A fantastic lecture to String theorists and Cosmologists

Cosmological Constant, String Landscape and Anthropic Principle

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1 Cosmological Constant Problem

Why the order of magnitude of the dark energy density is much smaller than quantum corrections in quantum field theory?

Corrections from the quantum fields $\sim M^4$

$$\frac{\Lambda}{M_p^4} \sim 10^{-120} \tag{1}$$

$$\frac{\Lambda}{M_{susy}^4} \sim 10^{-60}, \text{if } M_{susy} \sim 1 \text{ Tev} \tag{2}$$

· How to take the back reaction of gravity into account?

Holographic Dark Energy?

· Naturalness.

Supersymmetry?

$\Lambda = \Lambda(M_{susy})$?

Why the dark energy density is the same order as today's critical energy density of our universe? Called the cosmic coincidence problem.

· Anthropic Principle?

<**Mysteries**>

Mechanism due to the accelerating expansion of Universe:

Cosmological constant problem and inflation.

· The potential of scalar fields?

Are scalar fields fundamental?

· Modification of Einstein's Gravity Theory?

M. Turner et al., astro-ph/0410031, astro-ph/0306438, ...

2 String Landscape

2.1 Hidden sectors

· At first, it was hoped that string theory would be unique and explain the various parameters that quantum field theory left unexplained.

· There are many hidden sectors in the whole String/M theory.
The typical compactification of heterotic or type II strings on a Calabi-Yau manifold has hundreds of scalar fields, larger gauge groups and more charged matter.
Far larger groups are possible with branes and non-perturbative gauge symmetry.

Hidden sectors, not directly visible to observation or experiment.

Examples of moduli are the size and shape parameters of the compact internal space.

The low energy properties of string theory can be approximated by field theory. But the low energy approximation may break down in some regions of the landscape.

· Landscape and cosmological constant

$$\mathcal{N}_{vac} \sim c^N, \tag{3}$$

where we assume a hidden sector allow c distinct vacua and we have N hidden sectors.

Banks, Dine and Seiberg, hep-th/0405159;

Brown and Teitelboim, Phys. Lett. B 195 (1987) 177

→ One can expect that vacua will exist with

$$\Lambda \sim \frac{M_p}{\mathcal{N}_{vac}}. \tag{4}$$

We require $\mathcal{N}_{vac} > 10^{120}$.

Bousso and Polchinski, hep-th/0004134

(first argued Flux compactification of string theory)

$$c \sim 10, N \sim 100 - 500 \tag{5}$$

· Anthropic principle helps us to select the acceptable vacua.

2.2 The trouble with de Sitter Space

· The second law of thermodynamics.

Dyson, Lindesay and Susskind, hep-th/0202163

Susskind, hep-th/0302219

The temperature and entropy of de Sitter Space

$$T = \frac{1}{2\pi R}, \tag{6}$$

$$S = \frac{\pi R^2}{G}. \tag{7}$$

The Poincare recurrence time

$$T_r = e^S. \tag{8}$$

On such long time scale the second law of thermodynamics will repeatedly be violated by large scale fluctuations.

The entropy has reached its maximum value and the second law forbids any further interesting history. But on a sufficiently long time scale, large fluctuations will occur. The phase point will return over and over to the neighborhood of any point in phase space including the original starting point. **Called Poincare recurrences.**

· Mathematical conflict between the symmetry of de Sitter Space and the finiteness of the entropy.

Goheer, Kleban, Susskind, hep-th/0212209

Finite entropy indicates that the spectrum of energy is discrete.

Mathematically, The symmetry algebra of de Sitter space can not be realized in a way which is consistent with the discreteness of the spectrum.

Physically, The discreteness of the spectrum means that there is a typical energy spacing of order

$$\triangle E \sim e^{-S}. \tag{9}$$

The discreteness of the spectrum can only manifest itself on time scales of order $(\triangle E)^{-1} \sim T_r$.

My comment: What is the meaning of the energy in de Sitter space?

2.3 de Sitter Space is unstable or meta-stable

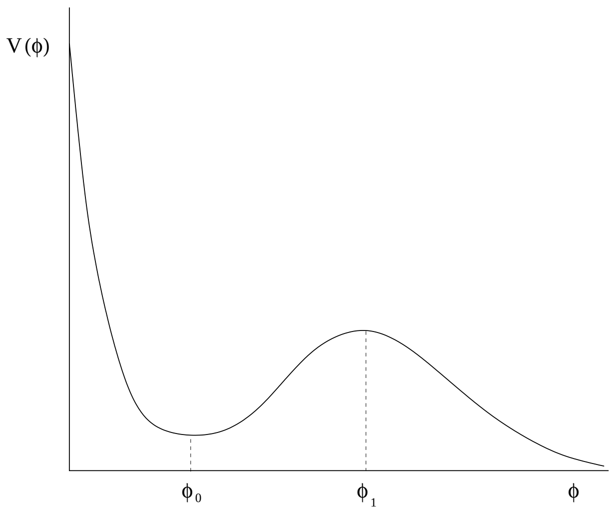

Figure 1.

The vacuum of the causal patch has a finite entropy and fluctuates up and down the walls of the potential.

Or the volume of the causal patch is finite and the total energy required for a tunnelling process is finite.

The probability of tunnelling can be described by Hawking-Moss instanton

$$\mathcal{P} \sim \exp(S_{top} - S_{ds}) \sim \exp\left(\frac{24\pi^2}{V(\phi_1)} - \frac{24\pi^2}{V(\phi_0)}\right). \tag{10}$$

The typical time scale for such a fluctuation to take place

$$T_t \sim \exp(S_{ds} - S_{top}) = e^{-S_{top}} T_r < T_r. \tag{11}$$

· The lifetime of the meta-stable vacuum is **always less than** the Poincare recurrence time.
· Other possibilities. If the cosmological constant is not very small it may tunnel over the nearest mountain to a neighboring valley of smaller positive cosmological constant. This will also take place on a time scale which is too short to allow recurrences.

2.4 Spontaneous decompactification

S. B. Giddings hep-th/0303031

S. B. Giddings and R. C. Myers, hep-th/0404220

· Dimensional reduction.

The action in $4+d$ dimensional space-time

$$S = \int d^{d+4}x\sqrt{-G}\{M_p^{d+2}\mathcal{R} + \mathcal{L}(\psi) + \hat{\mathcal{L}}(\psi,\mathcal{R})\}, \tag{12}$$

where $\hat{\mathcal{L}}(\psi,\mathcal{R})$ summarizes possible corrections.

Assume all the moduli except the overall volume modulus of the internal space are fixed.

$$ds^2 = ds_4^2 + R^2(x)g_{mn}(y)dy^m dy^n, \tag{13}$$

where $R(x) = e^{D(x)}$. The 4-dimensional effective action

$$S = M_4^2\int d^4x\sqrt{-g_4}\left[e^{dD(x)}\mathcal{R}_4 + d(d-1)(\nabla D)^2 e^{dD(x)} + e^{(d-2)D}\mathcal{R}_d\right]. \tag{14}$$

We choose new units by the Weyl re-scaling

$$g_{4\mu\nu} \to e^{-dD}g_{4\mu\nu}. \tag{15}$$

The action in 4 dimensional space-time

$$\begin{aligned} S &= \int d^4x\sqrt{-g_4}\Bigg\{M_4^2\mathcal{R}_4 - \frac{1}{2}M_4^2 d(d+2)(\nabla D)^2 \\ &+ \frac{1}{V_d}\int d^dy\sqrt{g_d}\left[\mathcal{L}(e^{2D}g_d,\psi) + M_p^{d+2}e^{-2D}\mathcal{R}_d + \hat{\mathcal{L}}\right]\Bigg\} \\ &= M_4^2\int d^4x\sqrt{-g_4}\Bigg\{\mathcal{R}_4 - \frac{1}{2}d(d+2)(\nabla D)^2 - V(D)\Bigg\}, \end{aligned} \tag{16}$$

where $V_d = R^d$ and

$$M_4^2 = M_p^{d+2}V_d. \tag{17}$$

· Potential of dilaton.

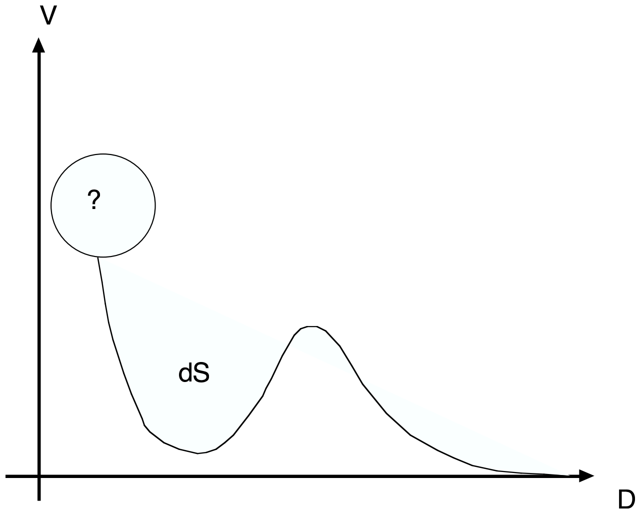

Figure 2.

Banks, Dine and Gorbatov (hep-th/0309170) argued that there are regions in the combined field space of the moduli and metrics where the dynamics becomes strongly coupled. Maybe the effective potential description of the landscape is not justified and cannot be trusted.

Key point: the effective potential for D contains a factor that falls like the inverse volume of the compact manifold.

$$\rho_d \sim V_d^{-(w_d+1)}. \tag{18}$$

If the potential to approach a constant, or grow, at large volume,

$$w_d \leq -2. \tag{19}$$

This violates the null dominant energy condition!

· Asymptotic dilaton dynamics.

Dine and Seiberg: there are always runaway solutions in string theory.

With canonically normalized dilaton,

$$\phi = \sqrt{d(d+2)}D, \tag{20}$$

we find the action of the form

$$S = M_4^2 \int d^4x\sqrt{-g}\left[\mathcal{R}_4 - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right], \tag{21}$$

with

$$V(\phi) = \frac{A}{2}e^{-\alpha\phi}. \tag{22}$$

In these conventions,

$$\alpha = \sqrt{\tfrac{d}{d+2}}\left(1+\tfrac{2q}{d}\right) \quad \text{q-flux in the extra dimensions,} \tag{23}$$

$$= \sqrt{\tfrac{d}{d+2}}\left(1+\tfrac{d+3-p}{d}\right) \quad \text{wraped p-brane, filled in 4 dimensional space-time,} \tag{24}$$

$$= \sqrt{\tfrac{d+2}{d}} \quad \text{internal curvature.} \tag{25}$$

If Universe is dominated by the energy density of ϕ,

$$a \propto t^{1/\alpha^2}, \tag{26}$$

$$w_\phi = \frac{2}{3}\alpha^2 - 1. \tag{27}$$

If $w_\phi < -1/3$, we require $\alpha < 1$. Only for a space-filling brane, $p = d+3$, we have

$$\alpha_{(d+3)-brane} = \sqrt{\frac{d}{d+2}} < 1. \tag{28}$$

Friedmman-Robertson-Walker Universe with $k = 0, \pm 1$

$$ds_4^2 = -dt^2 + a^2(t)d\chi^2, \phi = \phi(t). \tag{29}$$

Equations of motion,

$$H^2 = \frac{\rho_\gamma}{6} + \frac{\dot{\phi}^2}{12} + \frac{V}{6}, \tag{30}$$

$$\dot{H} + \frac{\gamma\rho_\gamma}{4} = -\frac{\dot{\phi}^2}{4}, \tag{31}$$

$$\ddot{\phi} = -3H\dot{\phi} - \frac{dV}{d\phi}, \tag{32}$$

where $\gamma_{matter} = 1$, $\gamma_{radiation} = \frac{4}{3}$, $\gamma_{curvature} = \frac{2}{3}$, and $\gamma_\phi = \frac{2\alpha^2}{3}$.

The solution of these equations

$$ds^2 = -dt^2 + a_0^2[c(t - t_0)]^{2\beta} d\chi^2, \tag{33}$$

$$e^\phi = [c(t - t_0]^{2/\alpha}, \tag{34}$$

where $\beta = \max\left(\frac{2}{3\gamma}, \frac{1}{\alpha^2}\right)$ and $c = \frac{\alpha}{2}\sqrt{\frac{A}{3\beta - 1}}$.

· Expansion to higher dimensions.

Recall a different rescaling,

$$ds^2_{d+4} = [c(t-t_0]^{-2/\lambda}\left[-dt^2 + a_0^2[c(t-t_0]^{2\beta}d\chi^2\right] + [c(t-t_0]^{4/(d\lambda)}ds^2_d, \qquad (35)$$

where

$$\begin{aligned} \lambda = & \quad 1+\tfrac{2q}{d} \quad \text{q-flux}, & (36)\\ = & \quad 1+\tfrac{d+3-p}{d} \quad \text{p-brane}, & (37)\\ = & \quad 1+\tfrac{2}{d} \quad \text{internal curvature.} & (38) \end{aligned}$$

For $\lambda > 1$,

$$ds^2_{d+4} = -d\tau^2 + a_0^2(\tilde{c}\tau)^{2(\beta\lambda-1)/(\lambda-1)}d\chi^2 + (\tilde{c}\tau)^{4/d(\lambda-1)}, \qquad (39)$$

where

$$\begin{aligned} \tilde{c} &= \frac{\lambda-1}{\lambda}c, & (40)\\ \tau &= \frac{1}{c}\frac{\lambda}{1-\lambda}[c(t-t_0)]^{1-1/\gamma}. & (41) \end{aligned}$$

The compact dimensions expand more slowly, if

$$(\beta\lambda - 1)d \geq 2, \qquad (42)$$

which means $q \geq 1$ or $p \leq d+1$.

2.5 Summary

· How to describe the String Landscape?

· If there is String Landscape, how to select the vacua?

· t' Hooft: Super-theory is not super any longer.

· Witten: Not understanding why the cosmological constant is zero, or extremely small, is in my opinion the key obstacle to making the models of particle physics that can be derived from string theory more realistic. We need the cosmological constant as a clue to understand particle physics better.

· Maybe Landscape is a temporary picture. We will find there is only one reasonable vacuum after we know how to break Supersymmetry exactly.

3 Anthropic principle

3.1 Anthropic principle

Weinberg, Phys. Rev. Lett. 59 (1987) 2607

Martel, Shapiro and Weinberg, astro-ph/9701099

No one askes the cosmological problem if the astronomers can not exist at all.

· Anthropic bound on the Cosmological Constant. At recombination,

$$\frac{729\rho_\Lambda}{500\bar{\rho}} \leq \delta^3. \tag{43}$$

Martel, Shapiro and Weinberg: $\mathcal{P}(\rho_\Lambda > \rho_{\Lambda *}) \simeq 90\%$.

· Aguirre et al. stressed that life may exist in universes with some of the cosmological parameters differing from those in our own subuniverse by several orders of magnitude, if larger density fluctuations than those in our universe are allowed.

Aguirre, astro-ph/0106143

Rees, astro-ph/0401424

Tegmark and Rees, astro-ph/9709058

· For cosmological constant

Graesser, Hsu, Jenkins and Wise (hep-th/0407174): $\mathcal{P}(\rho_\Lambda > \rho_{\Lambda *}) \simeq 99.9\%$

· For Holographic dark energy

$$\frac{\rho_\Lambda^{rec}}{\bar{\rho}} < 0.103\delta. \tag{44}$$

Huang and Li (hep-th/0410095): $\mathcal{P}(\rho_\Lambda > \rho_{\Lambda *}) \simeq 73.6\%$

Anthropic Principle Favors the Holographic Dark Energy.

3.2 Supersymmetry Breaking in the Anthropic Landscape

Susskind, hep-th/0405189

Douglas, hep-th/0405279, 0409207

· The distribution in string theory

$$\rho \sim M_{susy}^{2\alpha} d\left(M_{susy}^2\right). \tag{45}$$

In general, $\alpha = 2n_F + n_D - 1 \geq 1$ is surely satisfied by almost all string models and it will favor high scale breaking.

· Hierarchy Problem.

Correction for the mass of Higgs μ: $\mu^2 = \mu_0^2 + M_{susy}^2$

Correction for the cosmological constant: $\Lambda = \Lambda_0 + xM_p^4 + yM_{susy}^4$

· The distribution

$$\text{Susskind:} \quad P(M_{susy}, \mu, \Lambda) = \frac{\Lambda\mu^2}{M_{susy}^6} \tag{46}$$

$$\text{Douglas:} \quad P(M_{susy}, \mu, \Lambda) = \frac{\Lambda\mu^2}{M_{susy}^2 M_p^4} \tag{47}$$

Douglas argue that the spectrum of x is featureless with no singularity at $x = 0$. Later, Susskind agree with him.

Only the gauge hierarchy fine-tuning favors low energy supersymmetry breaking.

· Huang and Li's new proposal ($M_p = 1$)

Naturalness: Supersymmetry will be enhanced if we set $\Lambda = 0$.

Similarly with Banks (hep-th/0007146), we assume $\Lambda \sim M_{susy}^n$.

The distribution of M_{susy} and Λ

$$d\rho(M_{susy}, \Lambda) = \delta(\Lambda - M_{susy}^n) f(\Lambda, M_{susy}) d\Lambda dM_{susy}. \tag{48}$$

Integrating out Λ,

$$\begin{aligned} d\rho(M_{susy}) &\sim f(M_{susy}^n, M_{susy}) dM_{susy} \\ &\sim M_{susy}^{2\alpha} dM_{susy}^2, \end{aligned} \tag{49}$$

Therefore

$$f(M_{susy}^n, M_{susy}) = M_{susy}^{2\alpha+1}. \tag{50}$$

Integrating out M_{susy}, we obtain the distribution of Λ,

$$d\rho(\Lambda) \sim \frac{d\Lambda}{\Lambda^{1-\frac{2\alpha+2}{n}}}. \tag{51}$$

If $n > 2\alpha + 2$, a low scale breaking of supersymmetry will be favored from the viewpoint of Anthropic principle. In other words, Anthropic principle selects $n > 2\alpha + 2$.

Banks proposed $n = 8$ and $M_{susy} \simeq 1$ Tev.

3.3 Quantum Gravity and Quantum cosmology

· Quantum cosmological ($M_p = 1$)

Hartle and Hwaking (Phys. Rev. D 15 (1983) 2960):

$$P \sim \exp\left(\frac{24\pi^2}{V(\phi)}\right). \tag{52}$$

It favors $\Lambda = 0$. But Λ may not vanish at all and how about Inflation?

A. Linde (gr-qc/9802038, hep-th/0408164):

$$P \sim \exp\left(-\frac{24\pi^2}{V(\phi)}\right). \tag{53}$$

Hsu and Zee (hep-th/0406142) (Holographic dark energy)

$$P \sim \exp\left(-\Lambda L^4 - \frac{M_p^4}{\Lambda}\right). \tag{54}$$

Maximize P, we find $\Lambda \sim M_p^2 L^{-2}$.

Firouzjahi, Sarangi and Tye (hep-th/0406107) (Cosmic Landscpae)

$$\Psi \sim \exp \mathcal{F}, \tag{55}$$

where

$$\begin{aligned} \mathcal{F} &= -S_E - D, \\ D &= c\left(\frac{M_s}{2\pi}\right)^9 V_9 + ..., \end{aligned} \tag{56}$$

where D may come from: (i) destructive interference due to small fluctuations of large phases, (ii) quantum decoherence and (iii) space-like brane in string theory.

$$\mathcal{F} = \frac{3\pi}{2G_N\Lambda} - c\frac{g_s^2}{48\pi G_N l_s(\Lambda/3)^{3/2}}. \tag{57}$$

With $c \sim 10^{-3}$, maximizing $\mathcal{F}$ yields $\sqrt{\Lambda} \sim 10^{14}$ Gev and $\mathcal{F} \sim 10^{18}$.

Spontaneous creation of inflationary universes in the cosmic landscape.

· Why Quantum cosmology?

The initial conditions for inflation: inflation appears in a patch of a closed universe or of an infinite flat or open universe if this patch is sufficiently homogeneous and its size is greater than the size of the horizon, $\triangle l \leq H_{-1} \sim V^{-1/2}$, where $V(\phi)$ is a falt inflationary potential.

Inflation: Universe could grows up from a microscopic size l_p (Linde, Phys. Lett. B 129 (1983) 177). Such a tiny region of space can apear as a result of quantum fluctuation of metric (**Quantum Gravity**).

· Some Thoughts.

Quantum gravity dominate the world with size l_p or energy scale M_p.

Chaotic inflation?

Quantum gravity has to face (Quantum) cosmology.

Can we find the direct evidence in the accelerator? If not, how can we check the theory of Quantum Gravity?

Thank you!