$

Explicit Forms for the Regularized ILIs I_0^R and I_2^R

$$\mathbf{I_2^R = \frac{-i}{16\pi^2}\ \{\ M_c^2 - \mu^2 [\ \ln \frac{M_c^2}{\mu^2} - \gamma_w + 1 + y_2(\frac{\mu^2}{M_c^2})\ ]\ \}}$$

$$\mathbf{I_0^R = \frac{i}{16\pi^2}\ [\ \ln \frac{M_c^2}{\mu^2} - \gamma_w + y_0(\frac{\mu^2}{M_c^2})\ ]}$$

$$\mathbf{y_0(x) = \int_0^x d\sigma\ \frac{1-e^{-\sigma}}{\sigma}, \quad y_1(x) = \frac{e^{-x} - 1 + x}{x}, \quad y_2(x) = y_0(x) - y_1(x),}$$

$$M_c^2 = \lim_{N, M_R} M_R^2 / \ln N, \quad \mu^2 = \mu_s^2 + \mathcal{M}^2, \quad \gamma_w = \gamma_E = 0.5772\cdots$$

Two Intrinsic Mass Scales

- $\mathrm{M_c}$- **UV Cut-off scale & Characteristic Energy Scale (CES)**
- μ_{s}- **IR Cut-off scale & Sliding Energy Scale (SES)**

Regulating Distribution Functions

The Regularized ILIs in Proper-Time Formalism

$$\mathbf{I}^{\mathbf{R}}_{-2\alpha} = \frac{\mathbf{i}(-1)^{\alpha}}{\Gamma(\alpha+2)} \lim_{N\to\infty} \int \frac{\mathbf{d^4k}}{(2\pi)^4} \int_0^{\infty} \mathbf{d}\tau\ \mathcal{W}_{\mathbf{N}}(\tau; \mathbf{M_c}, \mu_{\mathbf{s}})\ \tau^{\alpha+1} \mathbf{e}^{-\tau(\mathbf{k^2}+\mathcal{M}^2)}$$

Conditions for the Regulating Distribution Function $\mathcal{W}_{\mathbf{N}}(\tau;\mathbf{M_c},\mu_{\mathbf{s}})$

- $\mathcal{W}_{\mathbf{N}}(\tau=\mathbf{0};\mathbf{M_c},\mu_{\mathbf{s}})=\mathbf{0}$
  so as to eliminate the singularity at $\tau=0$ which corresponds to the UV divergence for the momentum integration;

- $\mathcal{W}_{\mathbf{N}}(\tau=\infty;\mathbf{M_c},\mu_{\mathbf{s}}=\mathbf{0})=\mathbf{1}$
  for ensuring the regulating distribution function not to modify the behavior of original theory in the IR regime;

- $\mathcal{W}_{\mathbf{N}}(\tau;\mathbf{M_c}\to\infty,\mu_{\mathbf{s}}=\mathbf{0})=\mathbf{1}$
  which ensures the proper-time formalism to recover the original ILIs in the physical limits;

- $\lim_{\mathbf{N}\to\infty}\mathcal{W}_{\mathbf{N}}(\tau;\mathbf{M_c},\mu_{\mathbf{s}}=\mathbf{0})=\mathbf{1}$ **for** $\tau\geq 1/M_c^2$
  $\lim_{\mathbf{N}\to\infty}\mathcal{W}_{\mathbf{N}}(\tau;\mathbf{M_c},\mu_{\mathbf{s}}=\mathbf{0})=\mathbf{0}$ **for** $\tau< 1/M_c^2$**,**
  so that M_c acts as the UV cutoff scale.

The Simple Form of $\mathcal{W}_N(\tau; M_c, \mu_s)$

$$\mathcal{W}_N(\tau; M_c, \mu_s) = e^{-\tau\mu_s^2}\left(1 - e^{-\tau M_R^2}\right)^N = \sum_{l=0}^{N}(-1)^l \frac{N!}{(N-l)!\ l!}\ e^{-\tau(\mu_s^2 + l M_R^2)}$$

$$M_R^2 = M_c^2 h_w(N)\ln N, \qquad h_w(N)1 \quad h_w(N\to\infty) = 1$$

The Regularized ILIs after integrating over τ

$$I^R_{-2\alpha} \;=\; \lim_{N, M_l^2}\int \frac{d^4k}{(2\pi)^4}\sum_{l=0}^{N} c_l^N \frac{i(-1)^\alpha}{(k^2 + \mathcal{M}^2 + M_l^2)^{\alpha+2}}$$

$$=\; i(-1)^\alpha \int \frac{[d^4k]_l}{(2\pi)^4}\frac{1}{([k^2]_l + \mathcal{M}^2)^{\alpha+2}}$$

which exactly reproduces the prescription for the new SPLR method.

Generalization to More Closed Loops

UV-divergence preserving parameter method

$$\frac{1}{a^{\alpha}b^{\beta}} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\int_0^{\infty} du \frac{u^{\beta-1}}{[a+bu]^{\alpha+\beta}}$$

Here u is the UV-divergence preserving integral variable. As a consequence, the UV divergence for the momentum integration transfers into the one for u integration.

General Form for n-fold ILIs

After safely performing (n-1) convergent integrations over the momentum

k_i **(**$i = 1, \cdots, n-1$**),**

$$\mathbf{I}^{(n)}_{\Delta} = \prod_{i=1}^{n-1} \int_0^{\infty} \mathbf{du_i} \frac{\mathbf{F_{is}(x_{lm})}}{(\mathbf{u_i} + \rho_i)^{\Delta_{is}}} \; \mathbf{I}^{(1)}_{\Delta_n}(\mu_n^2)$$

$$\mathbf{I}^{(n)}_{\Delta_{\mu\nu}} = \prod_{i=1}^{n-1} \int_0^{\infty} \mathbf{du_i} \frac{\mathbf{F_{is}(x_{lm})}}{(\mathbf{u_i} + \rho_i)^{\Delta_{is}}} \; \mathbf{I}^{(1)}_{\Delta_n\mu\nu}(\mu_n^2)$$

with Overall 1-fold ILIs

$$\mathbf{I}^{(1)}_{\Delta_n}(\mu_n^2) = \int \mathbf{d^4k_n} \frac{1}{(\mathbf{k_n^2} + \mathcal{M}^2 + \mu_n^2)^{\Delta_n}}$$

$$\mathbf{I}^{(1)}_{\Delta_n\mu\nu}(\mu_n^2) = \int \mathbf{d^4k_n} \frac{\mathbf{k_{n\mu}k_{n\nu}}}{(\mathbf{k_n^2} + \mathcal{M}^2 + \mu_n^2)^{\Delta_n+1}}$$

Important Features

The functions ρ_i and μ_n^2 have the limits

$$\boldsymbol{\rho_i \to 0, \quad \mu_n^2 \to 0 \quad \text{for} \quad u_1, \cdots, u_{n-1} \to \infty}$$

- **The integral over the n-th loop momentum k_n describes the overall divergent property of n-loop diagrams**
- **The sub-integrals over the variables u_i $(i = 1, 2, \cdots, n-1)$ characterize the UV divergent properties for the one-loop, two-loop, $\cdots$, (n-1)-loop sub-diagrams respectively**

Important Theorems

The Key Theorem

"In the Feynman loop integrals the overlapping divergences which contain overall divergences and also divergences of sub-integrals will become factorizable in the corresponding ILIs."

The Main Theorem

"All the overlapping divergent integrals can be made to be harmless via appropriate subtractions."

General Prescription of Loop Regularization

Universally replace in the n-fold ILIs the n-th loop momentum square k_n^2 and the integrating measure $\int d^4k_n$ as well as the UV-divergence preserving integral variables u_i $(i = 1, \cdots, n-1)$ and $\int du_i$ by the regularizing ones $[k_n^2]_l$ and $\int [d^4k_n]_l$ as well as $[u_i]_l$ and $\int [du_i]_l$ with n being arbitrary:

$$\mathbf{k_n^2} \quad \rightarrow \quad [\mathbf{k_n^2}]_\mathbf{l} \equiv \mathbf{k_n^2} + \mathbf{M_l^2} = \mathbf{k_n^2} + \mu_\mathbf{s}^\mathbf{2} + \mathbf{l M_R^2} \ ,$$

$$\int \mathbf{d^4k_n} \quad \rightarrow \quad \int [\mathbf{d^4k_n}]_\mathbf{l} \equiv \lim_{\mathbf{N}, \mathbf{M_R^2}} \sum_{\mathbf{l}=\mathbf{0}}^{\mathbf{N}} \mathbf{c_l^N} \int \mathbf{d^4k_n}$$

$$\mathbf{u_i} \quad \rightarrow \quad [\mathbf{u_i}]_\mathbf{l} \equiv \mathbf{u_i} + \mathbf{M_l^2}/\mu_\mathbf{s}^\mathbf{2} = \mathbf{u_i} + \mathbf{1} + \mathbf{l M_R^2}/\mu_\mathbf{s}^\mathbf{2}$$

$$\int \mathbf{du_i} \quad \rightarrow \quad \int [\mathbf{du_i}]_\mathbf{l} \equiv \lim_{\mathbf{N}, \mathbf{M_R}} \sum_{\mathbf{l}=\mathbf{0}}^{\mathbf{N}} \mathbf{c_l^N} \int \mathbf{du_i}$$

Low Energy Dynamics of QCD

Well-Known Phenomena in Real Would

- Hadrons are considered to be the bound states of quarks and gluons
- Lightest pseudoscalar mesons are successfully described by current algebra with PCAC
- The chiral symmetry $\mathbf{U(3)_L \times U(3)_R}$ is strongly broken down
- QCD was motivated from the studies of hadrons at low energy
- Low energy dynamics of QCD remains unsolved due to nonperturbative effects of strong interactions

Important Issues on Low Energy Dynamics of QCD

- **How the chiral symmetry is dynamically broken down**
- **How the instanton plays the role as a quantum topological solution of nonperturbative QCD**
- **Whether the effective meson theory should be realized as a linear σ model or a non-linear σ model**
- **Whether the lowest lying $U(3)_V$ nonet scalar mesons corresponds to the chiral partners of the lowest lying nonet pseudoscalar mesons**
- **Whether there exist other lighter isospinor scalar mesons κ_0 (900MeV) as the lowest lying scalar mesons**

- **Does the singlet scalar meson $f_0(400-1200)$ (or the σ) truly exist, why it is so light.**

Answers to the Questions

Based on Three Basic Realistic Assumptions

- **The (approximate) chiral symmetry of QCD Lagrangian**
- **Bound State Solutions of Nonperturbative QCD**
- **Important Instanton Effects**

Applications of Loop Regularization Method

- **Quantum Chiral Dynamics is realized nonlinearly**
- $\mathbf{SU(3)_L \times SU(3)_R}$ **Symmetry is Dynamically spontaneous broken down**
- **Gap Equations as Minimal Conditions of Effective Potential**
- **Nonet scalar mesons play the role of composite Higgs Bosons**
- **Nonet scalar mesons are the chiral partners of the lowest lying nonet pseudoscalar mesons**
- **Chiral Effective Field Theory has the same parameters as QCD (5 parameters:** $\mathbf{v_o}$ **(**$\mathbf{\Lambda_{QCD}}$**),** μ_s **(**$\mathbf{g_s(\mu)}$**),** $\mathbf{m_u}$**,** $\mathbf{m_d}$**,** $\mathbf{m_s}$**)**

Consistent Predictions

- **18 mass spectrum for nonet scalar and pseudoscalar mesons**
- **2 (6) mixing angles among the neutral mesons**
- **2 Intrinsic Mass Scales** $M_c \sim 1$ **GeV and** $\mu_s \sim \Lambda_{QCD}$
- **Quark condensate** $< \bar{q}q >$ **or vacuum expectation value** v_o
- κ_0 **(900MeV) is the lowest lying isospinor scalar mesons**
- **Lightest** σ **and heaviest** η' **all are duo to Instanton Effects**

Effective Chiral Lagrangian

QCD Lagrangian for Light Quarks

$$\mathcal{L}_{QCD} = \bar{q}\gamma^{\mu}(i\partial_{\mu} + g_s G_{\mu}^{a} T^{a})q - \bar{q}Mq - \frac{1}{2}trG_{\mu\nu}G^{\mu\nu}$$

$$q = (u, d, s), \qquad M = diag.(m_1, m_2, m_3) \equiv diag.(m_u, m_d, m_s)$$

Approximate Global Chiral Symmetry

$$U(3)_L \times U(3)_R, \qquad m_i << \Lambda_{QCD}(i = 1, 2, 3)$$

Instanton Effects via t'Hooft Determination

$$\mathcal{L}^{inst} = \kappa_{inst} e^{i\theta_{inst}} \det(-\bar{q}_R q_L) + h.c., \qquad \kappa_{inst} \sim e^{-8\pi^2/g^2}$$

$$\mathbf{U(1)_L \times U(1)_R \rightarrow U(1)_V}$$

Effective Four Quark Interactions-NJL at low energy

$$\mathcal{L}^{4q} = \frac{1}{\mu_f^2}(\bar{q}_{Li} q_{Rj})(\bar{q}_{Rj} q_{Li}) + h.c.$$

Effective Lagrangian for Quarks and Bound States

Integrating over the gluon field and considering the bound state solution

$$\begin{aligned}\mathcal{L}_{eff}(q, \bar{q}, \Phi) &= \bar{q}\gamma^{\mu} i\partial_{\mu} q + \bar{q}_L \gamma_{\mu} \mathcal{A}_L^{\mu} q_L + \bar{q}_R \gamma_{\mu} \mathcal{A}_R^{\mu} q_R - [\ \bar{q}_L(\Phi - M) q_R + h.c.\ ] \\ &+ 2\mu_f^2 tr\left(\Phi M^{\dagger} + M\Phi^{\dagger}\right) - \mu_f^2 tr \Phi\Phi^{\dagger} + \mu_{inst}\left(\det \Phi + h.c.\right)\end{aligned}$$

$\Phi_{ij}-$ **the effective meson fields**

$$\Phi_{ij} \simeq -\frac{1}{\mu_f^2}\bar{q}_{Rj}q_{Li} + 2M_{ij}, \qquad <\phi>\mu_{inst} << \mu_f^2$$

$$\Phi(x) = \xi_L(x)\phi(x)\xi_R^\dagger(x), \quad U = \xi_L(x)\xi_R^\dagger(x) = \xi_L^2(x) = e^{i\frac{2\Pi(x)}{f}}$$

$$\phi^\dagger(x) = \phi(x) = \sum_{a=0}^{a=9}\phi^a(x)T^a, \quad \Pi^\dagger(x) = \Pi(x) = \sum_{a=0}^{a=9}\Pi^a(x)T^a$$

Generating Functionals for effective chiral Lagrangian of mesons

$$\begin{aligned}\frac{1}{Z}\int \mathcal{D}G_\mu \mathcal{D}q\mathcal{D}\bar{q}e^{i\int d^4x\mathcal{L}_{QCD}} &= \frac{1}{\bar{Z}}\int \mathcal{D}\Phi\mathcal{D}q\mathcal{D}\bar{q}e^{i\int d^4x\mathcal{L}_{eff}(q,\bar{q},\Phi)} \\ &= \frac{1}{Z_{eff}}\int \mathcal{D}\Phi e^{i\int d^4x\mathcal{L}_{eff}(\Phi)}\end{aligned}$$

Chiral Effective Lagrangian

Applying the Schwinger's proper time technique to the determinant of Dirac operator and adopting the Loop Regularization method for momentum integrals

$$\begin{aligned}
\mathcal{L}_{\rm eff}(\Phi) &= \frac{1}{2}\frac{N_c}{16\pi^2}\mathrm{tr}T_0\,[\, D_\mu\hat{\Phi}D^\mu\hat{\Phi}^\dagger + D_\mu\hat{\Phi}^\dagger D^\mu\hat{\Phi} \\
&- (\hat{\Phi}\hat{\Phi}^\dagger - \bar{M}^2)^2 - (\hat{\Phi}^\dagger\hat{\Phi} - \bar{M}^2)^2\,] \\
&+ \frac{N_c}{16\pi^2}M_c^2\ \mathrm{tr}T_2\,[\,(\hat{\Phi}\hat{\Phi}^\dagger - \bar{M}^2) + (\hat{\Phi}^\dagger\hat{\Phi} - \bar{M}^2)\,] \\
&+ \mu_m^2\mathrm{tr}\left(\Phi M^\dagger + M\Phi^\dagger\right) - \mu_f^2\mathrm{tr}\Phi\Phi^\dagger + \mu_{\rm inst}\left(\det\Phi + h.c.\right)
\end{aligned}$$

with

$$\hat{\Phi} = \Phi - M, \qquad \bar{M} = V - M = \mathrm{diag.}(\bar{m}_1, \bar{m}_2, \bar{m}_3),$$

$$\mathbf{V} = \mathbf{diag.}(\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}), \qquad \bar{\mathbf{m}}_\mathbf{i} = \mathbf{v_i} - \mathbf{m_i}$$

$$\mathbf{T_0} = \mathbf{diag.}(\mathbf{T_0^{(1)}}, \mathbf{T_0^{(2)}}, \mathbf{T_0^{(3)}}), \qquad \mathbf{T_2} = \mathbf{diag.}(\mathbf{T_2^{(1)}}, \mathbf{T_2^{(2)}}, \mathbf{T_2^{(3)}})$$

and

$$\begin{aligned}
\mathbf{T_0^{(i)}}(\frac{\mu_\mathbf{i}^2}{\mathbf{M_c^2}}) &= \ln \frac{\mathbf{M_c^2}}{\mu_\mathbf{i}^2} - \gamma_\mathbf{w} + \mathbf{y_0}(\frac{\mu_\mathbf{i}^2}{\mathbf{M_c^2}}), \quad \mu_\mathbf{i}^2 = \mu_\mathbf{s}^2 + \bar{\mathbf{m}}_\mathbf{i}^2 \\
\mathbf{T_2^{(i)}}(\frac{\mu_\mathbf{i}^2}{\mathbf{M_c^2}}) &= \mathbf{1} - \frac{\mu_\mathbf{i}^2}{\mathbf{M_c^2}}[\ \ln \frac{\mathbf{M_c^2}}{\mu_\mathbf{i}^2} - \gamma_\mathbf{w} + \mathbf{1} + \mathbf{y_2}(\frac{\mu_\mathbf{i}^2}{\mathbf{M_c^2}})\ ]
\end{aligned}$$

Dynamically Spontaneous Symmetry Breaking

Dynamically Generated Effective Potential

$$\mathbf{V_{eff}(\Phi)} = -\mathbf{tr}\hat{\mu}_{\mathbf{m}}^{\mathbf{2}}\left(\mathbf{\Phi M^\dagger + M\Phi^\dagger}\right) + \frac{\mathbf{1}}{\mathbf{2}}\mathbf{tr}\hat{\mu}_{\mathbf{f}}^{\mathbf{2}}(\mathbf{\Phi\Phi^\dagger + \Phi^\dagger\Phi})$$
$$+ \frac{\mathbf{1}}{\mathbf{2}}\mathbf{tr}\lambda[\,(\hat{\mathbf{\Phi}}\hat{\mathbf{\Phi}}^\dagger)^{\mathbf{2}} + (\hat{\mathbf{\Phi}}^\dagger\hat{\mathbf{\Phi}})^{\mathbf{2}}\,] - \mu_{\mathbf{inst}}\left(\det\mathbf{\Phi} + \mathbf{h.c.}\right)$$

with $\hat{\mu}_f^2$, $\hat{\mu}_m^2$ and λ the three diagonal matrices

$$\hat{\mu}_{\mathbf{f}}^{\mathbf{2}} = \mu_{\mathbf{f}}^{\mathbf{2}} - \frac{\mathbf{N_c}}{\mathbf{8}\pi^{\mathbf{2}}}\left(\mathbf{M_c^2T_2} + \bar{\mathbf{M}}^{\mathbf{2}}\mathbf{T_0}\right)$$
$$\hat{\mu}_{\mathbf{m}}^{\mathbf{2}} = \mu_{\mathbf{m}}^{\mathbf{2}} - \frac{\mathbf{N_c}}{\mathbf{8}\pi^{\mathbf{2}}}\left(\mathbf{M_c^2T_2} + \bar{\mathbf{M}}^{\mathbf{2}}\mathbf{T_0}\right), \quad \lambda = \frac{\mathbf{N_c}}{\mathbf{16}\pi^{\mathbf{2}}}\mathbf{T_0}$$

Spontaneous Symmetry Breaking

Vacuum Expectation Values (VEVs)

$$\Phi(x) = \xi_L(x)\phi(x)\xi_R^\dagger(x), \quad \phi(x) = V + \varphi(x), \quad V =< \phi >= \mathrm{diag.}(v_1, v_2, v_3)$$

Minimal Conditions/Generalized Gap Equations

$$-\left(\hat{\mu}_f^2\right)_i v_i + \left(\hat{\mu}_m^2\right)_i m_i - 2\lambda_i \bar{m}_i^3 + \mu_{inst}\bar{v}^3/v_i = 0, \quad i = 1, 2, 3, \quad \bar{v}^3 = v_1 v_2 v_3$$

Expanding in terms of the quark masses m_i

$$\left(\hat{\mu}_f^2\right)_i = \bar{\mu}_f^2 + 2\mu_{fo}\tilde{m}_i[\ 1 + \sum_{k=1} \alpha_k \left(\frac{\tilde{m}_i}{\mu_o}\right)^k\ ]$$

$$\left(\hat{\mu}_m^2\right)_i = \bar{\mu}_m^2 + 2\mu_{fo}\tilde{m}_i[\ 1 + \sum_{k=1} \alpha_k \left(\frac{\tilde{m}_i}{\mu_o}\right)^k\ ]$$

$$\lambda_i = \bar{\lambda} - \lambda_o \sum_{k=1} \beta_k \left(\frac{\tilde{m}_i}{\mu_o}\right)^k, \qquad \lambda_o = \frac{N_c}{16\pi^2}$$

$$\mu_o^2 = \mu_s^2 + v_o^2, \qquad \tilde{m}_i = m_i[1 + m_i/(2v_o)]$$

The Unknown 5 + 3 Parameters $\bar{\mu}_f^2$, μ_{inst}, v_o, μ_s, $\bar{\lambda}$, m_u, m_d, m_s

Three Constraints

$$3v_o(1 - \epsilon_o)/4 \simeq v_{inst}$$

$$2\bar{\lambda}(v_{inst}v_3 - v_o^2) \simeq \bar{\mu}_f^2$$

$$10\bar{\lambda}v_o^2 + 2\lambda_o v_o^2(1 - \frac{2v_o^2}{\mu_o^2}) + 4\bar{\mu}_f^2 \simeq \bar{\mu}_m^2$$

with

$$\epsilon_o = \frac{\lambda_o}{\bar{\lambda}}\left[\ \left(\frac{2v_o^2}{\mu_o^2} - 1\right)\left(1 - \frac{1}{3}\frac{v_o^2}{\mu_o^2}\right) - \frac{1}{3}\frac{2v_o}{\mu_o}\alpha_1(1-r) + r + \left(\frac{2v_o^2}{\mu_o^2} - \frac{1}{3}\frac{\bar{\lambda}}{\lambda_o}\right)\frac{m_s}{v_o}\ \right]$$

$$\mu_{fo} \equiv 2\lambda_o v_o(1-r), \quad \mu_{inst} \equiv 2\bar{\lambda} v_{inst}$$

Here r and α_1 are given by

$$r = \frac{\mu_s^2}{\mu_o^2} - \frac{\mu_o^2}{M_c^2}[1 + \frac{\mu_s^2}{\mu_o^2} + O(\frac{\mu_o^2}{M_c^2})]$$

$$\alpha_1(1-r) = \frac{2v_o}{\mu_o}\ [\ \frac{\mu_s^2}{2\mu_o^2} + O(\frac{\mu_o^2}{M_c^2})\ ]$$

Parameter Definitions

$$\bar{\mu}_f^2 = \mu_f^2 - \frac{N_c}{8\pi^2}\left(M_c^2 T_2(\frac{\mu_o^2}{M_c^2}) + v_o^2 T_0(\frac{\mu_o^2}{M_c^2})\right)$$

$$\bar{\mu}_m^2 = \mu_m^2 - \frac{N_c}{8\pi^2}\left(M_c^2 T_2(\frac{\mu_o^2}{M_c^2}) + v_o^2 T_0(\frac{\mu_o^2}{M_c^2})\right)$$

$$\bar{\lambda} = \frac{N_c}{16\pi^2} T_0(\frac{\mu_o^2}{M_c^2}), \qquad T_0(\frac{\mu_o^2}{M_c^2}) = \ln\frac{M_c^2}{\mu_o^2} - \gamma_w + y_0(\frac{\mu_o^2}{M_c^2})$$

Gap Equation without Instanton ($v_{inst} = 0$)

$$\frac{N_c}{8\pi^2\mu_f^2}\left[\, M_c^2 - \mu_o^2\left(\ln\frac{M_c^2}{\mu_o^2} - \gamma_w + 1 + y_2(\frac{\mu_o^2}{M_c^2})\right)\,\right] = 1$$

Lightest Composite Higgs Bosons & Mass Formulations

The Composite Higgs Bosons - Scalar Mesons and Would-be Goldstone Bosons-Pseudoscalar Mesons

$$\sqrt{\mathbf{2}}\varphi = \begin{pmatrix} \frac{a_0^0}{\sqrt{2}} + \frac{1}{\sqrt{6}} f_8 + \sqrt{\frac{1}{3}} f_s & a_0^+ & \kappa_0^+ \\ a_0^- & -\frac{a_0^0}{\sqrt{2}} + \frac{1}{\sqrt{6}} f_8 + \sqrt{\frac{1}{3}} f_s & \kappa_0^0 \\ \kappa_0^- & \bar{\kappa}_0^0 & -\frac{2}{\sqrt{6}} f_8 + \sqrt{\frac{1}{3}} f_s \end{pmatrix},$$

$$\sqrt{\mathbf{2}}\mathbf{\Pi} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{1}{\sqrt{6}} \eta_8 + \sqrt{\frac{1}{3}} \eta_0 & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{1}{\sqrt{6}} \eta_8 + \sqrt{\frac{1}{3}} \eta_0 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta_8 + \sqrt{\frac{1}{3}} \eta_0 \end{pmatrix},$$

Mass Formulations: Pseudoscalar Mesons at Leading Order

$$\mathbf{m^2_{\pi^\pm} \simeq \frac{2\mu_P^3}{f^2}(m_u + m_d)}$$

$$\mathbf{m^2_{K^\pm} \simeq \frac{2\mu_P^3}{f^2}(m_u + m_s)}$$

$$\mathbf{m^2_{K^0} \simeq \frac{2\mu_P^3}{f^2}(m_d + m_s)}$$

$$\mathbf{m^2_{\eta_8} \simeq \frac{2\mu_P^3}{f^2}[\ \frac{1}{3}(m_u + m_d) + \frac{4}{3}m_s\ ] = \frac{1}{3}(4m_K^2 - m_\pi^2)}$$

$$\mathbf{m^2_{\eta_8\eta_0} \simeq -\frac{2\mu_P^3}{f^2}\frac{\sqrt{2}}{3}[\ 2m_s - (m_u + m_d)\ ] = -\frac{2\sqrt{2}}{3}(m_K^2 - m_\pi^2)}$$

$$\mathbf{m^2_{\eta_0} \simeq \frac{2\mu_P^3}{f^2}\frac{2}{3}(m_u + m_d + m_s) + \frac{12\bar{v}^3}{f^2}\mu_{inst} = \frac{1}{3}(2m_K^2 + m_\pi^2) + \frac{24\bar{v}^3}{f^2}\bar{\lambda}v_{inst}}$$

$$\mu_P^3 = (\bar{\mu}_m^2 + 2\bar{\lambda}v_o^2)v_o \simeq 12\bar{\lambda}v_o^3 \simeq 3v_o f^2$$

Scalar Mesons - Lightest Composite Higgs Bosons

$$m^2_{a_0^\pm} \simeq m^2_{a_0^0} \simeq 2(2\bar{m}_u + \bar{m}_d)\bar{m}_u + 2v_{inst}v_3 \sim 8v_o^2$$

$$m^2_{k_0^\pm} \simeq 2(2\bar{m}_u + \bar{m}_s)\bar{m}_u + 2v_{inst}v_2 \sim 8v_o^2$$

$$m^2_{k_0^0} \simeq 2(2\bar{m}_d + \bar{m}_s)\bar{m}_d + 2v_{inst}v_1 \sim 8v_o^2$$

$$m^2_{f_8} \simeq \bar{m}_u^2 + \bar{m}_d^2 + 4\bar{m}_s^2 + \frac{2}{3}v_{inst}(2v_1 + 2v_2 - v_3) \sim 8v_o^2$$

$$m^2_{f_s} \simeq 2(\bar{m}_u^2 + \bar{m}_d^2 + \bar{m}_s^2) - \frac{4}{3}v_{inst}(v_1 + v_2 + v_3) \sim 2v_o^2$$

$$m^2_{f_s f_8} \simeq \sqrt{2}(2\bar{m}_s^2 - \bar{m}_u^2 - \bar{m}_d^2) - \frac{\sqrt{2}}{3}v_{inst}(2v_3 - v_1 - v_2) \sim 0$$

Mixing Angles

$$\tan 2\theta_P = 2\sqrt{2}[1 - \frac{9v_{inst}v_3}{m_K^2 - m_{\pi^2}}]^{-1}$$

$$\tan 2\theta_S = \frac{2m^2_{f_s f_8}}{m^2_{f_s} - m^2_{f_8}}$$

Two More Constraints

Normalization Condition from Kinetic Term

$$\bar{\lambda} v_o^2 = f^2/4, \quad T_0 v_o^2 = \frac{(4\pi f)^2}{4N_c} \equiv \bar{\Lambda}_f^2 \simeq (340\text{MeV})^2$$

Trace of Mass Matrix

$$v_{inst}v_3 = \frac{1}{6}(m^2_{\eta_8} + m^2_{\eta_0} - 2m^2_K) = \frac{1}{6}(m^2_\eta + m^2_{\eta'} - 2m^2_K)$$

Numerical Results

Input Parameters

$$f_\pi = 94\text{MeV} \qquad v_o = 340\text{MeV}$$

$$m_u \simeq 3.8\text{MeV} \qquad m_d \simeq 5.7\text{MeV} \qquad m_s/m_d \simeq 20.5$$

Output Predictions

$$\mu_f \simeq 144\text{MeV}, \qquad \mu_{inst} \simeq 8.0\text{MeV}$$

$$M_c \simeq 922\text{MeV}, \qquad \mu_s \simeq 333\text{MeV}$$

$$< \bar{u}u > \simeq < \bar{d}d > \simeq < \bar{s}s > = -(242\text{MeV})^3$$

$$m_\pi \simeq 139\text{MeV}, \qquad m_\pi|_{exp} \simeq 139\text{MeV}$$

$$m_{K^0} \simeq 500\text{MeV}, \qquad m_{K^0}|_{exp} \simeq 500\text{MeV}$$

$$\mathbf{m_{K^\pm} \simeq 496MeV} \qquad \mathbf{m_{K^\pm}|_{exp} \simeq 496MeV}$$

$$\mathbf{m_\eta \simeq 503MeV}, \qquad \mathbf{m_\eta|_{exp} \simeq 548MeV}$$

$$\mathbf{m_{\eta'} \simeq 986MeV}, \qquad \mathbf{m_{\eta'}|_{exp} \simeq 958MeV}$$

$$\mathbf{m_{a_0} \simeq 978\ MeV}, \qquad \mathbf{m_{a_0}^{exp.} = 984.8 \pm 1.4\ MeV \quad PDG}$$

$$\mathbf{m_{\kappa_0} \simeq 970\ MeV}, \qquad \mathbf{m_{\kappa_0}^{exp.} = 797 \pm 19 \pm 43\ MeV \quad E7912}$$

$$\mathbf{m_{f_0} \simeq 1126\ MeV}, \qquad \mathbf{m_{f_0}^{epx.} = 980 \pm 10\ MeV \quad PDG}$$

$$\mathbf{m_\sigma \simeq 677\ MeV}, \qquad \mathbf{m_\sigma^{exp.} = (400 - 1200)\ MeV \quad PDG}$$

$$\theta_\mathbf{P} \simeq -\mathbf{18^o}, \qquad \theta_\mathbf{S} \simeq -\mathbf{18^o}$$

$$\eta_\mathbf{8} = \cos\theta_\mathbf{P}\ \eta + \sin\theta_\mathbf{P}\ \eta'$$

$$\eta_\mathbf{0} = \cos\theta_\mathbf{P}\ \eta' - \sin\theta_\mathbf{P}\ \eta$$

$$\mathbf{f_8} = \cos\theta_\mathbf{S}\ \mathbf{f_0} + \sin\theta_\mathbf{S}\ \sigma$$

$$\mathbf{f_s} = \cos\theta_\mathbf{S}\ \sigma - \sin\theta_\mathbf{S}\ \mathbf{f_0}$$

Conclusions & Remarks

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Conclusions & Remarks

- **Symmetry-Preserving Loop Regularization is a Useful Tool for Underlying and Effective QFTs**

- **Low Energy Dynamics of QCD Can Well be Described by Nonlinearly Realized Effective Chiral Theory**

- **Dynamically Spontaneous Symmetry Breaking Mechanism Can well be Established in Strong Interactions**

- **Lightest Scalar Mesons are Truly the Composite Higgs Bosons that Have First been Observed in Nature**

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- **The Symmetry Breaking Mechanism may be applicable to Other Theories, Such as: Electroweak symmetry breaking , Symmetry breaking in GUTs / SUSY**