Pentaquark States

—— A Brief Overview

X.C. Song ITP PKU

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一. Experimental facts and Motivation

二. Some Theoretical Models

三. Several Theoretical Ideas

四. Discussions

中国科技大学 上海研究院
USTC-SIAS
USTC SHANGHAI INSTITUTE FOR ADVANCED STUDIES
中国上海浦东秀浦路99号
Add:No.99,South xiupu,shanghai
TEL: +86 21 68120000 FAX: +86 21 68121320
P.C.:201315 http://www.ustc-sias.sh.cn

1. Experimental Facts

LEPS 2003

2.4 GeV

$$\gamma + n \longrightarrow K^- + \underbrace{K^+ + n}_{\Theta^+}$$

$M = (1540 \pm 10)$ MeV

$\Gamma \leq 25$ MeV 4.6σ

DIANA $K^+ Xe \longrightarrow \underbrace{K^0 p}\, Xe'$ 1539 ± 2, < 9 4.4σ

CLAS $\gamma d \longrightarrow K^- \underbrace{K^+ n}\, p$ 1542 ± 5 ~ 21 MeV 5.3σ

SAPHIR . . . .

Θ $K^+ n$, $K^0 p$ Exotic Baryon !!!

$B = 1$, $S = +1$ $u^2 d^2 \bar{s}$

Ξ^{--} NA49 at CERN

$M = 1862$ $s^2 d^2 \bar{u}$

$\Gamma < 18$

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$Y = B + S$ $\qquad$ $Q = I_3 + \frac{1}{2} Y$

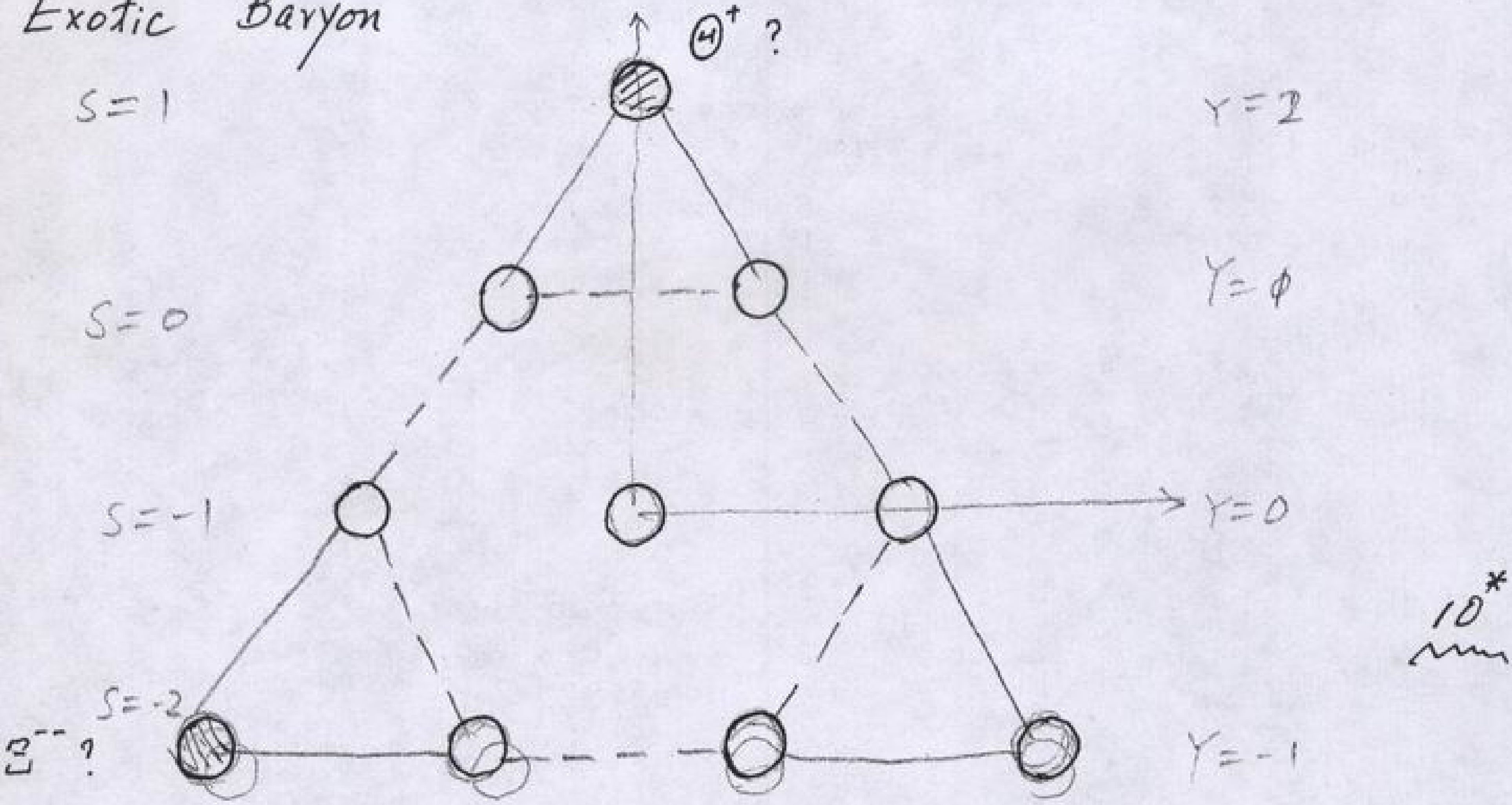
Baryon $J^P = \frac{1}{2}^+$ $\qquad$ $B=1$ $\qquad$ $J^P = \frac{3}{2}^+$

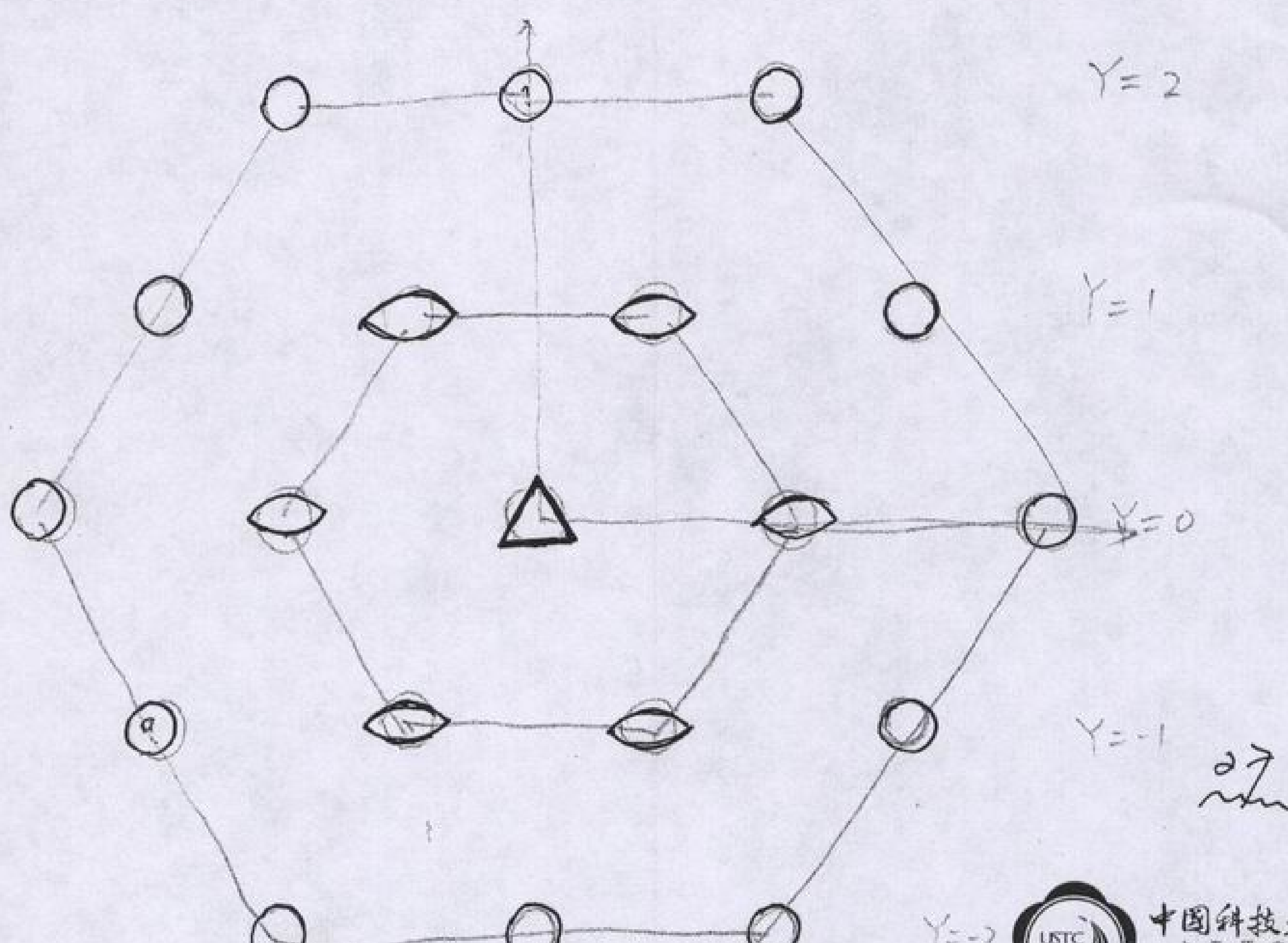
$S=0$		n	p		$Y=1$	Δ^-	Δ^0	Δ^+	Δ^{++}	$S=0$
$S=-1$	Σ^-	Λ Σ^0		Σ^+	$Y=0$	Σ^{*-}	Σ^{*0}	Σ^{*+}		$S=-1$
$S=-2$		Ξ^-	Ξ^0		$Y=-1$	Ξ^{*-}	Ξ^{*0}			$S=-2$
					$Y=-2$	Ω^-				$S=-3$

Meson $J^P = 0^-$ $\qquad$ $B=0$ $\qquad$ $J^P = 1^-$

$S=1$		K^0	K^+		$Y=1$	K^{*0}	K^{*+}		$S=1$
$S=0$	π^-	η π^0		π^+	$Y=0$	ρ^-	ω ρ^0	ρ^+	$S=0$
$S=-1$		K^-	$\bar{K}^0$		$Y=-1$	K^{*-}	$\bar{K}^{*0}$		$S=-1$

Exotic Baryon





二. Theoretical Models

2.1 Skyrmion model

Skyrmion PRS (London) $\underline{A}$ 260 ('61) 127

Adkins, Nappi, Witten NP B228 ('83) 552

The canonical form is the SU(2) hedgehog imbedded in SU(3)

$$U(\vec{r}) = \exp\{ i\vec{\lambda}\cdot\hat{r}\, F(r)\}$$

$$\vec{\lambda} = (\lambda_1, \lambda_2, \lambda_3) = \begin{pmatrix} \vec{\tau} & 0 \\ 0 & 0 \end{pmatrix}$$

F(r) represents the radial shape of the soliton with B.C

$$F(r=0) = \pi$$

$$F(r\to\infty) = 0$$

so that the topological winding number is equal to 1 which is identified with the baryon number B.

"Rigid rotation quantization" to specify spin and flavor

Chemtob NP B256

Lowest-lying states $\underline{8}$ $\frac{1}{2}^+$ Consistent with quark model (1985) 800.

$\underline{10}$ $\frac{3}{2}^+$

Next possible states $\underline{\overline{10}}$ $\frac{1}{2}^+$ $\in \Theta$

$\uparrow$

$N^*(1710)$

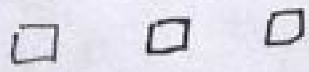
DPP Z Phys. A 359 (1997) 305 predict

$M = 1530 \quad \Gamma \leq 15\ \mathrm{MeV}$

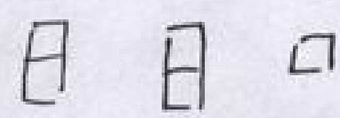
Yan '95

Ma '99

2.2 Quark Model

q 

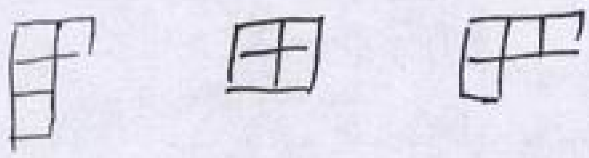
3 $\underset{(2)}{\underline{3}}$ 2

$\bar{q}$ 

3^* $\underset{(2)}{\underline{3^*}}$ 2

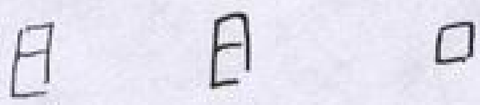
Consider Θ^+ $uudd\bar{s}$

tetra-quark system combining with $\bar{q}(3^*, 3^*, 2)$ to form Θ^+

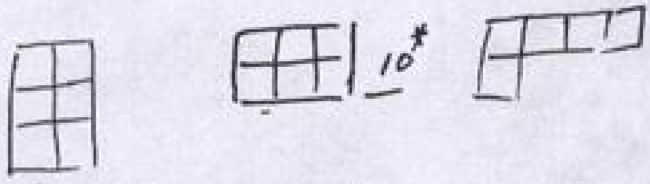
q^4: 

3 $\underset{(1)}{\underline{6^*}}$ 3

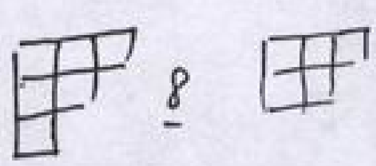
spin-orbit symmetry

$\bar{q}$ 

3^*

$q^4\bar{q}$ 

$\underline{10^*}$ 4 $J=\frac{3}{2}$



$\underline{8}$ 2 $J=\frac{1}{2}$

1

2.2a Diquark assumption

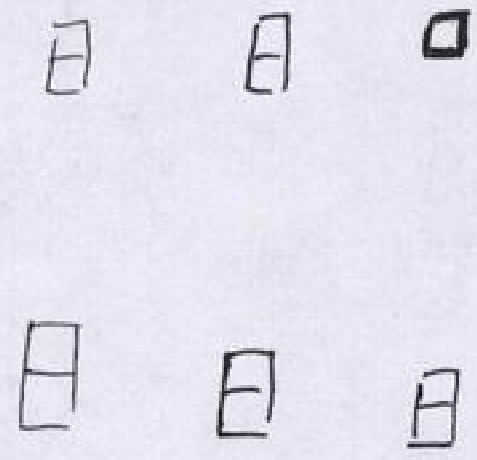
Jaffe Wilczek PRL 91 (2003) 55

$$\bar{s}$$

ud ud

(ud) − (ud)

3^* 3^* 2



3^* 3^* 1

$3^* \times 3^*$ $\underline{3^* \times 3^*}$ 1×1

$\cup$ $\cup$ $\cup$

3 6^* 1

反称 对称 对称

(ud)整体作为玻色体系必须对称

必须有空间反对称

p波

(udud) $(3, 6^*, 3)$

$\bar{s}$ $(3^*, 3^*, 2)$

4⁺ $\begin{pmatrix} 1, & 10^* & 4 \\ & 8 & 2 \end{pmatrix}$ $\begin{matrix} \frac{3}{2}^+ \\ \frac{1}{2}^+ \end{matrix}$

色交换作用 $\vec{\lambda}_c \cdot \vec{\lambda}_c$

味交换作用 $\vec{\lambda}_f \cdot \vec{\lambda}_f$

自旋交换作用 $\vec{\sigma} \cdot \vec{\sigma}$

quark-instanton coupling KMT PRL 37 (1976) 8.

2.2.b Cluster assumption

Karliner Lipkin PL B575 (2003), 249

$$ud \; \cdots \overset{l=1}{\cdots\cdots} \; ud\bar{s}$$

2.2.c Chiral Bag Model

Hosaka PL B571 (2003), 55.

(2.1) and (2.2) favor Θ^+ spin parity

$$\frac{1}{2}^+$$

2.3. QCD Sum Rules

Zhu PRL 91 (2003), 232002

Matheus et al., PL B578 (2004), 323

Sugiyama PL B581 (2004), 167

$J^P = 0^+$

$$\bar{S}^a = \varepsilon^{abc}(u_b^T C d_c)$$

$J^P = 0^-$

$$\bar{S}_5^a = \varepsilon^{abc}(u_b^T C \gamma^5 d_c)$$

$$\bar{U}^a = \varepsilon^{abc}(d_b^T C s_c) \qquad \bar{D}^a = \varepsilon^{abc}(s_b^T C u_c) \qquad \bar{U}_5^a = \varepsilon^{abc}(d_b^T C \gamma^5 s_c) \qquad \bar{D}_5^a = \varepsilon^{abc}(s_b^T C \gamma^5 u_c$$

$$\Theta^+ = \varepsilon_{abc} \bar{S}^a \bar{S}_5^b C (\bar{s}^T)^c$$

$$(NK)^+ = \varepsilon^{abc}\left([u_a^T C \gamma^5 d_b] u_c (\bar{s}^d \gamma^5 d_d) + (u \leftrightarrow d) \right)$$

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step 1 Chose the appropriate local interpolating operator, e.g.,

$$J(x) = \varepsilon_{abc} \bar{S}^a \bar{S}^b_5 C (\bar{S}^c)^T$$

$$\bar{S}^a = \varepsilon^{abc} (u_b^T C d_c), \quad \bar{S}^b_5 = \varepsilon^{bcd} (u_c^T C \gamma^5 d_d)$$

or

$$\eta_0(x) = \varepsilon^{abc} (u_a^T C \gamma^5 d_b) \left\{ u_e \bar{S}^e i\gamma^5 d_c - (u \leftrightarrow d) \right\}$$

Such a choice is evidently not unique,

different choice are even not orthogonal.

step 2 Using the operator product expansion to calculate the correlation function

$$\Pi(q) = i \int d^4x \, e^{ipx} \langle 0 | T(J(x) J(0)) | 0 \rangle$$

step 3 Equating the function $\Pi(q)$ at the hadron level as sum of some spectral density on the one hand and the condensate expression at the quark-gluon level on the other hand to give the sum rules

$$\int dq_0 \, W(q_0) \, \rho_{Phen.}(q_0) = \int dq_0 \, W(q_0) \, \rho_{OPE}(q_0)$$

step 4 Inserting the numerical value of various condensates from other experimental data to calculate the mass, width or other quantities.

$$\begin{aligned} \langle s\bar{s} \rangle &= \cdots \\ \langle g_s^2 GG \rangle &= \cdots \\ m_s &= \cdots \end{aligned} \quad \Longrightarrow \quad \begin{aligned} M &= \cdots \\ \Gamma &= \cdots \end{aligned}$$

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2.4 Lattice QCD

Csikor et al., J. High Energy Phys. 11 (2003), 070

Sasaki hep-lat/0310014

The majority of the QCD calculation, (2.3) & (2.4), agree that the ground state is characterized by $J=1/2$ $T=0$ and negative parity,

while the positive parity state has a mass that is greater at least by a few hundred MeV.

2.5 Pentaquark Baryon in String Theory

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Ξ. Several Ideas

Pauli Matrices

$$\sigma_1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} & -i \\ i & \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \qquad \sigma_0 = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

Orthonomality

$$\mathrm{Tr}(\sigma_a \sigma_b) = 2\delta_{ab} \qquad (\sigma_a)_i{}^j (\sigma_b)_j{}^i = 2\delta_{ab}$$

Completeness

$$\sum_{a=0}^{3} \sigma_a \otimes \sigma_a = 2P \qquad \sum_{a=0}^{3} (\sigma_a)_i{}^j (\sigma_a)_k{}^l = 2P_{ik}{}^{jl} = 2\delta_i{}^l \delta_k{}^j$$

$$\sigma_1 \otimes \sigma_1 = \begin{pmatrix} & & & 1 \\ & & 1 & \\ & 1 & & \\ 1 & & & \end{pmatrix} \qquad \sigma_2 \otimes \sigma_2 = \begin{pmatrix} & & & -1 \\ & & 1 & \\ & 1 & & \\ -1 & & & \end{pmatrix}$$

$$\sigma_3 \otimes \sigma_3 = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix} \qquad \sigma_0 \otimes \sigma_0 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$\sum_{a=0}^{3} \sigma_a \otimes \sigma_a = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} = 2P$$

$$\boxed{\sum_{a=1}^{3} (\sigma_a)_i{}^j (\sigma_a)_k{}^l = 2\delta_i{}^l \delta_k{}^j - \delta_i{}^j \delta_k{}^l}$$

SU(3) Gell-Mann Matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & \\ 1 & 0 & \\ & & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & \\ i & 0 & \\ & & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_8 = \sqrt{\tfrac{1}{3}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_0 = \sqrt{\tfrac{2}{3}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

Orthonomality

$$\mathrm{Tr}(\lambda_a \lambda_b) = 2\delta_{ab} \qquad (\lambda_a)_i^{\ j} (\lambda_b)_j^{\ i} = 2\delta_{ab}$$

Completeness

$$\sum_{a=0}^{8} (\lambda_a)_i^{\ j} (\lambda_a)_k^{\ l} = 2 P_{ik}^{\ jl} = 2\delta_i^{\ l} \delta_k^{\ j}$$

$$\boxed{\sum_{a=1}^{8} (\lambda_a)_i^{\ j} (\lambda_a)_k^{\ l} = 2\delta_i^{\ l} \delta_k^{\ j} - \frac{2}{3}\delta_i^{\ j} \delta_k^{\ l}}$$

1. Color Singlet space for pentaquark states

QCD $SU(3)_c$

Ordinary hadrons

Baryon : qqq Meson : $q\bar{q}$

$3 \times 3 \times 3 = 1 + 8 + 8 + 10$ $3 \times 3^* = 1 + 8$

Exotic Baryons $qqqq\bar{q}$ # singlets ?

$3 \times 3 \times 3 \times 3 \times 3^*$

$= (3 \times 3 \times 3) \times (3 \times 3^*)$

$= (1 + 8 + 8 + 10) \times (1 + 8)$

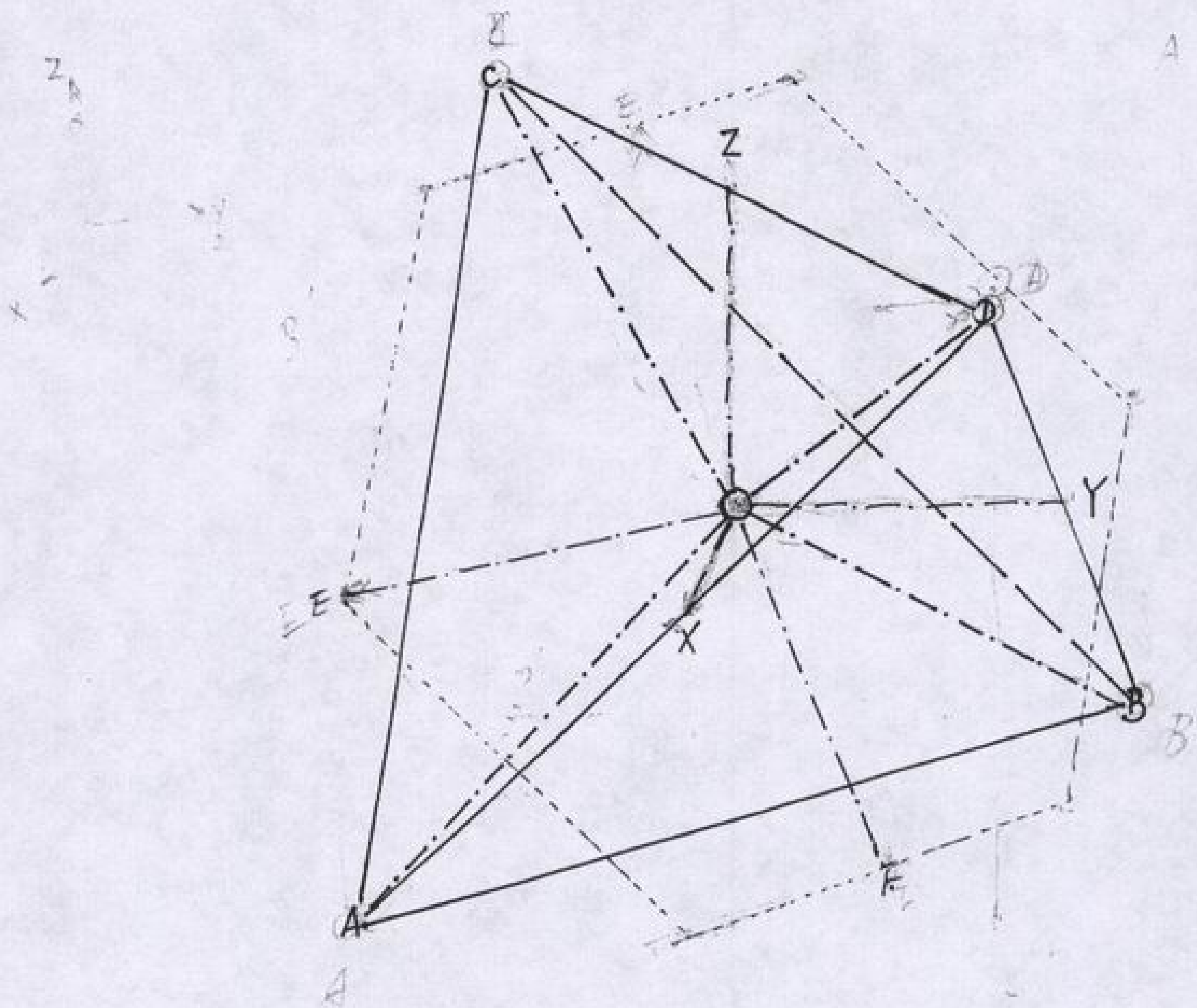
1 1 1 $\# = 3$

q^4 可以组成四个单态,

它们之间存在一个关系!!!

$$\boxed{\begin{array}{l} \varepsilon_{abc}\,\delta_d^{\,e} - \varepsilon_{abd}\,\delta_c^{\,e} + \varepsilon_{acd}\,\delta_b^{\,e} - \varepsilon_{bcd}\,\delta_a^{\,e} = 0 \quad * \\ \overrightarrow{OD} \quad + \quad \overrightarrow{OC} \quad + \quad \overrightarrow{OB} \quad + \quad \overrightarrow{OA} \quad = 0 \end{array}}$$

Color singlet space in 3-dim.



A(1,-1,-1)

X(1 0 0)

B(-1,1,-1

E(1,-1,-0)

Y(0 1 0)

C(-1,-1,1

F($\frac{1}{2}$, $-\frac{1}{2}$, -1)

Z(0 0 1)

D(1 1 1)

SU(2)

$$(\sigma_a \varepsilon)_{ij} \doteq (\sigma_a)_i{}^k \varepsilon_{kj} = (\sigma_a\varepsilon)_{ji} \qquad * \qquad \varepsilon^{ij}\varepsilon_{ik} = \delta^j_k$$

The antisymmetric combination has only on independent component $\forall a = 1,2,3$

$$(\sigma_a\varepsilon)_{ij} - (\sigma_a\varepsilon)_{ji} \doteq (T_a)_{ij} = -(T_a)_{ji} = 0$$

$$\tfrac{1}{2}\varepsilon^{ij}(T_a)_{ij} = \varepsilon^{ij}(\sigma_a)_i{}^k\varepsilon_{kj} = (\sigma_a)_i{}^k\delta^i_k = \mathrm{Tr}\,\sigma_a = 0.$$

Similarly, in SU(3),

$(\lambda_a\varepsilon)_{ijk} = (\lambda_a)_i{}^n\varepsilon_{njk}$ has 9 independent components $\forall a = 1-8$

And the totally antisymmetric combination has only one component

$$(\lambda_a)_i{}^n\varepsilon_{njk} + (\lambda_a)_j{}^n\varepsilon_{nki} + (\lambda_a)_k{}^n\varepsilon_{nij} = 0$$

Since

$$\varepsilon^{ijk}(\lambda_a\varepsilon)_{ijk} = \varepsilon^{ijk}(\lambda_a)_i{}^n\varepsilon_{njk} = 2\,\delta_n{}^i(\lambda_a)_i{}^n = 2\,\mathrm{Tr}\,\lambda_a = 0 \qquad \forall a = 1-8$$

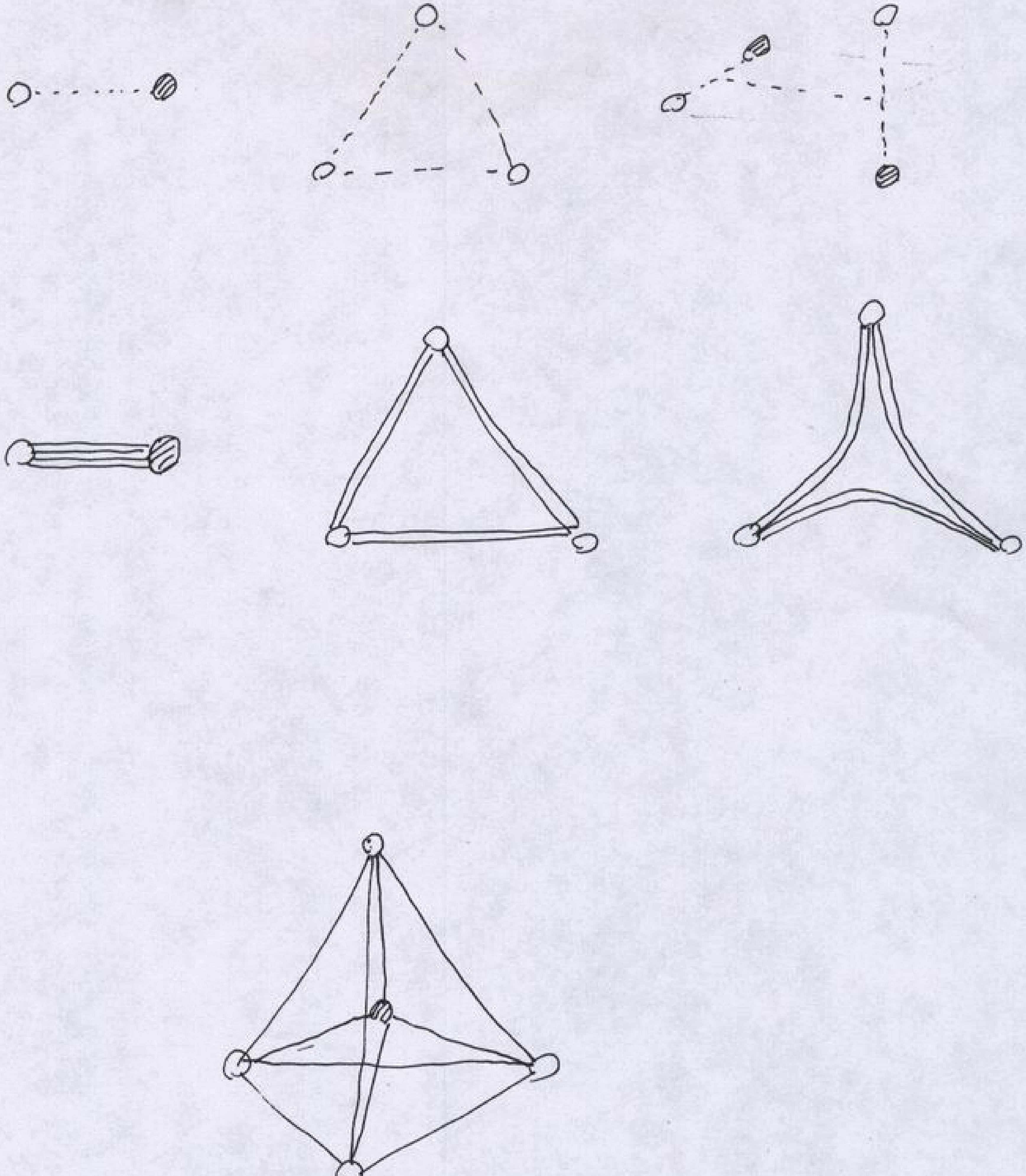
Multiplying $\sum_a (\lambda_a)_\ell{}^m$, we obtain

$$(2\delta_\ell{}^n\delta_i{}^m - \tfrac{2}{3}\delta_i{}^n\delta_\ell{}^m)\varepsilon_{njk}$$
$$+ (2\delta_\ell{}^n\delta_j{}^m - \tfrac{2}{3}\delta_j{}^n\delta_\ell{}^m)\varepsilon_{nki}$$
$$+ (2\delta_\ell{}^n\delta_k{}^m - \tfrac{2}{3}\delta_k{}^n\delta_\ell{}^m)\varepsilon_{nij} = 0$$

$$\varepsilon_{\ell jk}\delta_i{}^m + \varepsilon_{\ell ki}\delta_j{}^m + \varepsilon_{\ell ij}\delta_k{}^m - \delta_\ell{}^m\varepsilon_{kij} = 0$$

$$\boxed{\varepsilon_{ijk}\delta_\ell{}^m - \varepsilon_{ij\ell}\delta_k{}^m + \varepsilon_{ik\ell}\delta_j{}^m - \varepsilon_{jk\ell}\delta_i{}^m = 0}$$

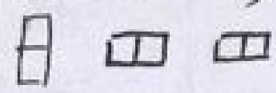
2. Speculation on diamond structure for $q^4\bar{q}$

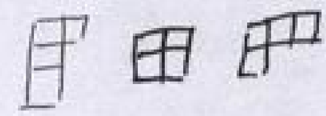


3. Further speculation on diquark structure

q $\qquad\qquad\qquad$ qq $\qquad\qquad\qquad$ $qqqq$

$(3 \quad 3 \quad 2)$ $\qquad\qquad$ $(3^* \quad 6 \quad 3)$ $\qquad\qquad$ $(3 \quad 6^* \quad 3)$





$(3^* \quad 3^* \quad 2)$ $\qquad\qquad$ $(3 \quad 6^* \quad 3)$ $\qquad\qquad$ $(3^* \quad 6 \quad 3)$

$\bar{q}$ $\qquad\qquad\qquad$ $\bar{q}\bar{q}$ $\qquad\qquad\qquad$ $\bar{q}\bar{q}\bar{q}\bar{q}$

Fierz identity

$$(\gamma_R)_\alpha{}^\beta = \left\{ \delta_\alpha{}^\beta, (\gamma_5)_\alpha{}^\beta, (\gamma_\mu)_\alpha{}^\beta, (\gamma_\mu\gamma_5)_\alpha{}^\beta, (\gamma_{\mu\nu})_\alpha{}^\beta \right\}$$

$$\delta_\alpha{}^\beta \delta_\gamma{}^\delta = \frac{1}{4} \sum_{RS} c_{RS} (\gamma_R)_\alpha{}^\delta (\gamma_S)_\gamma{}^\beta$$

$$c_{RS} = \delta_{RS} c_S$$

$$c_R = \left\{ 1, 1, \eta_{\mu\nu}, -\eta_{\mu\nu}, -\frac{1}{4}(\eta_{\mu\rho}\eta_{\nu\kappa} - \eta_{\nu\rho}\eta_{\mu\kappa}) \right\}$$

e.g.

$$\chi_\alpha(\bar{\lambda}\psi) = +\frac{1}{4} \chi_\beta \delta_\alpha^\beta \bar{\lambda}^\gamma \delta_\gamma{}^\delta \psi_\delta$$

$$= \frac{1}{4} \chi_\beta \bar{\lambda}^\gamma \psi_\delta \sum_R (\gamma_R)_\alpha{}^\delta (\gamma_R)_\gamma{}^\beta c_R$$

$$= -\frac{1}{4} \sum_R c_R (\gamma_R\psi)_\alpha (\bar{\lambda}\gamma_R\chi)$$

VII. Discussion

1. Does it exist?

2. What's its Propertie $I = ?$

   $J^P = ?$

3. Other multiquark states ?

4. Need much more idea in understanding the nonperturbative propertie of QCD.

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P.C.:201315 http://www.ustc-sias.sh.cn