

# Brane Creation & Annihilation

## Overview & Current Work

声  
焰  
新

Work done with  
Chen, Roy & Wang

中国科技大学交叉学科理论研究中心  
(& USTC-SIAS ?)

2004. 10. 27



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一. Overview

二. Set up

三. Current work (to appear)

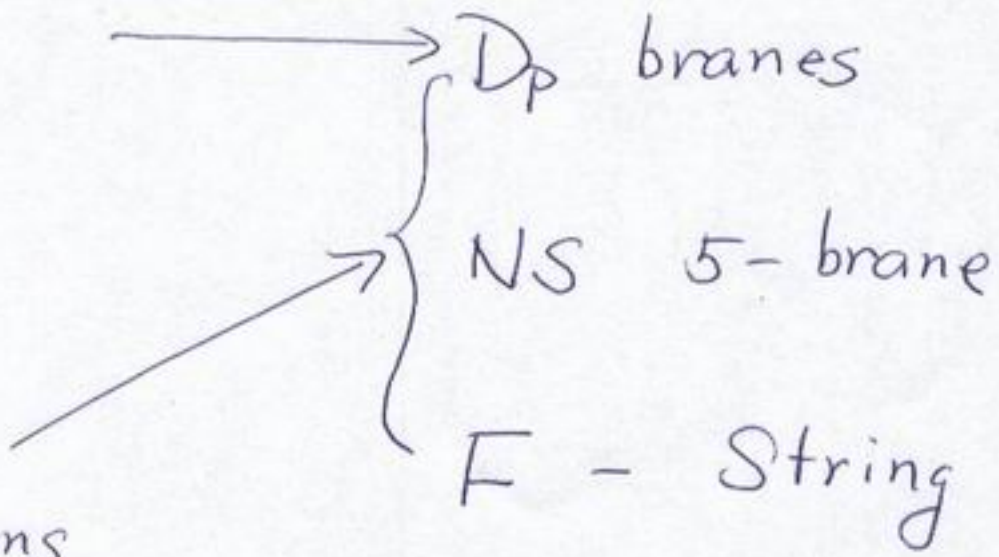
↑ Done with Chen, Roy & Wang



# Overview

String Theory  $\Rightarrow$  Existence of Extended Objects

Open string  $\oplus$   
T-duality



As Solitons

from the low-energy theory of superstring  
i.e., supergravity

Supergravities (low energy theories)  
contains more non-perturbative information  
than the worldsheet action of string



# (Closed) String theories (plus possible open strings)

low energy  
limit

Various Supergravities

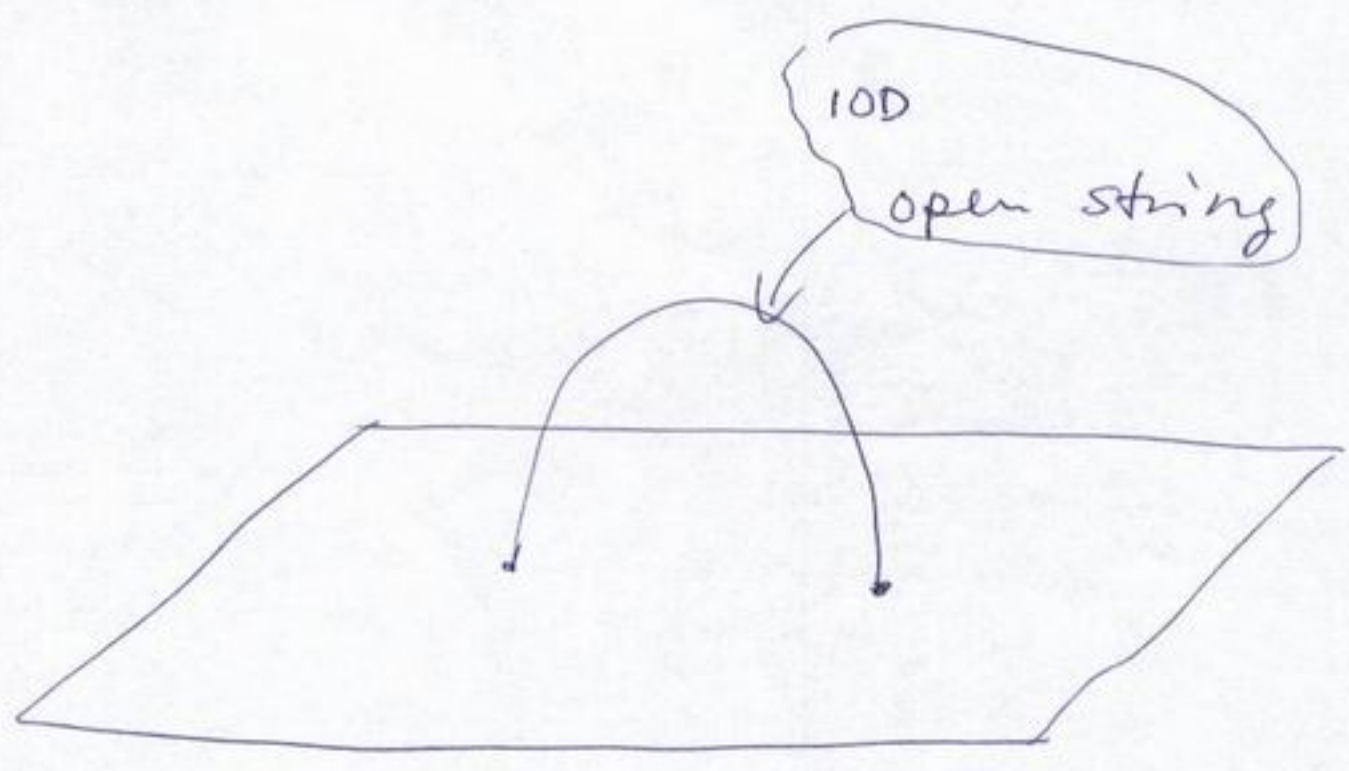
also low-energy theories of non-perturbative String theories

Non-perturbative objects

and other non-perturbative information

D-branes can be described by open string theories with the open string ends obeying the Dirichlet boundary conditions in the directions perpendicular to the brane





The low-energy limit of such open string which describes the dynamics of D-brane

⇒ DBI action

By the same token, we expect

DBI actions contains more,  
 in particular the corresponding non-perturbative  
 information, than the perturbative  
 open string

DBI  $\Rightarrow$  Non-perturbative  
 objects

& other interesting  
 non-perturbative  
 information



On the  $D_p$ -brane, low energy dynamics

$\leftrightarrow$  DBI (SUSY)

Field Content: SUSY Gauge Multiplet

$$\left\{ \begin{array}{l} \Phi^I, F_{\mu\nu} \\ \chi \end{array} \right. \quad \left( \cancel{\lambda} \Phi^I = \cancel{\lambda} X^I \right)$$

$$\cancel{\lambda} = 2\pi\alpha'$$



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Consider BPS configuration,

DBI  $\longrightarrow$  linearized DBI

$\Leftrightarrow$  Super Yang-Mills

$\Leftarrow$  Dimension Reduction  
of 10D Super Yang-Mills

$$\delta \chi = \Gamma_{\mu\nu} F^{\mu\nu} \epsilon + \frac{\Gamma^{\mu I}}{\lambda} \partial_\mu \Phi^I \epsilon \quad \left( \begin{array}{l} 10D \\ \text{notations} \end{array} \right)$$

$$\begin{cases} D_\mu F^{\mu\nu} = 0 \\ \square \Phi^I = 0 \end{cases}$$



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# BPS F-String

(C&M NPB 513 (98) 109 (9)  
Gibbons NPB 514 (98) 663)

$$X^9 = \frac{C_p}{r^{p-2}}, \quad A_0 = \frac{C_p}{r^{p-2}}$$

$$F_{0r} = -\partial_r A_0 = -\partial_r X^9$$

$$\begin{aligned} \delta \mathcal{X} &= \Gamma^{0r} F_{0r} \epsilon + \Gamma^{r9} \partial_r X^9 \epsilon \\ &= -\Gamma^9 (\Gamma^0 + \Gamma^9) \partial_r X^9 \epsilon \\ &= -\Gamma^9 \partial_r X^9 (\Gamma^0 + \Gamma^9) \epsilon = 0 \end{aligned}$$

$$(\Gamma^0 + \Gamma^9) \epsilon = 0 \quad \sim \quad (1 - \Gamma^0 \Gamma^9) \epsilon = 0$$

preserving half  
of SUSY

$$\int_{S^{p-1}} *F = g^n (2\pi)^{p-1}$$

~~$$E(\mathcal{E}) = \frac{1}{(2\pi)^p g} \int d^p r \frac{1}{2} (F_{0r}^2 + (\partial_r X^9)^2)$$~~  
~~$$= \frac{C_p^2 V_{p-1} (p-2)}{(2\pi)^p g} \int \frac{1}{r^2} dr$$~~



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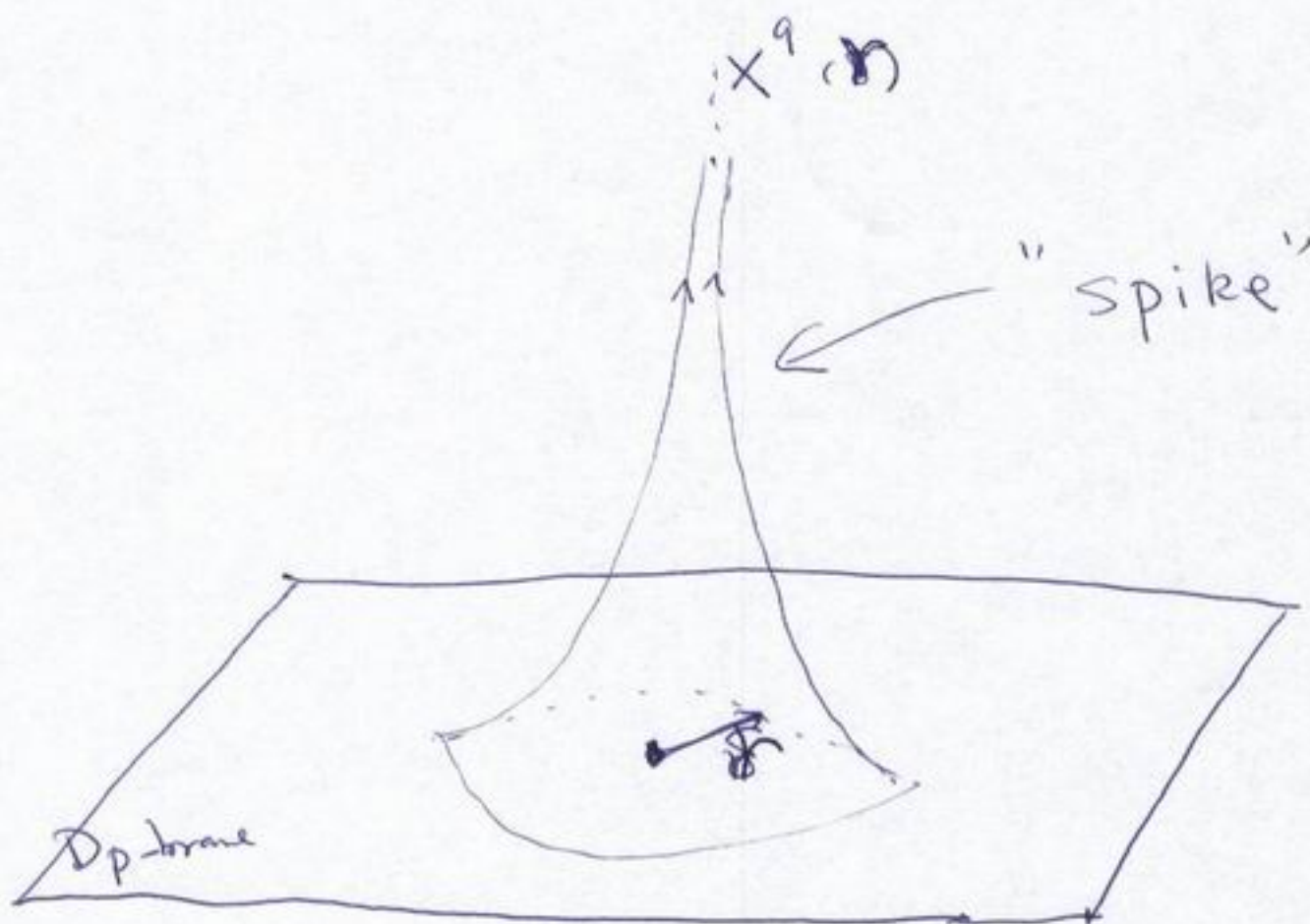
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$$E(\delta) = \frac{1}{(2\pi)^p g} \int d^p r \frac{1}{2} \left[ F_{or}^2 + (\partial_r X^9)^2 \right]$$

↑  
energy ( $r \geq \delta$ )

$$= \frac{1}{2\pi} X^9(\delta) = T_f X^9(\delta) \quad (\alpha' = 1)$$

$$u \quad \left\{ \frac{E(\delta)}{X^9(\delta)} = T_f \right. \quad (\delta \rightarrow 0)$$

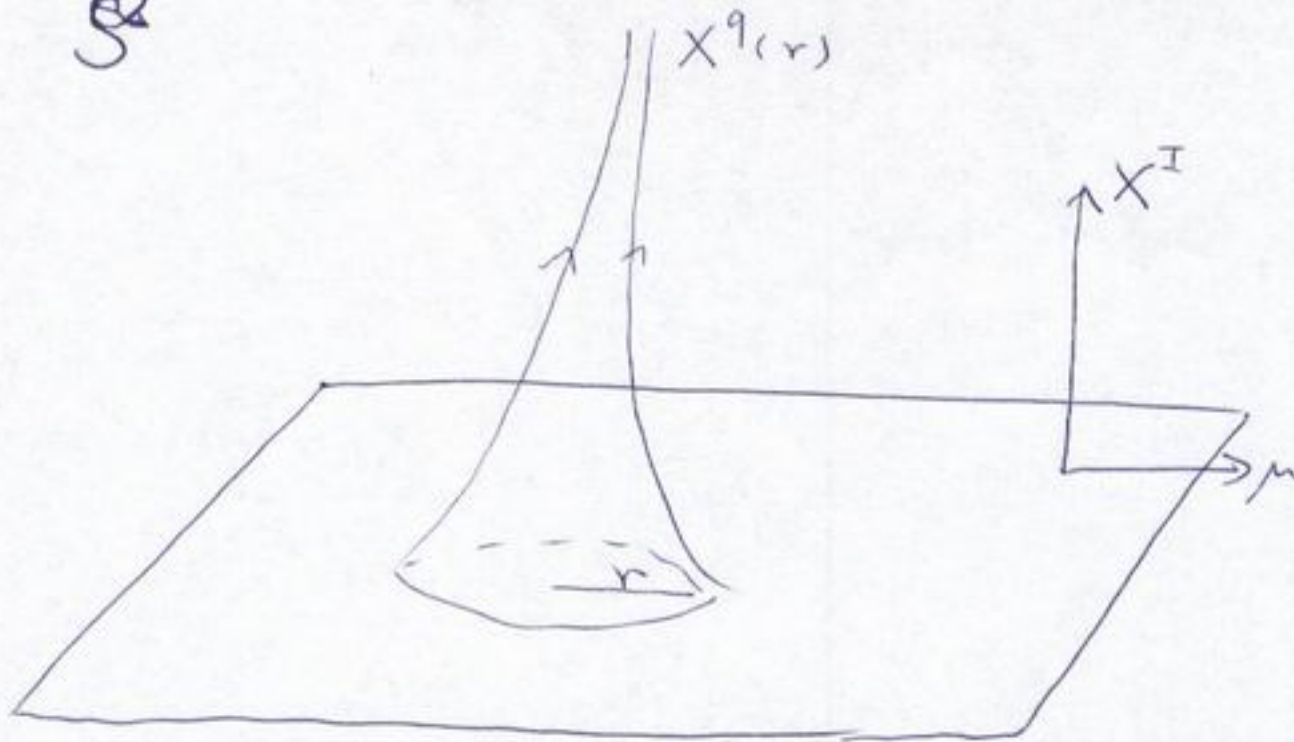


Note here

$$r^2 = X_1^2 + X_2^2 + \dots + X_p^2$$

All the spatial directions on the brane are used but we do have one  $X^9 \Rightarrow$  a string

$$\int_{S^2} F \propto M \quad (\text{magnetic flux})$$



$$F_{0\varphi} = \frac{N \Phi_m}{r}, \quad X^9 = \frac{N C_m}{r}$$

$$X^1, \dots, X^P$$

$$r = (X^{P-2})^2 + (X^{P-1})^2 + X^{P^2}$$

We have  $X^1, \dots, X^{P-3}$  directions combined

with  $\Rightarrow$  ISO(1, P-3)

$$R^{1, P-2} \oplus X^9$$

$\Rightarrow$  (P-2) spatial directions



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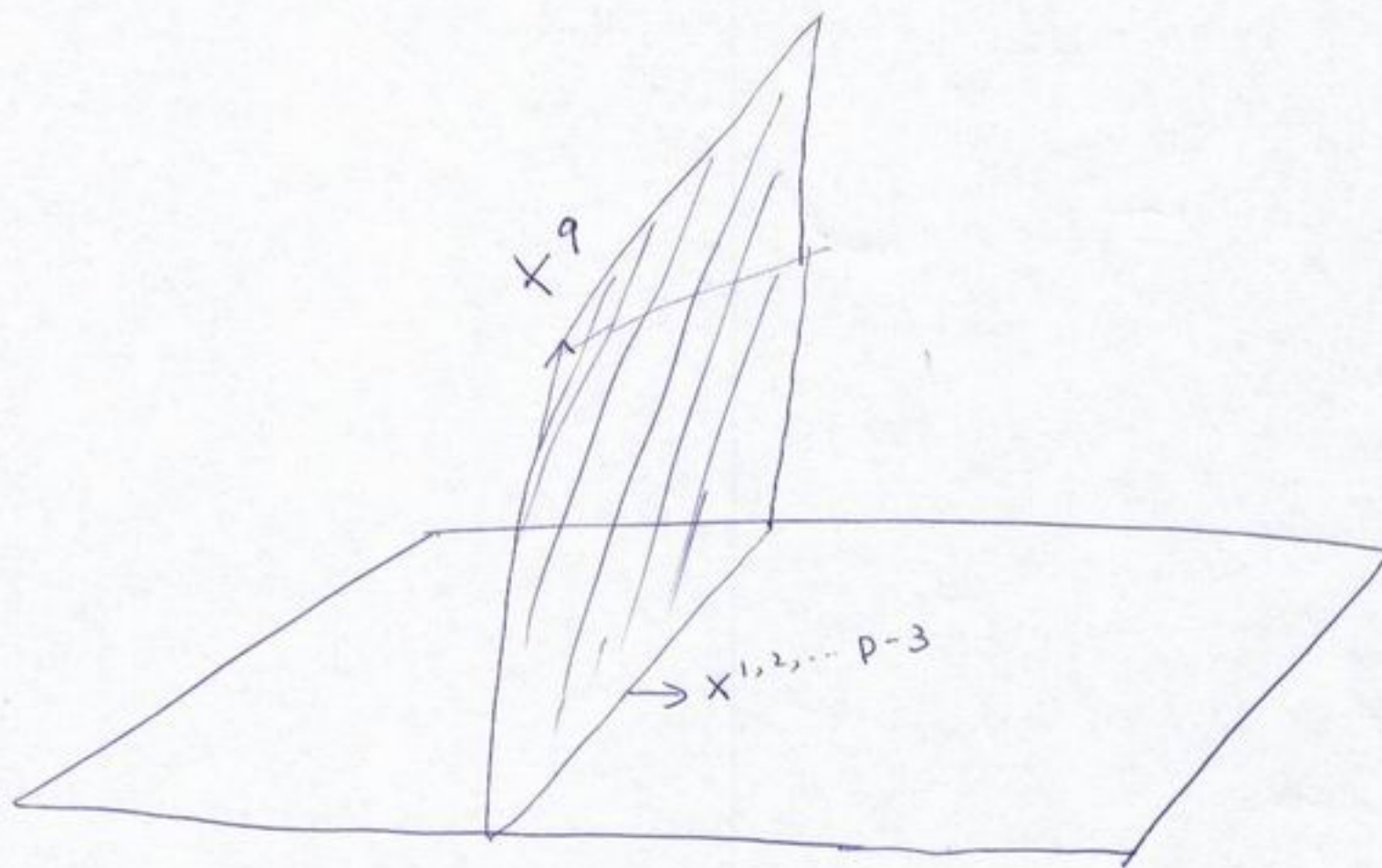
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A magnetic charge on  $D_p$

$\Rightarrow$  An object ending on  $D_p$  with ending with  $(p-3)$  spatial directions

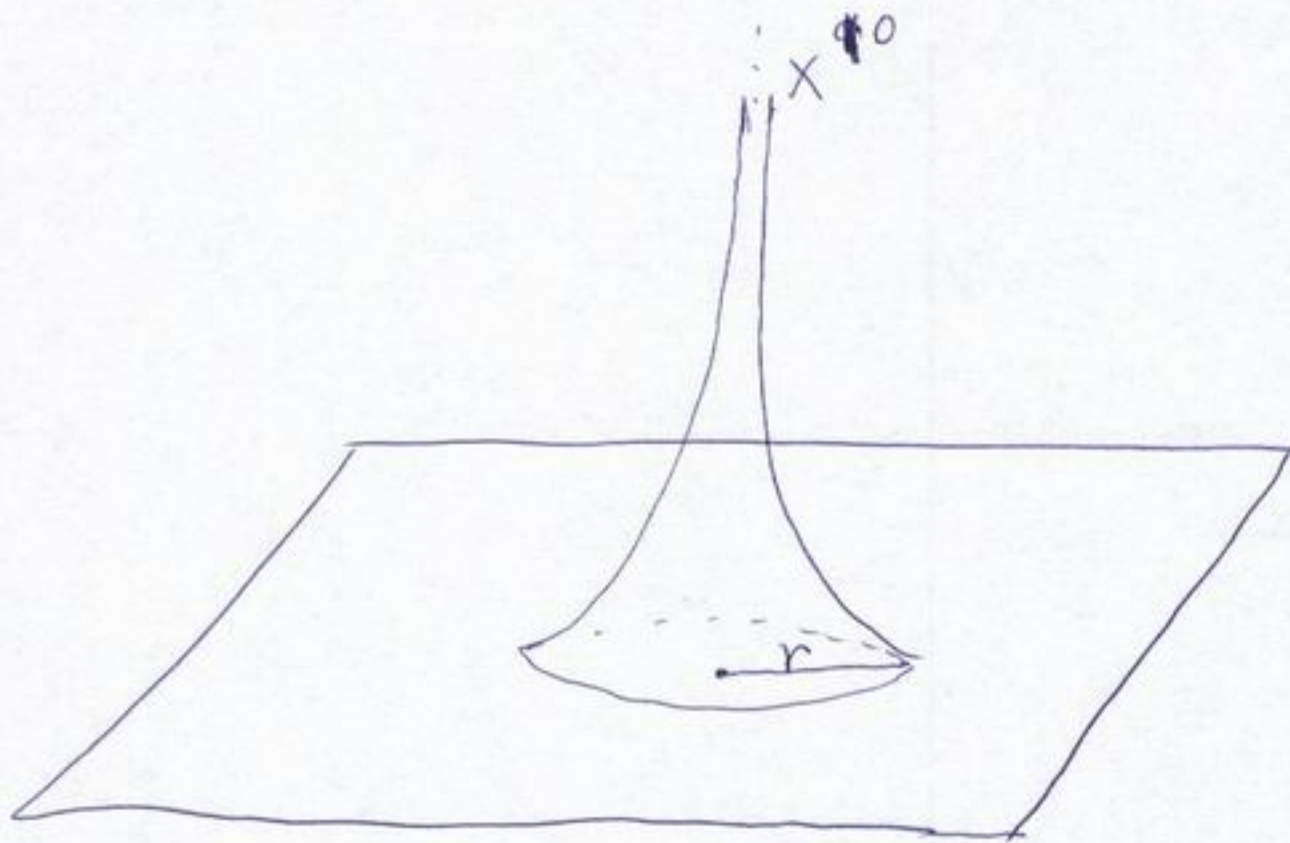
Since we have an additional  $x^9$

$\Rightarrow$  Open  $D_{(p-2)}$  brane



Similarly,

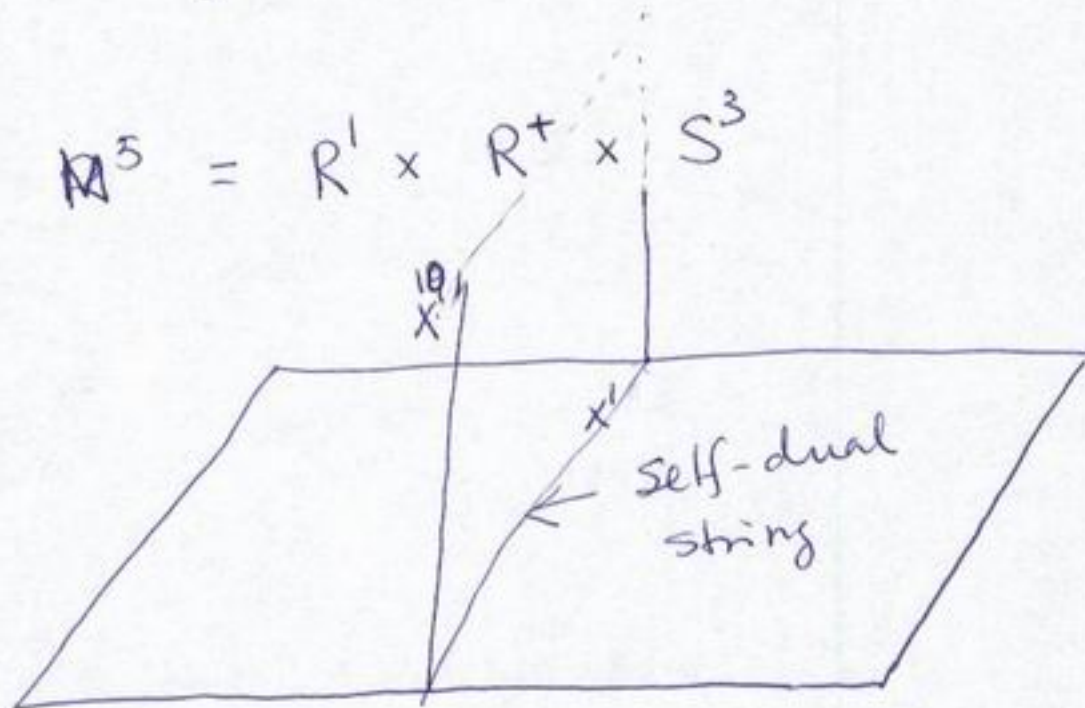
Howe, Lambert  
& West  
NPB 515 (98)  
203



$$r^2 = (x^3)^2 + (x^4)^2 + (x^5)^2 + (x^2)^2$$

$$\int_{S^3} H_3 = \int_{S^3} *H_3 \propto n$$

$$M^5 = R^1 \times R^+ \times S^3$$



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Another Picture of having other branes from a given one.

(Emparan  
PLB 423 (98) 71  
hep-th/9711106)

DBI Action

$$S = - T_p \int d^{p+1} \sigma \sqrt{-\det [G_{\mu\nu} + (F_{\mu\nu} + B_{\mu\nu}) \times \lambda]}$$

$$+ T_p \int \sum_{i=0}^{10} (C_i \wedge e^{\lambda(F+B)})_{p+1}$$

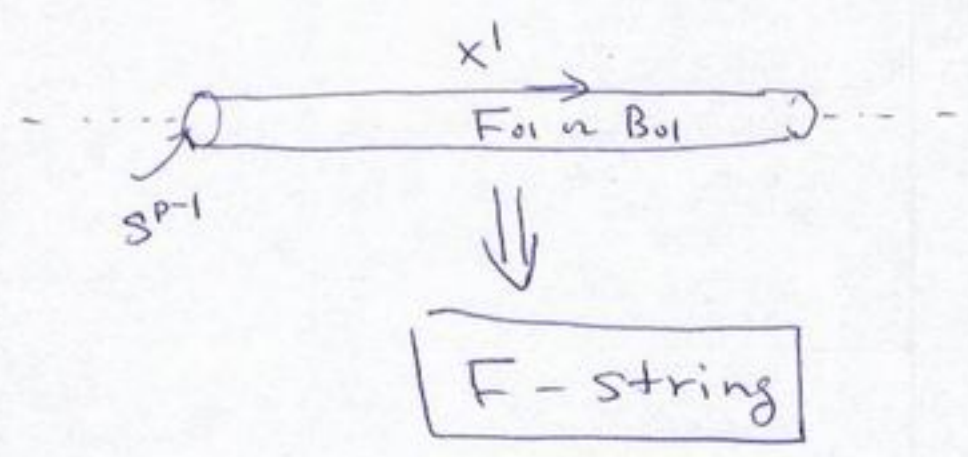
$B_{\mu\nu} \Rightarrow$  Electric Flux  $B_{01}$  (or  $F_{01}$ )

F-string  $\Leftrightarrow$  a  $D_p$ -brane with an Electric flux along  $x^1$   
with a topology  $R^1 \times S^{p-1}$  with  $S^{p-1}$  wrapping on trivial  $(p-1)$  cycle and vanishing side



$$\int_{R^1 \times S^{p-1}} j^{012 \dots p} d\sigma^{p+1} = 0 \quad (\text{net zero charge for } D_p)$$

$S^{p-1}$  wrapping on a trivial and vanishing size  $(p-1)$ -cycle



$D_{(p-2k)}$ -brane  $\iff$   $D_p$ -brane with a topology

$R^{p-2k} \times S^{2k}$  with  $S^{2k}$  wrapping on a vanishing size trivial  $2k$ -cycle

$$\int_{R^{p-2k} \times S^{2k}} j_R^{012 \dots p} d^p \sigma = 0 \quad (k \neq 0)$$

but

$$T_p \int_{M^{p+1}} C_{p+1-2k} \wedge F^{2k} = T_p \int_{M^{p+1-2k}} C_{p+1-2k} \wedge \int_{S^{2k}} F \wedge \dots \wedge F$$

$$= T_{p-2k} \int_{M^{p+1-2k}} C_{p+1-2k}$$

$$= T_{p-2k} \int_{R^{p+1-2k}} C_{p+1-2k}$$

$$\approx \int_{R^{p+1-2k}} \partial^{j_1 \dots (p-2k)} \neq 0$$

{ For example  $k=1$ ,  $F \leftrightarrow$  magnetic charge

Again another picture

Obtaining lower dimensional p-branes including F-string from a higher dimensional one is Sen's from a so-called non-BPS D-brane  $\approx$  D-brane - Anti D-brane system.

$$D_p - \text{Anti } D_p \xrightarrow{(-)F_L} \text{Non-BPS } D_{(p-1)} \xrightarrow[\text{solution}]{\text{Kink}} \text{BPS } D_{(p-2)}$$

$$\approx D_p - \text{Anti } D_p \xrightarrow[\text{solution}]{\text{Vortex}} \text{BPS } D_{(p-2)}$$





In summary,

low energy theory of D-brane (or its  
 variance such as non-BPS or brane-anti brane)  
 can give rise to F-string and other  
 lower dimensional D-branes

In other words

Given a  $D_p$ -branes

$\Rightarrow$  F or  $D_{p'}$  with  $p' < p$

Questions?

Given a  $D_p$

$\Rightarrow$   $D_{p'}$  with  $p' > p$ ?



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The answer is Yes!

F-string →  
in a R-R  
background

Spherical  $D_p$ -branes  
can be spontaneously  
nucleated in a uniform  
field Background (the  
analog of Schwinger pair  
creation process).

$D_{p'}$   
 $p' < p$   
in a  $F_{p+2}$   
background  
flux

Nothing  
in a  $F_{p+2}$   
" "

Emparan	PLB 423 (98) 71
	hep-th/9711106
⋮	hep-th/9810163
	hep-th/0204203



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F-string  $\rightarrow$  D2 brane  
(cylinder or torus)

Consider a (cylinder or torus) D2 brane  
with an F-string flux

$$S_2 = - \frac{1}{4\pi^2 g} \int d^3z \left( \sqrt{-\det(g_{\mu\nu} + 2\pi F_{\mu\nu})} + \frac{1}{8} \epsilon^{\alpha\beta\gamma} A_{\mu\nu\rho} \partial_\alpha X^\mu \partial_\beta X^\nu \partial_\gamma X^\rho \right)$$

Static gauge

$$X^0 = t, \quad 2\pi F_{t\alpha} = \epsilon$$

$$X^1 = z$$

$$X^2 = R \cos \sigma$$

$$X^3 = R \sin \sigma \quad (0 \leq \sigma \leq 2\pi)$$

R constant

rest  $X^M$  fixed.

$$\boxed{F_{0123} = -h = \text{const.}, \quad A_{012} = \frac{h}{2} X^3, \quad A_{013} = -\frac{h}{2} X^2}$$

$$\Rightarrow E = \frac{1}{2\pi g} \int dz \left( \sqrt{(R^2 + D^2)} - \frac{h}{2} R^2 \right)$$

$$\boxed{\frac{1}{2\pi g} D = \frac{\delta S_2}{\delta \epsilon}}$$

$$\boxed{D = ng}$$



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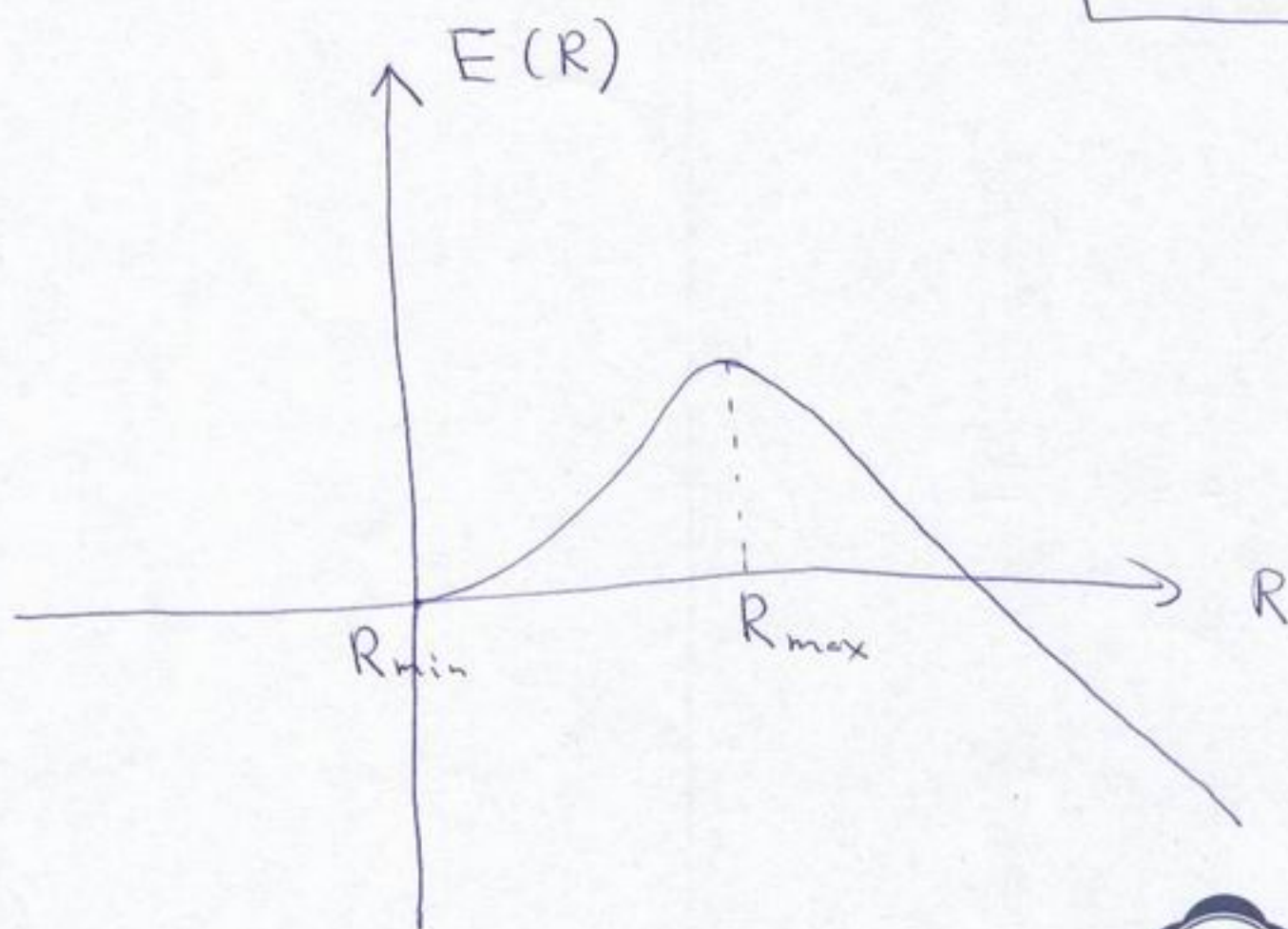
$$E = \frac{L}{2\pi g} \left[ \sqrt{R^2 + n^2 g^2} - \frac{h}{c} R^2 \right]$$

$$\frac{\delta E}{\delta R} = \frac{L}{2\pi g} \left[ \frac{R}{\sqrt{R^2 + n^2 g^2}} - hR \right] = 0$$

$$\Rightarrow R_{\min} = 0 \quad \sim \quad h \sqrt{R^2 + n^2 g^2} = 1$$

$$\sim R_{\max} = \sqrt{\frac{1}{h^2} - n^2 g^2}$$

$$\text{if } \boxed{hng < 1}$$



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at  $R = R_{min} = 0$

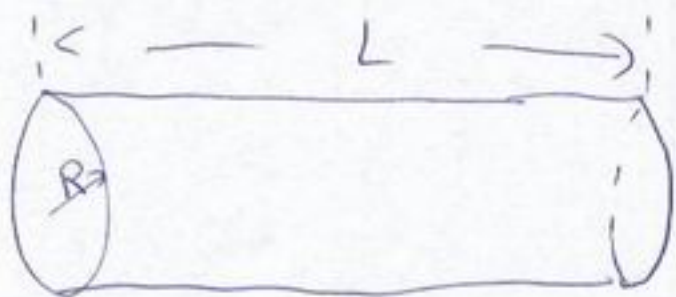
$$E = \frac{L}{2\pi} \cdot n g = \frac{L}{2\pi} n$$

$$\frac{E}{L} = \frac{1}{2\pi} n = T_f n \leftarrow n \text{ F-strings}$$

So  $R = R_{min} = 0$  represents  $n$  F-strings

These  $n$  F-strings can tunnel by instanton tunnelling to have  $R > 0 \Rightarrow$

a cylinder  $D_2$ -brane if  $L$  is a line-integral



a torus if  $z$  is  $2\pi L$  a circle



## Another Example

Do branes  
in a  $F_4$   
background

Myers dielectric  
effect

a spherical  
 $D2$  brane

or

Nothing  
in this  $F_4$

instanton  
tunnelling

$N$   $D0$  branes placed in a RR

$$F_{t\theta\varphi} = -2fr^2 \sin\theta \rightarrow C_{t\theta\varphi} = \frac{2fr^3}{3} \sin\theta$$

Assume such a stable spherical  $D2$  brane  
exist, then

$$N \text{ } D0 \text{ branes} \Leftrightarrow F_{\theta\varphi} = \frac{N}{2} \sin\theta$$

DBI action in background

$$C_{t\theta\varphi} = \frac{2fr^3}{3} \sin\theta$$



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$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) + \sum_{i=4}^9 dx_i^2$$

Static gauge

$$\sigma^0 = t$$

$$\sigma^1 = \theta$$

$$\sigma^2 = \varphi$$

$$S_2 = -T_2 \int d^3\sigma \left[ \sqrt{-\det(\eta_{\mu\nu} X^\mu_{,\alpha} X^\nu_{,\beta} g_{MN})} + 2\pi\alpha' F_{\mu\nu} \right] + T_P \int C_3$$

$$\Rightarrow E(r) = +T_2 \left[ 4\pi \sqrt{r^4 + \pi^2 \alpha'^2 N^2} - 2f \times \frac{4\pi}{3} r^3 \right]$$

$$= 4\pi T_2 \left[ \sqrt{r^4 + \pi^2 \alpha'^2 N^2} - \frac{2f}{3} r^3 \right]$$

$$\frac{\delta E(r)}{\delta r} = 0 \Rightarrow$$

$r = 0$ ,  
 $\varphi$   
Saddle  
point

$$r_{\min} = \sqrt{\frac{1}{2f^2} - \sqrt{\frac{1}{4f^4} - \pi^2 \alpha'^2 N^2}}$$

$$r_{\max} = \sqrt{\frac{1}{4f^4} + \sqrt{\frac{1}{4f^4} - \pi^2 \alpha'^2 N^2}}$$



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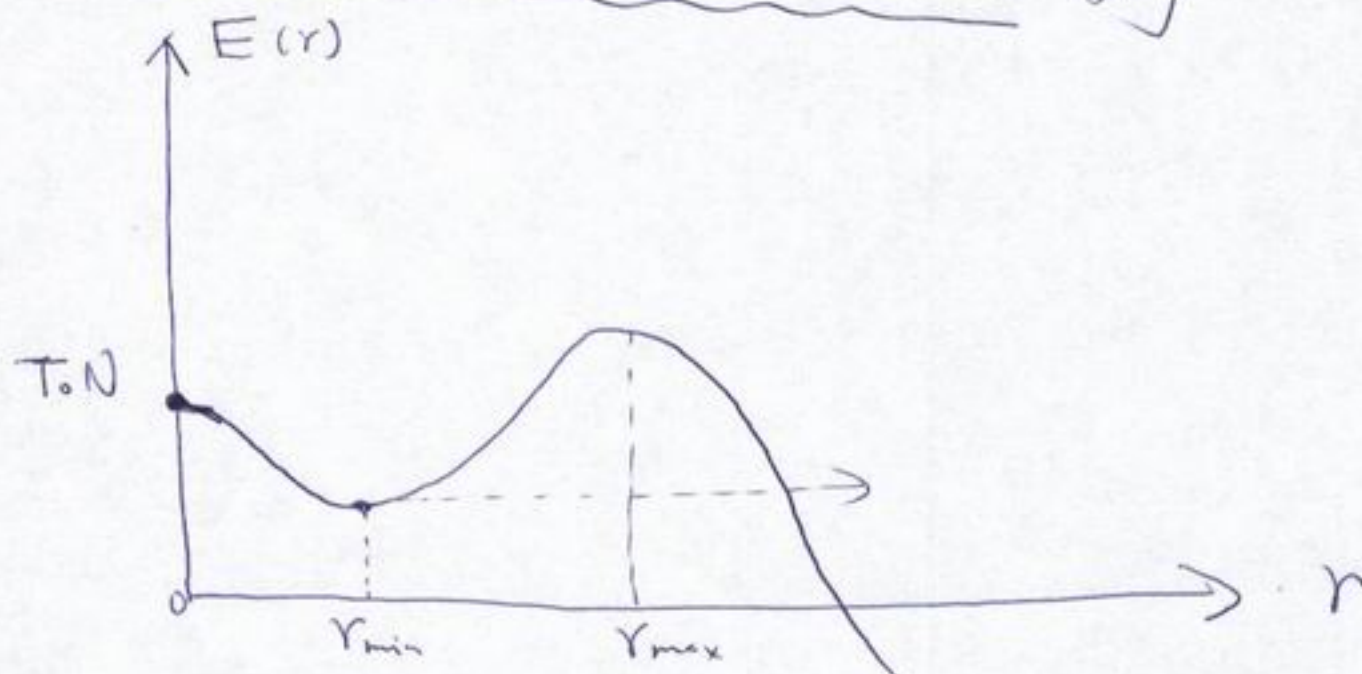
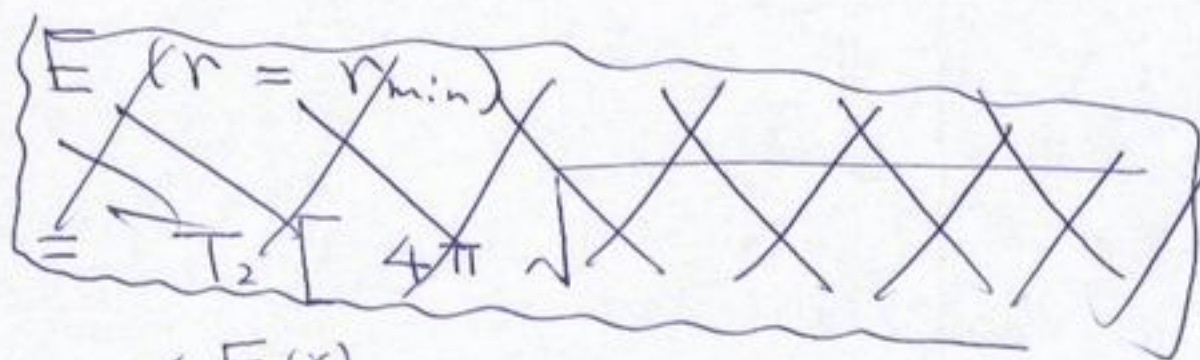
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$$\overline{T}_p = \frac{1}{g(2\pi)^p \alpha'^{\frac{p-1}{2}}}$$

$$\begin{aligned} E(r=0) &= 4\pi T_2 \cdot \pi \alpha'^2 N \\ &= \frac{N}{g \sqrt{\alpha'}} = \underline{T_0 N} \end{aligned}$$

$r=0$  is a saddle point, corresponding to  $N D_0$  branes

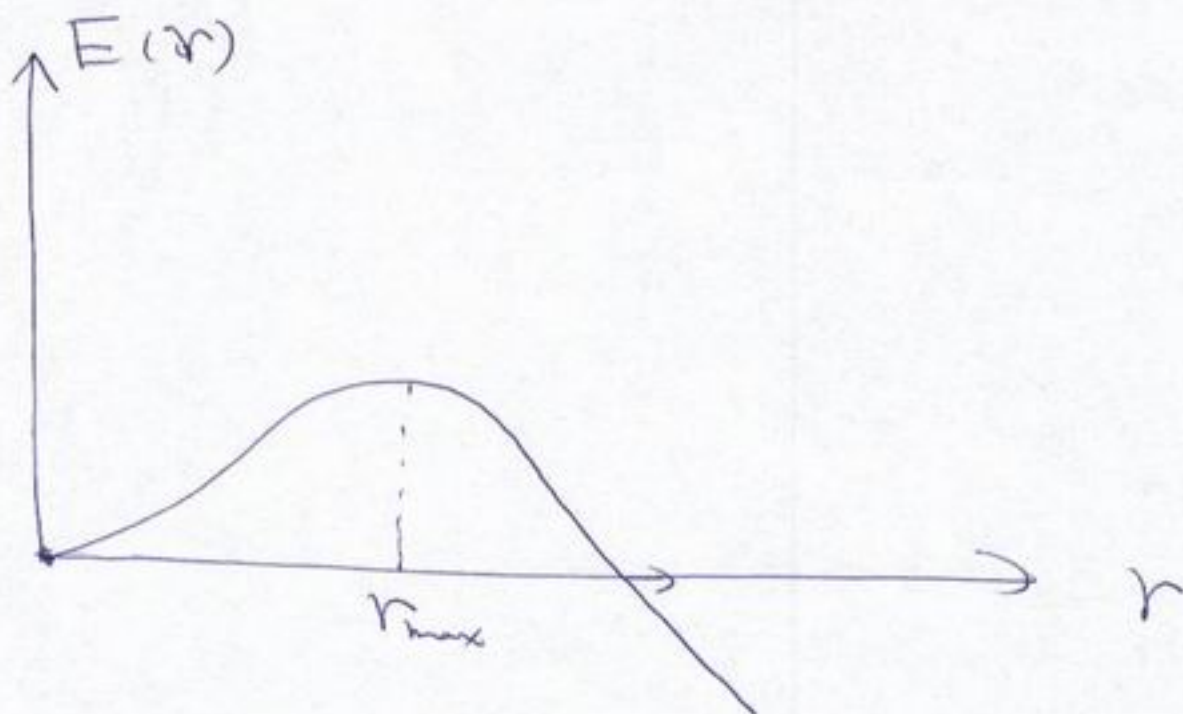




Nothing

$N = 0$  (No D0 branes)

$r_{min} = 0, \quad r_{max} = \frac{1}{f}$



Nothing tunnels to a spherical D2 brane

In Summary,

To create a higher dimensional Dp brane from lower dimensional ones,

EXIST TWO Processes,

Instanton Tunnelling

or Myers effects

Schwinger Type Pair Creation

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# Myers Effects

Myers JHEP9911 26  
 (99)022  
 hep-th/9910053

Abelian DBI  $\rightarrow$  Non-Abelian  
 DBI

$$S_{DBI} = -T_p \int d^{p+1}\sigma \text{STr} \left[ e^{-\phi} \sqrt{-\det(P[E_{ab} + E_{ai}(Q^{-1})^j_b] E_{jk} + \lambda F_{ab}) \det(Q^i_j)} \right]$$

$$S_{CS} = \mu_p \int \text{STr} \left( P \left[ e^{i\lambda \frac{1}{2} i_{\Phi}} (\sum C^{(n)} e^B) \right] e^{\lambda F} \right)_{(p+1)}$$

$$\lambda = 2\pi\alpha'$$

$$E_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}$$

$$Q^i_j \equiv \delta^i_j + i\lambda [ \Phi^i, \Phi^j ] E_{kj}$$

e.g.

$$i_{\Phi} i_{\Phi} C^{(p+3)} = \frac{1}{2} [ \Phi^i, \Phi^j ] \frac{1}{(p-1)!} \underbrace{C_{j i i_1 \dots i_{p-1}}}_{\substack{\uparrow \\ \text{interior product}}} dx^{i_1} \wedge \dots \wedge dx^{p+1}$$

Appearance  
 of a higher  
 dimensional  
 D(p+2) brane



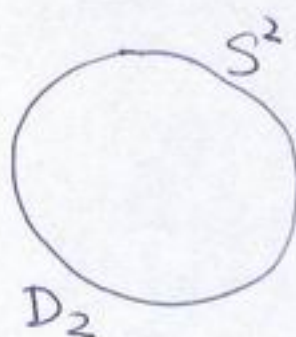
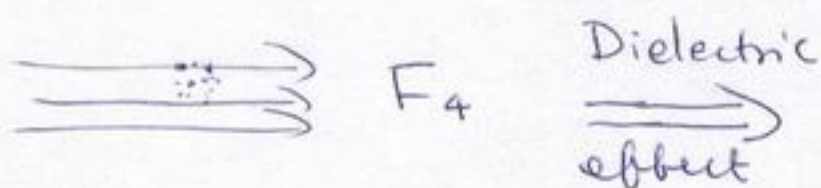
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e.g.

$N$  D0 branes placed in a  $F_4$  background

Static stable

$\Rightarrow$  a  $\wedge$  Spherical D2 brane



∴ Set up

Consider  $N$   $D(p-2)$  branes placed in a background

$$F_{(p-2+i)(p-2+j)(p-2+k)01\dots(p-2)}$$

$$= \begin{cases} 2f \epsilon_{ijk} \\ 0 \end{cases}$$

for  $i, j, k \in \{1, 2, 3\}$

Otherwise



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$f$  carries dimensions of  $\text{Length}^{-1}$

set  $B_{pc} = 0$ , flat Minkowski

and take static gauge

$$\sigma^a = x^a$$

$$a = 0, 1, \dots, (p-2)$$

$$\sqrt{\lambda} \Phi^I < 1$$

$$\frac{I = p-1, \dots, q}{a = 0, 1, \dots, p-2}$$

$$S_{DBI} = - T_{p-2} \int d^{p-1} \sigma \left( N + \frac{\lambda^2}{2} \text{Tr} D_a \Phi^I D^a \Phi^I + \frac{\lambda^4}{2} \text{Tr} F_{ab} F^{ab} - \frac{\lambda^2}{4} \text{Tr} [\Phi^I, \Phi^J]^2 \right)$$

$$S_{CS} = - \frac{i \mu_{p-2} \lambda^2}{3} \int d^{p-1} \sigma \text{Tr} \Phi^i \Phi^j \Phi^k F_{(p-2+i)(p-2+j)(p-2+k)} \quad (0, 1, \dots, p-1)$$



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In the following, we try also to set  $\dot{A}_a = 0$

to do so, we need to be careful since

$$D_a F^{ab} \sim [\Phi^I, D^b \Phi^I]$$

Only

$$\Rightarrow [\Phi^I, D^b \Phi^I] = 0 \Leftrightarrow [\Phi^I, \partial_a \Phi^I] = 0$$

We can set  $A_a = 0$  consistently.

$$S = S_{DBI} + S_{CS}$$

$$= - T_{p-2} \int d^{p-1} \sigma \left[ N + \frac{\lambda^2}{2} \text{Tr} \partial_a \Phi^I \partial^a \Phi^I - \frac{\lambda^2}{4} \text{Tr} [\Phi^I, \Phi^J]^2 \right]$$

$$- \frac{i \mu_{p-2} \lambda^2}{3} \int d^{p-1} \sigma \text{Tr} \Phi^i \Phi^j \Phi^k F_{(p-2+i)(p-2+j)(p-2+k) \dots (p-2)}$$

$$\mu_p = T_p = \frac{2\pi}{g_s (2\pi l_s)^{p+1}}$$

$$l_s^2 = \alpha'$$



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The corresponding Hamiltonian density is

$$\mathcal{H} = N T_{p-2} + \frac{\lambda^2 T_{p-2}}{2} \left[ \text{Tr} (\dot{\Phi}^I)^2 + \text{Tr} (\nabla \Phi^I)^2 \right] \\ - \frac{\lambda^2 T_{p-2}}{4} \text{Tr} [\Phi^I, \Phi^J]^2 \\ + \frac{i T_{p-2} \lambda^2}{3} \text{Tr} \Phi^i \Phi^j \Phi^k F_{(p-2+i)(p-2+j)(p-2+k) 01 \dots (p-2)}$$

Potential density

$$\mathcal{V} = - \frac{\lambda^2 T_{p-2}}{4} \text{Tr} [\Phi^I, \Phi^J]^2 \\ + \frac{i T_{p-2} \lambda^2}{3} \text{Tr} \Phi^i \Phi^j \Phi^k F_{(p-2+i)(p-2+j)(p-2+k) 01 \dots (p-2)}$$

EOM

$$\partial_a \partial^a \Phi^I - \left[ [\Phi^I, \Phi^J], \Phi^J \right] - i \Phi^j \Phi^k F_{(p-2+i)(p-2+j)(p-2+k) 01 \dots (p-2)} \\ = 0$$

⇒



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$$\Rightarrow \left\{ \begin{array}{l} \partial_a \partial^a \Phi^i - [ [\Phi^i, \Phi^J], \Phi^J] \\ \quad - 2f_i \Phi^j \Phi^k \epsilon_{ijk} = 0 \quad i, j, k \in \{1, 2, 3\} \\ \\ \partial_a \partial^a \Phi^l - [ [\Phi^l, \Phi^J], \Phi^J] = 0 \quad l = 4, \dots, 11-p \\ \quad \text{hr,} \end{array} \right.$$

In the following,

$$[ \Phi^i, \Phi^l ] = 0$$

the no mixing between  $i$  &  $l$  in the above,  
 $\Phi^i$  &  $\Phi^l$  are decoupled.

Static & spatial independent configuration

$$\partial_a \Phi^I = 0$$

$$\Rightarrow [ [\Phi^i, \Phi^j], \Phi^j ] + \underbrace{2f_i \Phi^j \Phi^k \epsilon_{ijk}} = 0$$

$$[ [\Phi^l, \Phi^m], \Phi^m ] = 0$$

$\Downarrow$

$$[ \Phi^l, \Phi^m ] = 0$$



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① If there is a background, then a static stable configuration, i.e.,

$N$   $D(p-2)$  branes  $\longrightarrow$  a spherical  $D_p$  brane can be created with

$$\underline{R^{p-2} \times S^2} \text{ topology}$$

(Myers effect)

② If no background presence,

the generated spherical  $D_p$  must be either time dependent  $\sim$  spatial

dependent  $\sim$  both with a topology

$$R^{p-2} \times S^2 \text{ with the radius of } S^2$$

either time dependent  $\sim$  spatial dependent  $\sim$  both.



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