

Conformal Scaling Gauge Theory Inflationary Dark Universe

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Introduction

- Recent developments in cosmology[1] indicate that the astrophysics reaches an epoch for exploring fundamental theory inaccessible to particle accelerators. Especially due to more accurate astrophysical data from Wilkinson Microwave Anisotropy Probe (WMAP)[2] and other planned future cosmological observations, it enables one to test various cosmological models.
- One of the basic ideas in modern cosmology is the inflation[3, 4, 5, 6, 1] which postulates that the universe at earlier times in its history was dominated by the potential or vacuum energy. The developments of inflationary cosmology involve various scalar fields and different types of potentials, while all proposed models attend to solve the difficult cosmological problems met in the standard Big Bang cosmology, such as the homogeneity, isotropy, horizon, flatness, structure formation, monopole *etc.*.

- It is intriguing that the constituents in today's universe are also dominated by the potential or vacuum energy, i.e., the so-called dark energy, about 73% by weight. The remaining part of universe is mainly filled by the so-called dark matter, about 23% by weight, while the density of ordinary baryonic matter known from the standard model in particle physics is less than 4% by weight.
- Namely all structure in the universe, from small scale to large scale, may be formed via a common origin based on the potential or vacuum energy. Therefore either the inflation happened in the small scale of universe at earlier times or the accelerated expansion observed in the large scale of today's universe can be driven by dynamical scalar fields or homogenous dark energy.
- Obviously, those scalar fields must be beyond the standard model in particle physics. It then rises some basic issues, such as:
 - whether the introduced scalar fields are the fundamental fields;
 - what are the deep physical reasons for introducing scalar fields;
 - how the scalar fields interact with ordinary matter fields;

which kind of potential form or vacuum structure is the true choice of nature; why the small and large scale behavior of universe is so analogous.

- In general, different motivations and ideas lead to different inflationary models. Nevertheless, recent progresses in more accurate observational cosmology provide a possibility to have an experimental verification for various inflationary models. It is expected that more precise cosmological observations will guide us to a more fundamental theory.

The large and small scales of universe are characterized by the corresponding low and high mass energy scales M_{HI} and \bar{M}_{Pl} respectively

$$M_{HI} \equiv H_0 = 2.0 \times 10^{-33} \text{eV}$$

is the Hubble constant and

$$\bar{M}_{Pl} = M_{Pl} / \sqrt{8\pi} = 2.44 \times 10^{18} \text{GeV} = 2.44 \times 10^{27} \text{eV}$$

is the reduced Planck mass ($M_{Pl} = 1.22 \times 10^{19}$ GeV). A mysterious observation is that the cosmological constant Λ or the homogeneous dark energy is found at the present epoch to be at the energy scale given by the relation

$$\frac{\bar{M}_{Pl}}{\Lambda} \simeq \frac{\Lambda}{M_{HI}}, \quad \text{i.e.} \quad \Lambda \simeq \sqrt{\bar{M}_{Pl} M_{HI}} = 2.21 \times 10^{-3} \text{eV} \quad (1)$$

- In general, the introduction of mass scale in a four dimensional space-time theory will spoil a conformal scaling global symmetry in the absence of gravity or destroy a conformal scaling gauge (or local) symmetry in the presence of gravity.
- It is noticed that a conformal scaling gauge symmetry, which differs from a conformal scaling global symmetry, implies the existence of a fundamental mass scale as one can always make a conformal scaling gauge transformation to fix the gauge so as to match the Einstein-Hilbert action of gravity.

- Namely, the conformal scaling gauge symmetry can be made to be broken down explicitly by an appropriate fixing gauge, which distinguishes from the spontaneous symmetry breaking of unitary gauge symmetries.

From the special feature of conformal scaling gauge symmetry, we are going to study a theory based on an assumption:

Conformal scaling gauge symmetry is a fundamental symmetry in the presence of gravity.

- It is then known that a scalar field $\Phi(x)$ is necessary for constructing a conformal scaling gauge invariant action of gravity.
- While such a scalar field is not a physical one due to a wrong sign of its kinetic term.
- To make the scalar field be a dynamically physical field, it is seen that a gauge

field is necessary to be introduced for obtaining a gauge invariant kinetic term for the scalar field.

- Thus the original Weyl's theory[7] with the existence of vector gauge boson becomes natural. The early works along the Weyl's theory may be found in refs.[8, 9].
- We will show that a conformal scaling gauge invariant potential action can in general be constructed by the scalar field and a real Wilson-like line element of gauge field.
- As a nontrivial background field solution of pure gauge has a minimal energy in gauge interactions, the evolution of universe is dominated by the potential energy of the pure gauge background field which is characterized by a scalar field.
- As a consequence, we naturally arrive at an exponential potential model of the scalar field after fixing the gauge to match the Einstein-Hilbert action of gravity.

- It will be seen that such a model will lead to an inflationary universe[3, 4, 5] via a chaotic type inflation[6].
- As the exponential potential model has an exact solution with a power-law inflation[10] for the scale factor, i.e., $a(t) \sim t^p$, its cosmological effects are solely characterized by the power-law index p .
- It will be shown that the power-law index in such an inflationary theory is determined by a basic gauge fixing parameter g_F via $p = 16\pi g_F^2 [1 + 3/(4\pi g_F^2)]$.
- When taking the Planck mass to be the gauge fixing scale of the scalar field, we have $g_F = 1$ and $p \simeq 62$. The resulting cosmological effects in such an inflationary theory will be discussed.

Conformal Scaling Gauge Theory of Gravity

The conformal scaling gauge invariant action for gravity requires the introduction of a fundamental scalar field $\Phi(x)$. It has the following known form

$$S_G = \int d^4\mathbf{x} \sqrt{-g} \kappa_0^2 \left\{ -\frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{12} \Phi^2 R \right\} \quad (2)$$

with κ_0 a constant. Where $g_{\mu\nu}$ is a metric which has the signature $(+ - - -)$. This action is invariant under the conformal scaling gauge transformation

$$\Phi(\mathbf{x}) \rightarrow \xi(\mathbf{x})\Phi(\mathbf{x}), \quad g_{\mu\nu}(\mathbf{x}) \rightarrow \xi^{-2}(\mathbf{x})g_{\mu\nu}(\mathbf{x}) \quad (3)$$

Note that the above kinetic term of the scalar field has a wrong sign for the scalar field to be as a physical one. To construct an invariant kinetic term of the

scalar field with a right sign, it is necessary to introduce a gauge field $A_\mu(x)$ with transformation property

$$A_\mu(x) \rightarrow A_\mu(x) + \xi^{-1}(x)\partial_\mu\xi(x) \quad (4)$$

Thus the invariant kinetic action can be written down as follows

$$S_K = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \kappa^2 g^{\mu\nu} D_\mu \Phi D_\nu \Phi - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} \right\} \quad (5)$$

with κ a constant satisfying the normalization condition of kinetic term

$$\kappa^2 - \kappa_0^2 = 1 \quad (6)$$

Where the covariant derivation and the gauge field strength are defined as

$$\mathbf{D}_\mu \Phi(\mathbf{x}) = (\partial_\mu - \mathbf{A}_\mu) \Phi(\mathbf{x}) \quad (7)$$

$$\mathbf{F}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu \quad (8)$$

We now consider a real Wilson-like line element associated the element in the conformal scaling gauge group for the path C_P going from x_P to x with the gauge field $A_\mu(x)$

$$\mathbf{G}_P(\mathbf{x}, \mathbf{C}_P; \mathbf{A}) = \mathbf{P} \exp \int_{\mathbf{x}_P}^{\mathbf{x}} d\mathbf{z}^\mu \mathbf{A}_\mu \quad (9)$$

with P the path-ordering operation. It transforms under the conformal gauge transformation as follows

$$\mathbf{G}_P(\mathbf{x}, \mathbf{C}_P; \mathbf{A}) \rightarrow \xi(\mathbf{x}) \mathbf{G}_P(\mathbf{x}, \mathbf{C}_P; \mathbf{A}) \xi^{-1}(\mathbf{x}_P) \quad (10)$$

Here x_P is a point at a small scale of Planck length $l_p = 1/M_{Pl}$. With the above Wilson-like line element, we can construct the following gauge invariant potential

$$S_V \equiv - \int d^4x \sqrt{-g} V(A_\mu, \Phi) \quad (11)$$

$$= - \int d^4x \sqrt{-g} \sum_{n=1}^4 \lambda_n \Phi^{4-n}(x) [G_P(x, C_P; A) \Phi(x_P)]^n \quad (12)$$

with λ_n being coupling constants.

It is known that the pure gauge solution has a minimal energy in gauge interactions. Taking the pure gauge as a nontrivial background field solution of gauge interactions and supposing that such a solution is dominated at earlier time of universe, thus the evolution of universe is described by the background field of pure gauge. Here, the pure gauge background field is simply characterized by a scalar field $\chi(x)$. In general we may rewrite the gauge field into two parts

$$\mathbf{A}_\mu = \partial_\mu \ln \chi + \mathbf{g} \mathbf{a}_\mu = \chi^{-1} \partial_\mu \chi + \mathbf{g} \mathbf{a}_\mu \quad (13)$$

where g is the coupling constant and a_μ represents the quantum fluctuation of gauge field in the background field of pure gauge. The conformal scaling gauge transformation property for the scalar field is $\chi(x) \rightarrow \xi(x)\chi(x)$, while the quantum field a_μ is unchanged under the transformation. For the pure gauge part, the Wilson-like line element $G_P(x, C_P; \partial_\mu \ln \chi)$ becomes independent of the gauge field and gets the following simple form

$$\mathbf{G}_P(\mathbf{x}, \mathbf{C}_P; \partial_\mu \ln \chi) = \frac{\chi(\mathbf{x})}{\chi(\mathbf{x}_P)} \quad (14)$$

Assume that at the point x_P the scalar field $\chi(x)$ and $\Phi(x)$ run into a conformal scaling gauge invariant fixed point so that their ratio is fixed to be a constant

$$\Phi(\mathbf{x}_P)/\chi(\mathbf{x}_P) = \mathbf{c}_0 \quad (15)$$

Thus an invariant action with a pure gauge background field can simply be

expressed as follows

$$\begin{aligned} \mathbf{S} &= \mathbf{S}_G + \mathbf{S}_K + \mathbf{S}_V \\ &\rightarrow \int d^4x \sqrt{-g} \left\{ \kappa_0 \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{12} \Phi^2 \mathbf{R} \right] \right. \\ &\quad \left. + \frac{1}{2} \kappa^2 g^{\mu\nu} D_\mu \Phi D_\nu \Phi - \sum_{n=1}^4 \lambda_n c_0^n \Phi^{4-n} \chi^n \right\} \end{aligned}$$

On the other hand, one can always make a conformal scaling gauge transformation to fix the gauge, so that the conformal scaling gauge symmetry becomes manifestly broken down in such a fixing gauge. In the presence of gravity, we can choose the most convenient gauge fixing condition in such a way that it leads to the Einstein-Hilbert action for gravity.

Regarding the Planck mass is a fundamental energy scale that characterizes the scaling behavior of universe, it is then natural to choose the following gauge fixing

condition

$$\Phi(x) \rightarrow \xi(x)\Phi(x) \equiv g_F M_{Pl} \quad (16)$$

$$\chi(x) \rightarrow \xi(x)\chi(x) \equiv \hat{\chi}(x) \quad (17)$$

$$g_{\mu\nu}(x) \rightarrow \xi^{-2}(x)g_{\mu\nu}(x) \equiv \hat{g}_{\mu\nu} \quad (18)$$

where the gauge fixing parameter g_F is a basic parameter introduced for a general case. With the above gauge fixing condition the action for gravity and pure gauge background field is simplified to be

$$S_0 = \int d^4x \sqrt{-\hat{g}} \left\{ \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - \frac{1}{2} \bar{M}_{Pl}^2 R \right\} \quad (19)$$

with

$$V(\phi) = \sum_{n=1}^4 V_n(\phi) \equiv \sum_{n=1}^4 V_n^0 e^{n\phi/M}, \quad V_n^0 = \lambda_n g_F^{4-n} c_0^n M_{Pl}^4 \quad (20)$$

$$\kappa g_F M_{Pl} \ln(\hat{\chi}(x)/M_{Pl}) = M \ln(\hat{\chi}(x)/M_{Pl}) \equiv \phi(x), \quad \phi(x_P) = 0 \quad (21)$$

where we have ignored the quantum gauge field a_μ , its effects are assumed to be unimportant during the inflationary epoch, especially for the case with a small gauge coupling constant $g \ll 1$.

Taking the above gauge fixing condition, together with the normalization condition for the general relativity of Einstein theory and the normalization condition for the kinetic term of scalar field, the relevant three basic parameters can be expressed in terms of one basic gauge fixing parameter g_F

$$\kappa_0 = \frac{1}{g_F} \sqrt{\frac{3}{4\pi}}, \quad \kappa = \sqrt{1 + \frac{3}{4\pi g_F^2}} \quad (22)$$

$$M = \kappa g_F M_{Pl} = M_{Pl} \sqrt{g_F^2 + \frac{3}{4\pi}} \quad (23)$$

Thus we arrive at an exponential potential model for the scalar field ϕ .

Power-Law Inflationary Universe

For an inflationary universe, the vacuum expectation values of the scalar field are in general time-dependent. Assuming that during that stage the universe is described in a good approximation by a Robertson-Walker metric

$$\hat{g}_{\mu\nu}(x) \equiv g_{\mu\nu}^{RW} + \tilde{g}_{\mu\nu}(x) \simeq g_{\mu\nu}^{RW} \quad (24)$$

Thus the time evolution of scalar field is governed by its equation of motion in the cosmological background. Namely they satisfy the following time-dependent minimal conditions

$$\phi'' + 3H\phi' + \frac{\partial V(\phi)}{\partial\phi} = 0 \quad \text{or} \quad \frac{d}{dt} \left[\frac{1}{2}\phi'^2 + V(\phi) \right] + 3H\phi'^2 = 0 \quad (25)$$

with the Hubble parameter

$$H^2 = \frac{1}{3\bar{M}_{Pl}^2} \left(\frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho \right) \quad (26)$$

which is obtained from the Einstein equation. Here $H(t) = a'/a$ with $a(t)$ the scale factor in Robertson-Walker model. ρ represents the energy density of radiation or matter, and satisfies the continuity equation

$$\frac{d}{dt}\rho + 3H(1 + \omega)\rho = 0 \quad (27)$$

Here $\omega = p/\rho$ is the ratio of pressure to density.

We assume that in the first stage of evolution the universe is mainly governed by the potential energy characterized by the effective potential $V(\phi)$. We also assume that the rate Γ of particle creation by ϕ in this stage is much smaller than the expansion of universe, i.e., $\Gamma \ll 3H$. On the other hand, it can be shown that for

$\lambda_1 \sim \lambda_2 \sim \lambda_3 \sim \lambda_4$ and $g_F \sim c_0 \sim 1$, the potential V_1 becomes dominated in the inflationary period. As $V_1(\phi)$ is an exponential potential V_1 , the vacuum state is known to get an exact solution

$$\phi(x) \simeq \phi(t) \simeq \phi_0 - 2M \ln(1 + M_o t), \quad \lambda_1 g_F^3 c_0 e^{\phi_0/M} (8\pi \bar{M}_{Pl})^2 = (3p - 1)p (28) \\ V(\phi) \simeq V_1(\phi) = (3 - 1/p) \bar{M}_{Pl}^2 \frac{(pM_o)^2}{(1 + M_o t)^2} = (3 - 1/p) \bar{M}_{Pl}^2 H^2(t) \quad (29)$$

with the power-law index p being determined by the gauge fixing parameter g_F

$$p = 16\pi \frac{M^2}{M_{Pl}^2} = 16\pi g_F^2 \left(1 + \frac{3}{4\pi g_F^2} \right) \quad (30)$$

which causes a power-law inflation

$$a(t) = a_0 (1 + M_o t)^p \quad H(t) = \frac{pM_o}{1 + M_o t} \quad (31)$$

Predictions and Evolution of Universe

As a predictive case of the theory, taking the Planck mass as the gauge fixing scale of the scalar field Φ , we have $g_F = 1$. Thus the power-law index is predicted to be

$$p = 16\pi \frac{M^2}{M_{Pl}^2} = 16\pi \left(1 + \frac{3}{4\pi}\right) \simeq 62 \quad (32)$$

It is easy to check that for such a large value of p the effective potential $V(\phi)$ goes via a slowly-rolling down process. This can be seen from the following condition

$$|\phi''|/|3H\phi'| = 1/p \ll 1 \quad (33)$$

We can now follow the standard approaches to compute various cosmological observables by considering the predictive case $g_F = 1$ and $p \simeq 62$. In the leading

approximation with ignoring possible quantum loop corrections of gauge field and scalar field, the spectral index of the scalar fluctuation in the power-law inflation is approximately determined by the parameter p

$$n_s \simeq 1 - \frac{2}{p} \simeq 30/31 \simeq 0.97 \quad (34)$$

The tensor to scalar ratio is also approximately determined by the parameter p

$$r \simeq 16 \left(\frac{\sqrt{3\pi}\phi'}{M_{Pl}H} \right)^2 = \frac{16}{p} = 8(1 - n_s) \simeq 0.26 \quad (35)$$

which is consistent with the current cosmological observations[11].

We now consider the constraint from the primordial density fluctuation spectrum

which is approximately estimated by

$$10^{-5} \frac{\delta\rho}{\rho} = \frac{C H^2}{2\pi \phi'} < 10^{-4} \quad (36)$$

with C the normalization constant $C = 1/2\sqrt{\pi}$. It is assumed that the power-law inflation is changed when $\Gamma 3H = 3p/t$, i.e., the particle creation rate Γ due to the variation of $\phi(x)$ becomes larger than the expansion rate H . Thus the power-law inflation which is supposed starting from the Planck time $t_I = 1/M_o = 1/\bar{M}_{Pl} = \bar{t}_P \simeq 2.7 \times 10^{-43} \text{sec}$ will hold until the time $t_\Gamma = \frac{3p}{\Gamma}$. Combining the above requirement from primordial density fluctuation spectrum, we obtain the following constraint for an inflationary epoch

$$\frac{1}{4} \left(\frac{p}{\pi}\right)^{3/2} 10^4 < \frac{t_\Gamma}{\bar{t}_P} \frac{1}{4} \left(\frac{p}{\pi}\right)^{3/2} 10^5 \quad (37)$$

Numerically, we have

$$2.2 \times 10^5 < \frac{t_\Gamma}{t_P} 2.2 \times 10^6 \quad (38)$$

In this case, the inflationary universe has a size much larger than the observable part

$$10^{229} < \frac{a(t_\Gamma)}{l_0} 10^{291} \quad (39)$$

The reheating temperature T_{rh} may approximately be estimated at the epoch when the total energy density changes from being dominated by the effective potential $V(\phi)$ to being radiation domination. Namely

$$\rho_r = g^*(T) \frac{\pi^2}{30} T^4 \simeq 3 \frac{M_{Pl}^2}{32\pi t^2} \quad (40)$$

with $g^*(T)$ is the effective number of helicity states and is of order 10^2 . Suppose that the reheating process takes place rapidly, namely $t_{rh} \sim t_\Gamma$, the reheating temperature is found to be

$$2.6 \times 10^{14} \text{ GeV} T_{rh} < 0.85 \times 10^{15} \text{ GeV} \quad (41)$$

which is not much lower than the scale of grand unification.

With the above constraint, the horizon problem is well solved since it only needs to satisfy the following condition

$$\left(\frac{a(t_\Gamma)}{a_0} \right)^{(p-1)/p} \simeq \left(\frac{t_\Gamma}{\bar{t}_P} \right)^{p-1} > 5 \times 10^{29} \frac{T_{rh}}{\bar{M}_{Pl}} 5.3 \times 10^{25} \quad (42)$$

a numerical low bound for t_Γ is found to be

$$\frac{t_\Gamma}{\bar{t}_P} > 2.6 \quad (43)$$

which is much weaker than the one from the primordial density fluctuation spectrum.

It is seen from the above demonstration that the power-law inflation with $g_F = 1$ and $p \simeq 62$ remains consistent with the current cosmological data though it seems only marginally allowed. For the gauge fixing scale to be slightly higher than the Planck scale, for instance $g_F = \sqrt{2}$, one has $p \simeq 113$. Repeating the same computations, we are led to the following results

$$n_s \simeq 1 - \frac{2}{p} \simeq 0.98, \quad r \simeq \frac{16}{p} \simeq 0.14 \quad (44)$$

which is firmly in the allowed region from the current cosmological observations[11].

Note that in computing the time period t_Γ of inflation, it is assumed that a rapid and successful reheating process takes place after the inflation driven by the potential energy $V(\phi)$, and the created particles are to be thermalized in a short

time ($\ll 1/H(t_\Gamma)$). The power-law inflation is in general changed due to the variation of the field ϕ via the particle creation. Therefore, around the epoch when $\Gamma \sim 3H$, one may solve two correlated continuity equations

$$\frac{d}{dt} \left[\frac{1}{2} \dot{\phi}^2 + V(\phi) \right] + 3H \dot{\phi}^2 = -\Gamma \Delta \quad (45)$$

$$\frac{d}{dt} \rho_r + 4H \rho_r = \Gamma \Delta \quad (46)$$

with $\Delta = \dot{\phi}^2$ [12, 13]. The magnitude of Γ relies on the fields which interact with the scalar field ϕ and the coupling strength among them. In general, one also needs to consider the effect of quantum gauge field a_μ in the reheating process. Here we shall not discuss in detail for the process of reheating, which is beyond our aim in the present paper.

Dark Matter and Dark Universe

Before proceeding, we would like to point out that the conformal scaling gauge field A_μ do not directly couple to fermionic fields $\psi_i(x)$. This is because the fermionic action

$$\mathbf{S}_F = \int d^4\mathbf{x} \sqrt{-\mathbf{g}} \{ \bar{\psi}_i \gamma^a \mathbf{e}_a^\mu (i\partial_\mu + \omega_\mu^{bc} \Sigma_{bc}) \psi_i + \mathbf{H.c.} \} \quad (47)$$

is invariant under the conformal scaling gauge transformation when the fermion fields transform as $\psi_i \rightarrow \xi^{3/2}(x)\psi_i$.

There is also no Yukawa coupling terms $\bar{\psi}_i \psi_i \Phi$ for the ordinary fermion fields in the standard model of particle physics. This is because the fermion fields in the standard model are chiral with gauge symmetry $SU(2)_L \times U(1)_Y \times SU(3)_c$. Nevertheless, the scalar field Φ may have interaction with singlet fermions beyond

the standard model of particle physics, such as the right-handed Majorana neutrinos with $y_N \bar{N}_R N_R^c \Phi$.

In general, the scalar field Φ and gauge field A_μ can interact with the ordinary matter fields only via a gravitational interaction. It naturally avoids an observable strongly interacting fifth force in the conformal scaling gauge theory. There is also no Yukawa coupling terms $\bar{\psi}_i \psi_i \Phi$ for the ordinary fermion fields in the standard model of particle physics. This is because the fermion fields in the standard model are chiral with gauge symmetry $SU(2)_L \times U(1)_Y \times SU(3)_c$.

Nevertheless, the scalar field Φ may have interaction with singlet fermions beyond the standard model of particle physics, such as the right-handed Majorana neutrinos with $y_N \bar{N}_R N_R^c \Phi$. In general, the scalar field Φ and gauge field A_μ can interact with the ordinary matter fields only via a gravitational interaction. It naturally avoids an observable strongly interacting fifth force in the conformal scaling gauge theory.

More Interesting, the gauge field a_μ can be a candidate for dark matter. After

fixing the gauge,

$$\mathbf{A}_\mu = \partial_\mu \ln \chi + \mathbf{g} \mathbf{a}_\mu = \chi^{-1} \partial_\mu \chi + \mathbf{g} \mathbf{a}_\mu \quad (48)$$

with mass

$$\mathbf{M}_a = \mathbf{g} \mathbf{g}_F \mathbf{M}_{\text{Pl}} \quad (49)$$

The conformal scaling gauge symmetry allows the term

$$S_0 = - \int d^4x \sqrt{-g} \lambda_0 \Phi^4(x) \quad (50)$$

After fixing gauge, it provides a cosmological constant.

Conclusions

- Based on the conformal scaling gauge symmetry in the presence of gravity, there must exist a conformal scalar field $\Phi(x)$ and a gauge field A_μ .
- A gauge invariant potential action can in general be constructed by adopting a real Wilson-like line element of gauge field and the scalar field.
- The conformal scaling gauge symmetry has explicitly been broken down by fixing the gauge to match the Einstein-Hilbert action for gravity.
- A nontrivial background field solution of pure gauge has been found to result in an exponential potential model of scalar field, which leads to a power-law inflationary universe.

- The power-law index p is determined by the gauge fixing parameter g_F which is a unit when the gauge fixing scale is taken to be the Planck mass.
- The model is consistent with the current cosmological observations for a natural large value of the power-law index $p \simeq 62 \sim 113$ which is corresponding to the gauge fixing parameter $g_F = 1 \sim \sqrt{2}$. In this case, the spectral index of the scalar fluctuation and the tensor to scalar ratio are predicted to be $n_s \simeq 1 - 2/p \simeq 0.97 \sim 0.98$ and $r \simeq 16/p \simeq 0.26 \sim 0.14$ respectively.
- As the potential energy plays a role of dark energy, the ratio of pressure to energy is estimated to be $\omega \simeq -n_s \simeq -0.97 \sim -0.98$.
- The expected reheating temperature T_{rh} is below the grand unification scale and has a typical value: $T_{rh} \simeq 10^{14}$ GeV.
- It is noted that in this paper we have only considered the simple cases at leading approximation and ignored quantum corrections of gauge field as well

as other possible interactions. Some important issues such as the ending of inflation and the process of reheating have not been studied as they are beyond our purposes in the present paper. Also the question on the unexpectedly small anisotropy of CMB at large angles[2] needs to be understood. Nevertheless, it remains unclear whether it is a real anomaly or it is just a manifestation of cosmic variance[14].

- The gauge field a_μ can be a good candidate for dark matter.

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