Spinning Strings and AdS/CFT

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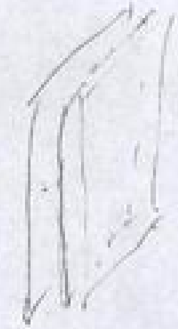
Spinning Strings and AdS/CFT

I. Brief review of AdS/CFT correspondence:

N D3 branes:



N

· open string point of view:

D3: SYM

N D3: U(N) SYM. $\mathcal{N} = 4$

t' Hooft limit: $g_{YM} \to 0$, $N \to \infty$ with $\lambda = g_{YM}^2 N$ fixed.

· closed string point of view:

D3: soliton solution in IIB supergravity

$\to AdS_5 \times S^5$ near ~~throat~~ horizon

$$g_{YM}^2 = 4\pi g_s$$

$$R^4 = 4\pi g_s N l_s^4 = g_{YM}^2 N l_s^4 = \lambda l_s^4$$

$$\alpha' = l_s^2$$

AdS_5/CFT correspondence. (Maldacena, Witten, Polyakov, et. al)

IIB on $AdS_5 \times S^5$ w/ g_s, radius R & N units of 5-form flux F_5

$$\wr\wr$$

$\mathcal{N}=4$ SYM w/ $U(N)$ & g_{YM}

with: $g_{YM}^2 = 4\pi g_s$

$R^4 = 4\pi g_s N l_s^4 = \lambda l_s^4$

i) $N \to \infty$ $g_s \to 0$ $\lambda = g_{YM}^2 N$ fixed.

$\therefore$ $\mathcal{N}=4$ SYM in the t' Hooft limit.

$\simeq$ classical type IIB strings on $AdS_5 \times S^5$

ii) low energy effective description of string theory.

IIB compactified on $AdS_5 \times S^5$

$1 \ll \frac{R^4}{l_s^4} = \lambda$ i.e curvature small.

$\therefore$ weakly coupled string theory

$\sim$ strongly coupled YM

Check on AdS/CFT:

1). Symmetries: $\begin{cases} AdS_5: & SO(4,2) \text{ conformal} \\ 5+1: & \end{cases}$

$\begin{cases} AdS_5 \times S^5 & \text{in } \underline{IIB} \text{ maximal SUSY} \\ N=4 & \text{SUSY} \end{cases}$

$SO(6)$ on S^5 $\sim$ $SU(4)_R$ symmetry

$SL(2, \mathbb{Z})$ symmetry

$$\tau \equiv \frac{4\pi i}{g_{YM}^2} + \frac{\theta}{2\pi} = \frac{i}{g_s} + \frac{\chi}{2\pi}$$

2) Some correlation functions, related to anomalies.

$SU(4)_R$; conformal (Weyl) anomaly

3). The spectrum of chiral operators

4) Partition function

$$e^{-I_{SUGRA}} \simeq Z_{string} = Z_{gauge} = e^{-W}$$

$$Z_{bulk}[\phi(\vec{x}, z)|_{z=0} = \phi_0(\vec{x})] = \langle e^{\int d^4x\, \phi_0(\vec{x}) \cdot O(\vec{x})} \rangle_{\text{field theory}}$$

changes in the boundary conditions of AdS correspond to change in the Lagrangian of the field theory.

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- solve the EOM of IIB SUGRA
- New maximally SUSY solution with non-trivial curvature

  Penrose limit of $AdS_5 \times S^5$

- Penrose limit: enlarge a small neighborhood around a particular null geodesic to full space washing away all the rest of the original background
- Penrose limit is a truncation of the physical spectrum of the original theory: focus only on the excitations that are confined to live close to the null geodesics

  $\longrightarrow$ semi-classical approximation exact!

- $E \sim -i\frac{\partial}{\partial t} \longrightarrow \Delta$

  $J \sim -i\frac{\partial}{\partial \psi} \longrightarrow U(1)_J$ charge (R-charge)

  $\underbrace{SU(2)_V \times SU(2)_H}_{SO(4)} \times U(1)_J \subset SU(4)$

  $\downarrow$ rotation of π

  $\sim$ the shift of x^-

II. plane-wave limit: (BMN)

$$\begin{cases} ds^2_{AdS_5\times S^5} \to ds^2_{PW} = -4dx^+dx^- - \mu^2 (x^i)^2 (dx^+)^2 + (dx^i)^2 \qquad i=1,\dots,8 \\[2ex] F_{+1234} = F_{+5678} = 4\mu \end{cases}$$

$$H_{lc} = 2p^- = i\partial_{x^+} = \mu i(\partial_t + \partial_\psi) = \mu(E-J)$$

$$2p^+ = \frac{E+J}{\mu R^2}$$

$$\Rightarrow \quad \frac{H_{lc}}{\mu} = \Delta - J$$

Penrose limit: $R \to \infty$, $J \sim R^2$ $\Rightarrow \begin{cases} N \to \infty \\ J \sim \sqrt{N} \\ g_{YM} \text{ held fixed} \end{cases}$

$$\Delta \sim J$$

$$\lambda' = \frac{g_{YM}^2 N}{J^2}$$

$$E_{l.c} = \mu N_0 + \mu (N_n + \tilde{N}_n)\sqrt{1 + \frac{n^2}{(\alpha' p^+ \mu)^2}}$$

supergravity sector:

$\lvert 0, p^+\rangle$	$E_{lc} = 0$	$\mathrm{Tr}(Z^J)$
$\alpha_0^{+I}\lvert 0\, p^+\rangle$	$E_{lc} = \mu$	$\mathrm{Tr}(\phi_i Z^J)$ or $\mathrm{Tr}(D_\mu Z\, Z^{J-1})$
$\theta_0^{+}\lvert 0\, p^+\rangle$	$E_{lc} = \mu$	
$\alpha_0^{+I_1}\cdots\alpha_0^{+I_N}\lvert 0\, p^+\rangle$	$E_{l.c} = N\cdot\mu$	$\mathrm{Tr}(\chi_\alpha Z^J)$

BPS protected. $\Delta - J = 1$

Novel features: Double scaling limit

$$N \to \infty \qquad J \sim \sqrt{N} \qquad \lambda \to \infty$$

$$\lambda' = \frac{g_{YM}^2 N}{J^2}$$

Get contributions from graphs of all genera, due to a combinatorial abundance of non-planar graphs growing with J.

Non-planar sector $\sim$ string interactions

$$N^F (g_{YM}^2)^{E-V} = N^{V-E+F} (g_{YM}^2 N)^{E-V}$$

$$= N^{2-2g} \lambda^{E-V}$$

$$g_s \sim N^{-1}$$

$$\left\{ \begin{array}{l} \dfrac{1}{(\alpha' p^+ \mu)^2} = \dfrac{g_{YM}^2 N}{J^2} = \lambda' \to \text{classical quantity related to the worldsheet dynamics} \\ g_2 = \dfrac{J^2}{N} = 4\pi g_s (\mu \alpha' p^+)^2 \end{array} \right.$$

classify topology of diagrams

$\hookrightarrow$ quantum expansion on the string side.

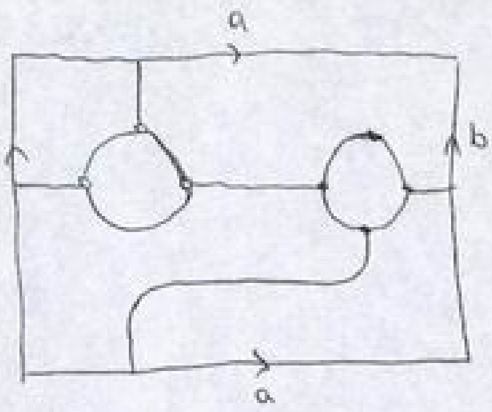
- String interaction

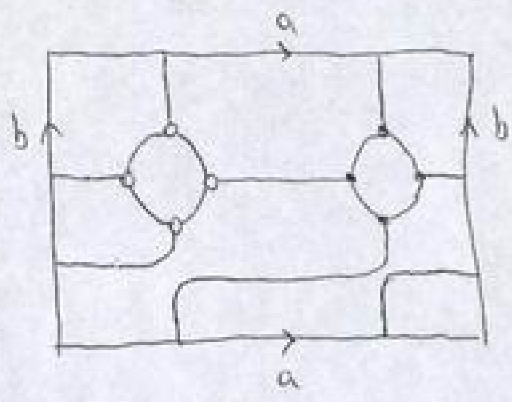
$\sim$ closed string field theory in the light-cone gauge

Example:

1) $\langle \mathrm{Tr}\, Z^J\, \mathrm{Tr}\, \bar{Z}^J \rangle_{mm} = J N^J \Big\{ 1 + \left[\binom{J}{4} + \binom{J}{3} \right] \frac{1}{N^2}$

$\quad + \left[21 \binom{J}{8} + 49 \binom{J}{7} + 36 \binom{J}{6} + 8 \binom{J}{5} \right] \frac{1}{N^4} + \dots$





$\binom{J}{3} \qquad\qquad \binom{J}{4}$

Using matrix model techniques

$$\Rightarrow \frac{1}{J N^J} \langle \mathrm{Tr}\, Z^J\, \mathrm{Tr}\, \bar{Z}^J \rangle_{mm} \xrightarrow{J,\,N\to\infty} 1 + \frac{1}{24}\frac{J^4}{N^2} + \frac{21}{8!}\frac{J^8}{N^4} + \dots$$

$$= \frac{2N}{J^2} \sinh\left(\frac{1}{2} \frac{J^2}{N} \right)$$

2) $O_p^J(x) = \mathrm{Tr}(\phi_1 Z^p \phi_2 Z^{J-p})(x)$

$\langle O_p^J \bar{O}_q^J \rangle = \mathrm{Tr}(\phi_1 Z^p \phi_2 Z^{J-p})\, \mathrm{Tr}(\bar{Z}^{J-q} \bar{\phi}_2 \bar{Z}^q \bar{\phi}_1)$

$\quad = \mathrm{Tr}(Z^p \phi_2 Z^{J-p} \bar{Z}^{J-q} \bar{\phi}_2 \bar{Z}^{q})$

$\quad = \mathrm{Tr}(\bar{Z}^q Z^p)\, \mathrm{Tr}(Z^{J-p} \bar{Z}^{J-q})$

III. $AdS_5 \times S^5$ string theory:

$$\begin{cases} \vec{X}_I \vec{X}_I = 1 & I = 1, \dots 6 \quad \Rightarrow S^5 \\ \eta^{MN} Y_M Y_N = -1 & \eta^{MN} = - + + + + - \quad \Rightarrow AdS_5 \end{cases}$$

$$I = T \int_0^{2\pi} d\sigma \int_0 d\tau \left(\partial Y^M \partial Y^N \eta_{MN} + \partial X^I \partial X^I + \text{fermions} \right)$$

where $T = \frac{R^2}{2\pi\alpha'} = \frac{\sqrt{\lambda}}{2\pi}$.

this is a 2-d CFT, integrable.

- Spectrum of string states?

$$E = E(Q_i, n_k, T)$$

Q_i: 2 + 3 Cartan charges of $SO(2,4) \times SO(6)$

$(\underbrace{S_1, S_2}_{AdS_5 \text{ spins}}; \underbrace{J_1, J_2, J_3}_{S^5 \text{ spins}})$ + higher charges n_k

- EOM
- Virasoro constraints
- Spinning strings

Duality.

quantum string states $\longleftrightarrow$ quantum SYM states.

$\mathrm{Tr}(\cdots)$

$$E_{AdS}(\lambda, J \ldots) = \Delta(\lambda, J \cdots) \ ?$$

$$\begin{cases} E = \sum_n \frac{c_n}{(\sqrt{\lambda})^n} & \alpha'\text{-expansion} \\ \Delta = \sum_n a_n \lambda^n & \text{perturbation theory} \end{cases}$$

How to check ?

- BPS sector: protected states.

  SUGRA modes $\longleftrightarrow$ chiral Primary operator's
  (point like strings) $\mathrm{Tr}\, \Phi_{\{i_1} \cdots \Phi_{i_n\}}$

- Near-BPS sector:

  Nearly point-like strings $\longleftrightarrow$ $\mathrm{Tr}(\Phi^J \ldots)$

- Non-BPS sector

  "large" closed strings $\longleftrightarrow$?

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key idea: (inspired by BMN on plane-wave limit
studied first by Gubser, Klebanov & Polyakov (2002))

look at subsectors of states with large (semi-classically) quantum number: $J \sim T \sim \sqrt{\lambda}$

$\Rightarrow$ new limit:

$$J \to \infty \qquad \tilde{\lambda} \equiv \frac{\lambda}{J^2} = \text{fixed}$$

Semi-classical string state + fluctuations

$$\longleftrightarrow \text{"Long" SYM operators } \mathrm{Tr}(\Phi \cdots \Phi)$$

$$E = J + f(\lambda, J)$$

Near BPS $\equiv$ BMN sector $\mathrm{Tr}(\Phi_1^J \Phi_2 \cdots)$

semi-classical state: point like

$$E = J$$

fluctuation spectrum: quantitative match

$$E_n = J + \sqrt{1+\tilde{\lambda} n^2}\, N_n + O\left(\frac{1}{J}\right)$$

Analytic in $\tilde{\lambda}$!

- 1-loop, 2-loop
- Impurity 2 case: all loops

(Zanon et. al)

Non-BPS states:

Quantitative agreement as in BMN case ?

Consider states with several large angular momenta !
(Tseytlin & Frolov)

Find Multi-spin solutions with large J in S^5:

$$\begin{cases} E = \sqrt{\lambda}\, \mathcal{E}(\omega_i) \\ J_i = \sqrt{\lambda}\, \omega_i \end{cases}$$

$$J = \sum_i J_i$$

$$E = E(J_i, \lambda) \longrightarrow \text{classical energy}$$

$$= J + c_1 \frac{\lambda}{J} + c_2 \frac{\lambda^2}{J^3} + \cdots$$

$$= J\left(1 + c_1 \tilde{\lambda} + c_2 \tilde{\lambda}^2 + \cdots\right)$$

Analytic in $\tilde{\lambda}$

$c_n = c_n\left(\frac{J_i}{J}\right)$ finite in the limit of $J_i \to \infty$ $\tilde{\lambda} = \text{fixed}$

String σ-model loop correction.

$$E_{total} = E + \mathcal{E}_1(\omega_i) + \frac{1}{\sqrt{\lambda}} \mathcal{E}_2(\omega_i) + \cdots$$

$$= E + \sum_{n=1}^{\infty} E_n$$

$$E_n = \frac{1}{(\sqrt{\lambda})^{n-1}} \mathcal{E}_n(\omega_i)$$

$$= \frac{d_{n1}\lambda}{J^{n+1}} + \frac{d_{n2}\lambda^2}{J^{n+3}} + \cdots$$

vanishing when $J \to \infty$

$$\therefore E_{tot} = J\left[1 + \sum_{k=1}^{\infty}\left(\frac{\lambda}{J^2}\right)^k\left(c_k + \sum_{n=1}^{\infty}\frac{d_{nk}}{J^n}\right)\right]$$

$\mathcal{N}=4$ SYM: superconformal, integrable?

$$\Delta = n + \gamma(\lambda)$$

↳ anomalous dimension

BPS: $\gamma(\lambda) = 0$

near BPS: (BMN) planar contribution ⇒ ~~pert~~ fluctuations

Non-BPS: first expanding in λ and then expanding in $\frac{1}{J}$.

$$\Delta(\lambda, J) = J + \sum_{k=1}^{\infty} f_k(J)\lambda^k$$

$$= J\left(1 + \sum_{k=1}^{\infty} a_k\left(\frac{\lambda}{J^2}\right)^k + \cdots\right)$$

Check: $c_k = a_k$?

1-loop: $c_1 = a_1$ without doubt!

2-loop: $c_2 = a_2$

higher-loop: ?

On string side, classical ~~solution~~ energies easy to calculate

while on SYM side, seems to be hard to compute.

Key: $\gamma(\lambda)$ = Hamiltonian of integrable spin chain

1-loop: $SO(6)$ $(SU(2,2|4))$ spin chain

nearest neighbor interaction

2-loop: elliptic spin chain

Next near-neighbour interaction

Examples:

1) circular string in $S^3 \subset S^5$ with $J_1 = J_2$

$$\begin{cases} \tilde{X}_1 = x_1 + i x_2 = \cos n\sigma \, e^{i\omega\tau} \\ \tilde{X}_2 = x_3 + i x_4 = \sin n\sigma \, e^{i\omega\tau} \end{cases}$$

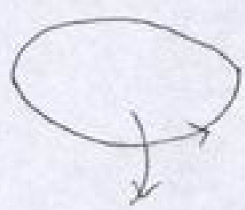
$t = k\tau$ $\qquad k^2 = \omega^2 + n^2$

$E = \sqrt{\lambda}\, k \qquad J = \sqrt{\lambda}\, \omega$

$$E_{class} = \sqrt{J^2 + n^2 \lambda}$$

$$= J\left(1 + \frac{n^2}{2}\tilde{\lambda} - \frac{n^4}{8}\tilde{\lambda}^2 + \dots\right)$$

- String stretched along a great circle of S^5
- rotates ω



- SYM:

$$O = \mathrm{Tr}\left(\Phi_1^{J_1} \Phi_2^{J_2}\right) + \dots$$

SU(2) sector: $\Phi_1 \; |\uparrow\rangle \qquad \Phi_2 : |\downarrow\rangle$

Hamiltonian: periodic 1-d chain $XXX_{1/2}$

$J = J_1 + J_2$ sites

Remarkable agreement with string theory!

2) Folded rotating string orbiting in S^5:

$$\begin{cases} \check{X}_1 = \cos\psi(\sigma)\, e^{i\omega_1\tau} \\ \check{X}_2 = \sin\psi(\sigma)\, e^{i\omega_2\tau} \end{cases}$$

in the circular case. $\psi(\sigma) = n\sigma$

s.t. ~~...~~ winding #.

$$\psi(\sigma+2\pi) = \psi(\sigma) + 2\pi n$$

in the folded case: $\psi(\sigma + 2\pi) = \psi(\sigma)$

$$E = J\,(1 + c_1\check{\lambda} + c_2\check{\lambda}^2 + \cdots)$$

$$c_n = c_n\left(\frac{J_1}{J_2}\right)$$ expressed in elliptic integral.

$E_{cir} > E_{folded}$.

- Same $\mathcal{O} = \mathrm{Tr}(\Phi_1^{J_1}\Phi_2^{J_2}) + \cdots$

but different distribution of the Bethe roots.

{ Beisert, Minahan, Staudacher, Zarembo

{ Beisert, Frolov, St—, A. Tseytlin

3) Folded (S, J) string: spin also in AdS_5 (14

$$\begin{cases} \tilde{Y}_1 = Y_1 + iY_2 = 0 & \tilde{X}_1 = 0 \\ \tilde{Y}_2 = Y_3 + iY_4 = \sinh\rho\, e^{i\phi_1} & \tilde{X}_2 = 0 \\ \tilde{Y}_0 = Y_5 + iY_0 = \cosh\rho\, e^{it} & \tilde{X}_3 = X_5 + iX_6 = e^{i\varphi_3} \end{cases}$$

$$\phi_1 = \omega_1 \tau \qquad \varphi_3 = \omega_3 \tau \qquad t = \tau$$

$$\rho(\sigma) = \rho(\sigma + 2\pi) \qquad \text{no winding}$$

- String stretched in the radial direction ρ of AdS
- rotates ω_1 in AdS_5 & a large circle of S^5

- SYM: $\mathcal{O} = \mathrm{Tr}(D^S \Phi_2^J) + \cdots$

$$\begin{cases} S = 0 \Rightarrow \text{point-like string i.e. BMN} \\ \omega_3 = 0 \Rightarrow \text{folded string in } AdS_5 \text{ (GKP)} \end{cases}$$

$$E_{dis} = J + S + \frac{\lambda}{J} c_1\left(\frac{S}{J}\right) + \cdots$$

$\mathcal{O}$: $SL(2)$ sector

$$a_1 = c_1$$

- Folded (S, J) $\sim$ Folded (J_1, J_2)

$\exists$ analytic continuation!

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Many other examples:

- $SU(3)$ sector three spins in S^5

  $Tr(\Phi_1^{J_1}\Phi_2^{J_2}\Phi_3^{J_3}) \sim (J_1 \quad J_2 \quad J_3)$

- Pulsating string:

Questions:

i) how to understand this agreement beyond specific examples, i.e. in a more universal way?

ii) which is the precise relation between profiles of string solutions and the structure of the dual SYM operat

iii) why the two limits: first $J\to\infty$ then $\tilde{\lambda}\to 0$ (string side)

&, $\tilde{\lambda}\to 0$ then $J\to\infty$ (SYM)

taken on the string and SYM sides

give equivalent results for the first two orders in expansion in $\tilde{\lambda}$?

why it doesn't work to all orders?

L16

why it works?

- "Derive" string σ-model from SYM?
- Precise relation between string profiles and structure of dual operators?

<u>key idea</u>: match effective 2-d actions on string and spin chain sides in the limit $J \to \infty$, $\lambda' \to 0$

Compare semiclassical (coherent) states of string to semiclassical (coherent) states of spin chain.

- Ferromagnetic spin chain: $J \to \infty$ limit with J_1/J_2 = fixed $\Rightarrow$ large clusters of spins

$$\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow \quad \downarrow\downarrow\downarrow \quad \uparrow\uparrow\cdots\uparrow$$

effective low energy description in continuum limit.

$\Rightarrow$ σ-model ("non relativistic")

(by: Kruczenski, A. Tseytlin, et al)

- Another way:

Match general solution integrable structures of string σ-model and Bethe ansatz

Coherent states:

1. Harmonic oscillator:

$$[a,\ a^{+}] = 1$$

$$a|u\rangle = u|u\rangle \qquad u \in \mathbb{C}$$

$$|u\rangle = R(u)|0\rangle$$

$$= e^{ua^{+} - u^{*}a}|0\rangle = e^{ua^{+}}|0\rangle$$

$$\sim \sum_{n=0} \frac{u^{n}}{\sqrt{n!}}|n\rangle$$

2. Alternative definition:

it is a state with minimal uncertainty for both

$$\hat{q} = \frac{1}{\sqrt{2}}(a + a^{+})$$

$$\hat{p} = -\frac{i}{\sqrt{2}}(a - a^{+})$$

$$(\Delta\hat{p}^{2})^{0} = (\Delta\hat{q}^{2}) = \frac{1}{2}$$

where $\Delta\hat{p}^{2} \equiv \langle u|\hat{p}^{2}|u\rangle - (\langle u|\hat{p}|u\rangle)^{2}$

$\therefore$ It is the "best" approximation to a classical state.

if $|u(t)\rangle = e^{-iHt}|u\rangle$

then $\langle u|\hat{q}|u\rangle = \frac{1}{\sqrt{2}}(u + u^{*})$

$\langle u|\hat{p}|u\rangle = -\frac{i}{\sqrt{2}}(u - u^{*})$ follow the classical traject.

Example: $SU(2)$ algebra

$$[S_3, S_\pm] = \pm S_\pm \qquad [S_+, S_-] = 2S_3$$

$s = 1/2$ repr. where $\vec{S} = \frac{1}{2}\vec{\sigma}$

1) Spin coherent state:

$$|u\rangle = R(u)|0\rangle$$

$$R = e^{uS_+ - u^* S_-}, \qquad |0\rangle = |\tfrac{1}{2}, \tfrac{1}{2}\rangle \uparrow$$

$$u \in \mathbb{C}$$

2) $|\vec{n}\rangle = R(\vec{n})|0\rangle \qquad |0\rangle : (0\ 0\ 1)$

$$\vec{n} = U^+ \vec{\sigma} U \qquad U = (u_1, u_2)$$

$\therefore$ $R(\vec{n})$ is an $SO(3)$ rotation from north pole to a generic point $\vec{n}$ on S^2

$$\langle\vec{n}|\vec{S}|\vec{n}\rangle = \frac{1}{2}\vec{n}$$

$$1 = \left(\frac{2s+1}{4\pi}\right)\int \sin\theta\, d\theta\, d\phi\, |\vec{n}\rangle\langle\vec{n}|$$

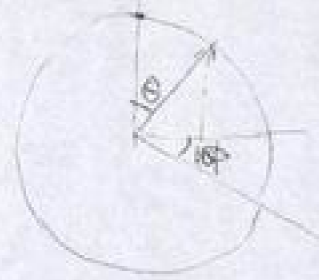
Similar definition of coherent states can be given in the case when $SU(2) \to G$

$$|\vec{n}\rangle = e^{iS_z\phi}\, e^{iS_y\theta}\, |s, s\rangle$$

$\uparrow$

$S_z = s$ total spin

$$\vec{n} = \left(\sin\theta\cos\phi \quad \sin\theta\sin\phi \quad \cos\theta \right)$$



$$1 = \left(\frac{2s+1}{4\pi}\right) \int \sin\theta\, d\theta\, d\phi\, |\vec{n}\rangle\langle\vec{n}|$$

$$\langle \vec{n} | \vec{S} | \vec{n} \rangle = \frac{1}{2}\vec{n}$$

$$Z = \int \mathcal{D}\vec{n}\, e^{iS(\vec{n})}$$

$\langle H \rangle$

$\downarrow$

$$S(\vec{n}) = s \sum_k \int dt \underbrace{\int_0^1 d\tau\, \vec{n}_k \cdot \left(\partial_t \vec{n}_k \times \partial_\tau \vec{n}_k \right)}_{\text{WZ term}} - \frac{\lambda S^2}{2} \int dt \sum_k \left(\vec{n}_k \cdot \vec{n} \ldots\right.$$

- WZ term: is equivalent to a magnetic charge s at the center of the S^2 over which $\vec{n}$ is moving

  charge s $\longrightarrow$ $(2s+1)$ levels

- This is equivalent to the Heisenberg chain exactly
- $J \to \infty$ continuum limit.

Ground state: all spins aligned parallel

Highly excited states: quantum number are large

$\longrightarrow$ well described by classical solutions of F.Q.M.: In quantum theory as superposition of a large # of eigenstates of the same approx energy phase space path integral $\longrightarrow$ integral over the overcomplete set of coherent states

$$(q,\ p) \longrightarrow u = \frac{1}{\sqrt{2}}(q + ip)$$

$$Z = \int [du]\, e^{iS[u]}$$

$$S = \int dt \left(\underbrace{\langle u | i \frac{d}{dt} | u \rangle}_{\uparrow} - \langle u | H | u \rangle \right)$$

"Wess-Zumino" or "Berry phase".

Analogue to $p\dot{q}$ term.

For Heisenberg spin chain, integration over $\vec{n}_\ell(t)$

$$S = \int dt \sum_{\ell=1}^{J} \left[\vec{C}(n_\ell)\cdot \dot{\vec{n}}_\ell - \frac{\lambda}{2(4\pi)^2}(\vec{n}_{\ell+1} - \vec{n}_\ell)^2 \right]$$

where $dC = \epsilon^{ijk} n_i\, dn_j \wedge dn_k$ (i.e. $\vec{C}$ is a monopole potential on S^2)

$$\vec{n} = (\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$$

$$\vec{C}\cdot d\vec{n} = \frac{1}{2}\cos\theta\, d\phi$$

$J \to \infty$ with fixed $\tilde{\lambda} = \frac{\lambda}{J^2}$

$$\vec{n}(t,\sigma) = \left\{ \vec{n}\left(t, \frac{2\pi}{J}\ell\right) \right\} \quad \ell = 1, \cdots J.$$

$\Rightarrow$

$$S = J \int dt \int_0^{2\pi} \frac{d\sigma}{2\pi} \left[\vec{C}\cdot \partial_t \vec{n} - \frac{1}{8}\tilde{\lambda}(\partial_\sigma \vec{n})^2 + \cdots \right]$$

higher derivative term $O\left(\frac{1}{J}\right)$

$\frac{1}{J}$: coupling.

$\therefore$ Quantum corrections suppressed in $J \to \infty$ limit

$\therefore$ The above action can be treated classically

E.O.M

$$\partial_t n_i = \frac{1}{2}\tilde{\lambda}\,\epsilon_{ijk}\, n_j\, \partial_\sigma^2 n_k .$$

Landau-Lifshitz eqs. for a classical ferromagnet, whose solution ~ exact eigenstates from ABAE (~~mag~~ macroscopic ferromagnet)

From string side

← collective coordinate

i) Isolate a "fast" coor. α whose momentum P_α large

ii) gauge fix $t \sim \tau$. $P_\alpha \sim J$.

iii) Expand the action in derivatives of "slow" or "transverse" coordinates (u, $\vec{n}$)

Example: SU(2) sector (circular string)

$$\tilde{X}_1 = X_1 + iX_2 = u_1 e^{i\alpha}$$

$$\tilde{X}_2 = X_3 + iX_4 = u_2 e^{i\alpha}$$

$\alpha \to S^1$

$\downarrow$

$u_i u_i^* = 1 \Rightarrow CP^1$

(Hopf S^1 fibration)

u_i: slow coordinates;

$$dX_i\, dX_i^* = (d\alpha + C)^2 + Du_i\, Du_i^*$$

$$C = -i u_i^*\, du_i$$

$$Du_i = du_i - iCu_i$$

pf: $n_i \equiv U^{\dagger}\sigma_i U \qquad U = (u_1 \ \ u_2)$

$$(ds^2)_{S^3} = (D\alpha)^2 + \frac{1}{4}\, dn_i\, dn_i$$

$$D\alpha = d\alpha + C(n)$$

↳ non-local WZ.

$$\Rightarrow \quad I = \sqrt{\lambda}\int d\tau \int_0^{2\pi} \frac{d\sigma}{2\pi}\, L$$

$$L = -\frac{1}{2}\sqrt{g}\, g^{pq}\left(-\partial_p t\, \partial_q t + D_p\alpha\, D_q\alpha + \frac{1}{4}\partial_p n_i\, \partial_q n_i\right)$$

t & α: "longitudinal" coordinates that reflect the redundancy of the reparametrization in string description.
should be ~~eliminated~~ gauged away.

n_i: "transverse" or physical coordinates

After imposing (non-conformal) gauge
(transla. inv.)

$t = \tau \qquad P_\alpha = \text{const} = J$ (the angular momentum is uniformly distributed)

$\Rightarrow$ Same action

- the agreement between the low-energy effective actions:
  not only the matching of energies of coherent states, but also
  the equivalence of the integrable structures

λ'^3 : $\begin{cases} J\to\infty \text{ then } \lambda'\to 0 \\ \lambda\to 0 \text{ then } J\to\infty \end{cases}$ need not to give the same result.

$$Q_{lk} \equiv I - P_{l,k} = \frac{1}{2}\left(1-\vec{\sigma}_l\cdot\vec{\sigma}_k\right)$$

↑

site l interact with site k

$D_0 = I$

$D_1 = 2Q_{l,l+1}$ nearest neibor interac

$D_2 = -2(4Q_{l,l+1} - Q_{l,l+2})$ next nearest

D_3 & D_4 containe $Q_{l,l+2}\,Q_{l+1,l+3}$ ...

i.e. Q^2 term

4-spin interactie, which does not contribute to anomalous dim. in the strict BMN limit.

⇒ interpolating functions:

Resolution: adding "wrapping" contributions to the dilation operator

(or modify Bethe-Ansatz relation)

- Still an open question!

High orders in λ:

- String side: including higher-order terms.

  eliminating higher powers of time-derivatives

  by field redefinitions.

- spin-chain: including 2-loop term (next near-neighbor)

  \+ non-trivial quantum correction

Result: up to 2-loop, everything is still fine.

3-loop: break down!

Generalization to other sectors:

- SU(3) sector.

- SL(2) sector.

- SO(6) sector

- phase-space action for "slow" variables on the string side

$$\cong$$

coherent-state action on the SYM (spin chain) side

$\Rightarrow$ i) how string action "emerge" on the conformal gauge theory

ii) string energies $\simeq$ SYM dimensions (first 2 order in $\frac{1}{\lambda}$)

iii) direct relation between the string profiles and the structure of coherent SYM operators

- Use the duality to uncover the structure of planar SYM theory to all orders in λ

BMN limit $\Rightarrow$ Dilatation operator to all orders ?

(Ryzhov & Tseytlin)

$$D = \Sigma Q + \Sigma QQ + \cdots$$

$$\downarrow$$

$$D^{(1)} = 2 \sum_{l=1}^{L} \sum_{k=1}^{\infty} f_k(\lambda)\, Q_{l,\, l+k}$$

$\uparrow$

hypergeometric function $\xrightarrow[\text{large } k]{} 0$

$\Rightarrow$ spin chain with short-range interactions

$f_k(\lambda)$: smoothly interpolates between the usual perturbative expansion at small λ and $f_k(\lambda) \sim \sqrt{\lambda}$ at large λ

V. Open spinning string. (with X.-J. Wang & Y.-S. Wu)

- setup: D-branes in D3-branes background

  i). D7-brane. $Usp(N)$. $\mathcal{N}=2$ SCFT with fund. matter

  ii) D5-brane: $AdS_4 \times S^2$ in $AdS_5 \times S^5$

  Defect CFT (De Wolfe & N. Mann)

D-branes: introduce fundamental quarks.

open string excitations between D7 and D3's

are in bi-fundamental repr.

- closed string states $\sim$ single trace operators.
- open string states $\sim$ $Q \Phi_{i_1} \cdots \Phi_{i_L} Q$
- $Q \Phi_{i_1} \cdots \Phi_{i_L} Q$: $\gamma(U)$ = Hamiltonian of integrable spin chains with boundary

$$Q \uparrow\uparrow \cdots \downarrow\downarrow \cdots \uparrow Q$$

Example:

1) open strings on plane waves and their TM duals

(D. Berenstein, et. al 2002)

open string version of BMN operators:

$$E_{open} = \Delta - J = 1 + \sum_n N_n \sqrt{1 + \frac{\lambda n^2}{4}}$$

where ground state $\Delta - J = 1$

Open SU(2) spin chain with boundary, few impurities

$$E_{open} = \frac{1}{2} E_{close}(2J), \qquad \Delta E_o = \Delta E_c(2J)$$

2) open spinning string

$\sigma \in [0, \pi]$

$$E_{open} = \frac{1}{2} E_{close}(2S_i, 2J_i \quad)$$

- folded spinning string $E_o = \frac{1}{2} E_c(2S, 2J)$
- folded circular string $E_o = \frac{1}{2} E_c(2J_1, 2J_2)$

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Doubling trick.

1. string:

A closed string can be constructed from two copies of open string

$$E_O = \frac{1}{2} E_C(2S_i, 2J_i)$$

2 Spin chain: from ABAE

Effectively, the length of the chains gets doubled and the impurities form mirror images due to the existence of the ends of the chain

$J \rightarrow 2J$

$J_i \rightarrow 2J_i$

(or $S_i \rightarrow 2S_i$).

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Other cases:

i) $Q D^s \Phi_1^L Q + \cdots \longrightarrow$ boundary $SL(2)$ spin chain.

ii) open pulsating string

iii) Doubling trick:

Remarks.

- thermodynamic limit, boundary conditions not essential
  $\longrightarrow$ higher-loop case.
  i) ϕ open plane-wave;
  ii) open spinning string;
- Coherent state construction.

Summary:

1. Spinning string: generalization of the non-BPS (BMN) to non-BPS sector of string/SYM states

→ nontrivial check on AdS/CFT

When and why this direct relation works or fails?

2. phase space action. coherent state

effective action match on both sides.

→ string action "emerges" on SYM

3. Uncover the structure of planar SYM to all orders?

→ interpolating functions $f(\lambda)$?

4. Integrable structure in SYM ⇒ ?

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