CMBA Multi-poles and The Upper Bound on the Number of E-foldings of Inflation

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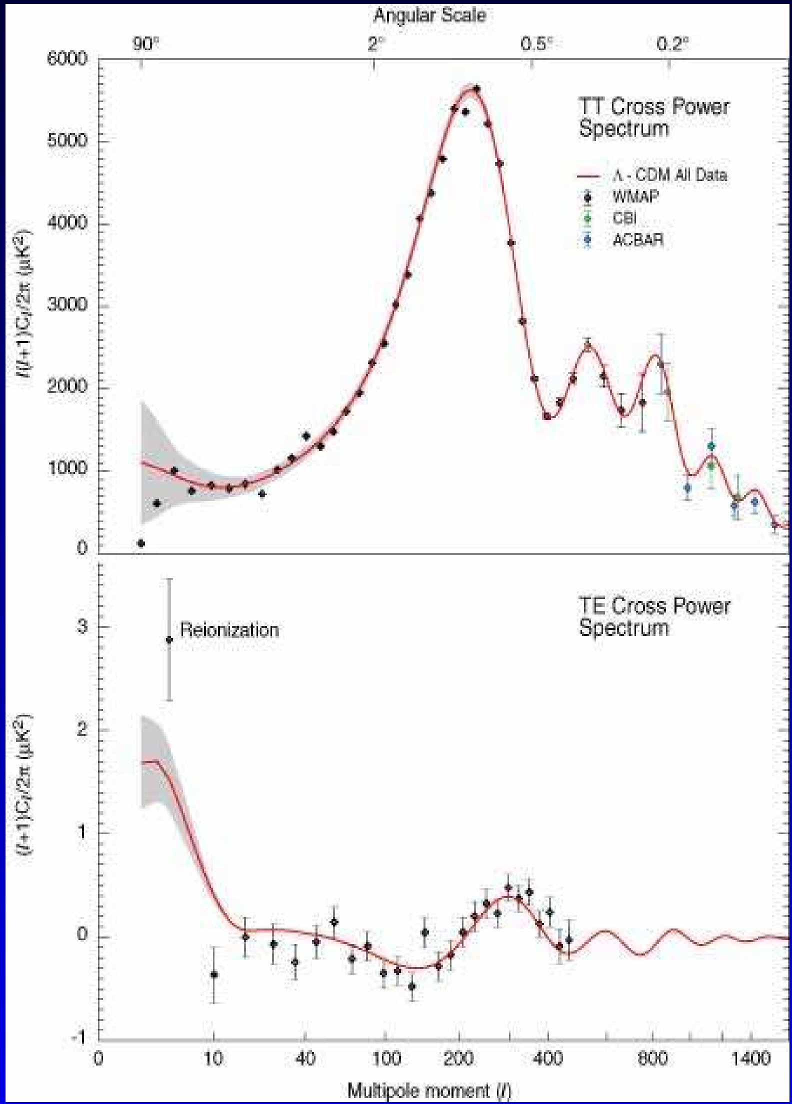
The Problem



astro-ph/0302207

Overview (1)

The essence of CMBA is photons distribution's deviation from the exact Bose-Einstein statistic:

$$f(t,\vec{x},\vec{p}) = \frac{1}{\left[\frac{p}{T(1+\Theta(t,\vec{x},\hat{p}))}\right] - 1}$$

In observations, we detect:

$$\begin{aligned} C(\theta) &= C(t,\vec{x},\theta)_{t=0,\vec{x}=0} \\ &=< \Theta(t,\vec{x},\hat{p}_1)\Theta(t,\vec{x},\hat{p}_2) >_{t=0,\vec{x}=0} \end{aligned}$$

Overview (2)

$$\Theta(t,\vec{x},\hat{p}) = \int d^3k e^{i\vec{k}\cdot\vec{x}}\Theta(t,\vec{k},\hat{p})$$

$$= \sum_{l=0}^{\infty}(-i)^l(2l+1)\int d^3k e^{i\vec{k}\cdot\vec{x}}\Theta_l(t,\vec{k})P_l(\hat{k}\cdot\hat{p})$$

$$\Theta_l(t,\vec{k}) = \Psi(\vec{k})\Theta_l(t,k)$$

$\Psi(\vec{k})$: primordial perturbation amplitude.

$\Theta_l(t,k)$: describe a given perturbation mode's evolution.

Overview (3)

$$< \Theta(t,\vec{x},\hat{p}_1)\Theta(t,\vec{x},\hat{p}_2) >$$

$$= \sum(-1)^l(2l+1)(2l'+1)\int d\vec{k}d\vec{k}' e^{i\vec{k}\cdot\vec{x}+i\vec{k}'\cdot\vec{x}}$$

$$< \Psi(\vec{k})\Psi(\vec{k}') > \Theta_l(t,k)\Theta_l(t,k')P_l(\hat{k}\cdot\hat{p}_1)P_l(\hat{k}\cdot\hat{p}_2)$$

$$\equiv P_\Psi(k)\delta(\vec{k}+\vec{k}')$$

$$= \sum[4\pi\int k^2 dk P_\Psi(k)\Theta_l^2(t,k)]\frac{2l+1}{4\pi}P_l(\hat{p}_1\cdot\hat{p}_2)$$

$$\equiv C_l$$

$P_\Psi(k)$: primordial power spectrum of perturbations.

$\Theta_l^2(t,k)$: photon's transition function, CMBfast.

PPSP & Inflation (1)

$$\Delta_{\mathcal{R}}^{\frac{1}{2}}(k) \approx \frac{k^{\frac{3}{2}}}{\sqrt{2\pi}} |\frac{u_k(\eta)}{z}|_{aH=k}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$H^2 = \frac{8\pi G}{3}[\frac{1}{2}\dot{\phi}^2 + V(\phi)]$$

$$u_k'' + (k^2 - \frac{z''}{z})u_k = 0$$

$$z = \frac{a\dot{\phi}}{H}, \; u_k(\eta)|_{k^{-1}/(aH)^{-1}\to 0} \to \frac{1}{\sqrt{2k}} e^{-ik\eta}$$

PPSP: primordial power spectrum of perturbations

PPSP & Inflation (2)

slow roll approximation:

$$
\begin{aligned}
u_k &= \frac{1}{\sqrt{2k}} e^{i(\nu+\frac{1}{2})\frac{\pi}{2}} \sqrt{\frac{\pi}{2}} (-k\eta)^{\frac{1}{2}} H_\nu^{(1)}(-k\eta) \\
\Delta_{\mathcal{R}}^{\frac{1}{2}}(k) &= \frac{k^{\frac{3}{2}}}{\sqrt{2\pi}} \left|\frac{u_k(\eta)}{z}\right|_{aH=k} \\
&= \frac{k^{\frac{3}{2}}}{\sqrt{2\pi}} \frac{1}{\sqrt{2k}} \sqrt{\frac{\pi}{2}} H_\nu^{(1)}(1) \left[\frac{aH\dot{\phi}}{H^2}\right]_{aH=k}^{-1} \\
&= \frac{1}{\sqrt{8\pi}} |H_\nu^{(1)}(1)| \frac{H^2}{\dot{\phi}} |_{aH=k}
\end{aligned}
$$

PPSP & Inflation (3)

numerical recipes:

$$\frac{d\phi}{da} = (aH)^{-1} f$$

$$\frac{df}{da} = -(aH)^{-1}(-3Hf - V'(\phi))$$

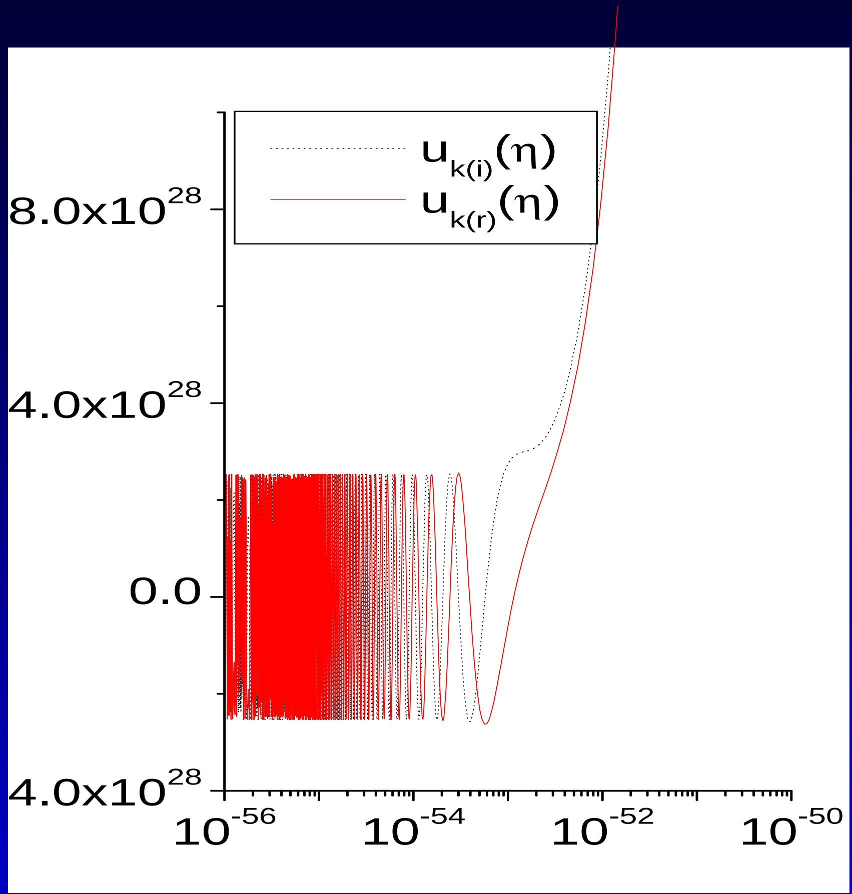
$$\frac{du_k}{da} = (a^2H)^{-1} w_k$$

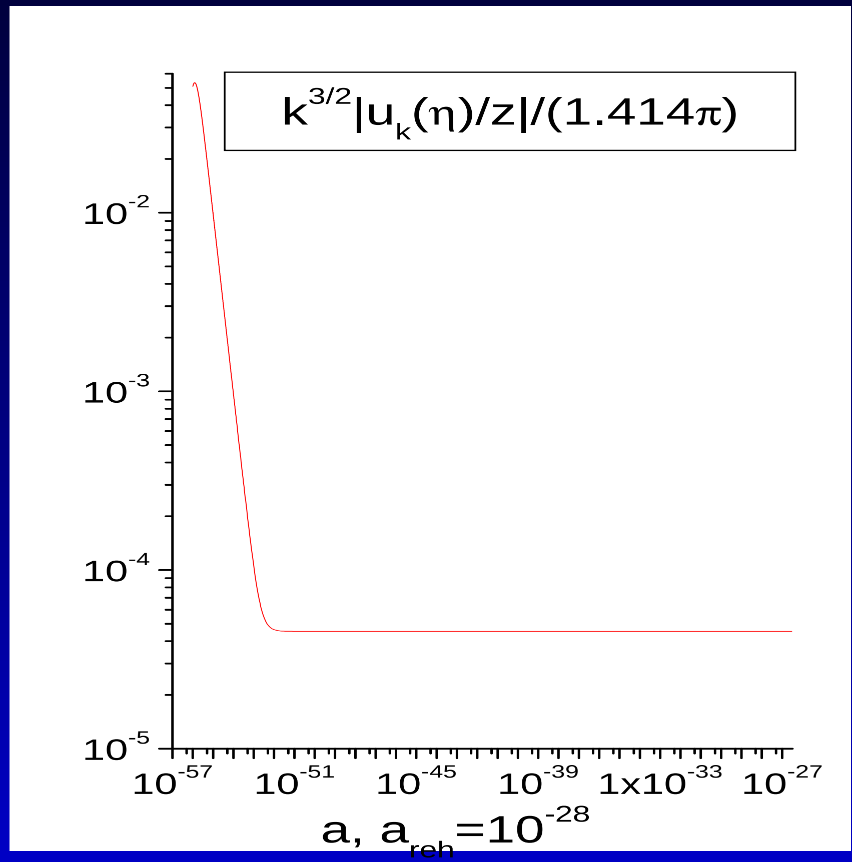
$$\frac{dw_k}{da} = -(a^2H)^{-1}(k^2 - \frac{z''}{z})u_k$$

$$H^2 = \frac{1}{3M_{pl}^2}(\frac{1}{2}f^2 + V(\phi))$$

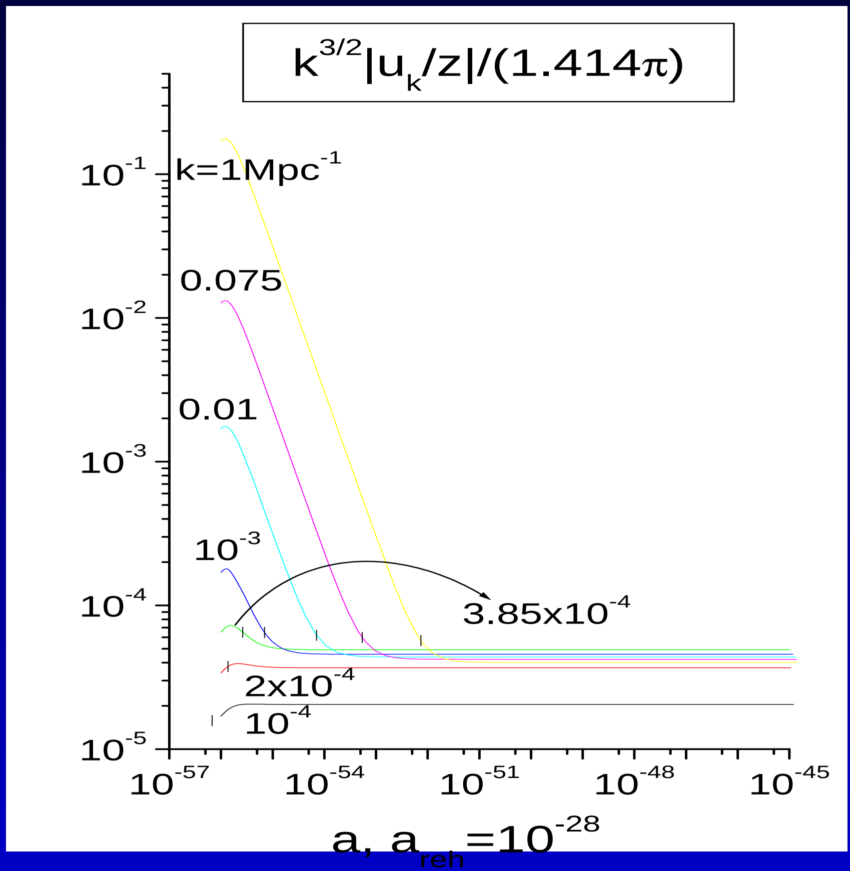
$$\frac{z''}{z} = a^2[2H^2 - V''(\phi) - \frac{7}{2M_{pl}^2}f^2 - \frac{2}{M_{pl}^2}\frac{fV'}{H} + \frac{1}{2M_{pl}^4}\frac{f^4}{H^2}]$$

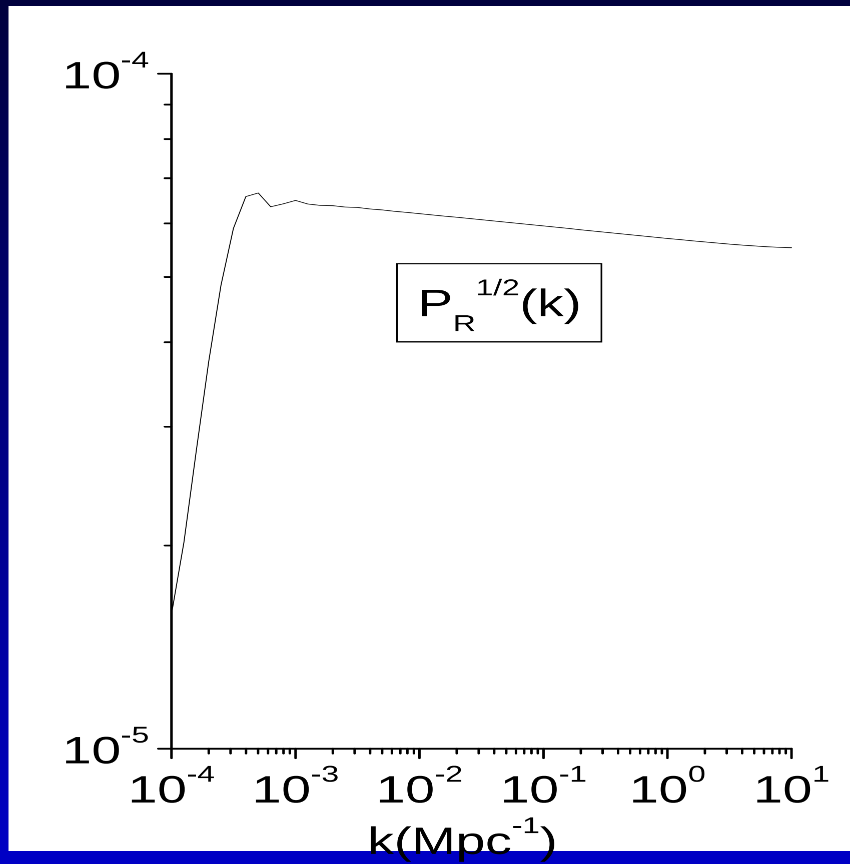
Numerical Results (1)





Numerical Results (2)





PPSP: Why Suppress? (1)

$$u_k(\eta)|_{k^{-1}/(aH)^{-1}\to 0} \to \frac{1}{\sqrt{2k}} e^{-ik\eta}$$

$a_{i.b.} = [e^{N_e}\frac{T_{reh}}{T_{tod}}]^{-1} = 10^{-56}$, $N_e = 65$, $T_{reh} = 10^{15}Gev$, $T_{tod} = 10^{-4}ev$. In numerical treatments, we take $a = 10^{-56}$ as the integration starting point, think that at this point $k^{-1}/[aH]^{-1} \to 0$ and write the initial condition as:

$$u_k(\eta)|_{a=10^{-56}} = \frac{1}{\sqrt{2k}} e^{-ik\eta}$$

PPSP: Why Suppress? (2)

In Linde's ϕ^2 model,

$$[aH]^{-1}_{i.b.} = [10^{-56}\{\frac{1}{3M_{pl}^2}\Lambda^4_{infl}\}^{1/2}]^{-1} = [7802Mpc]^{-1}$$

for perturbation modes of $k \sim (10^{-4}, 10^{-3})Mpc^{-1}$, when inflation begins, $k^{-1}/(aH)^{-1} = O(1) \nrightarrow 0$, so

$$u_k(\eta)|_{a=10^{-56}} = \frac{1}{\sqrt{2k}}e^{-ik\eta}$$

is a wrong initial condition.

PPSP's Calculation: Correct Way

If we let $a_{i.b} = 10^{-58}$, then still in Linde's ϕ^2 model,

$$[aH]^{-1}_{i.b.} = [10^{-58}\{\frac{1}{3M_{pl}^2}\Lambda^4_{infl}\}^{1/2}]^{-1} = [67Mpc]^{-1}$$

for perturbation modes of $k \sim (10^{-4}, 10^{-3})Mpc^{-1}$, when inflation begins, $k^{-1}/(aH)^{-1} < 10^{-2} \approx 0$, so

$$u_k(\eta)|_{a=10^{-58}} = \frac{1}{\sqrt{2k}}e^{-ik\eta}$$

could be taken as a correct initial conditions.

Upper Bound on N_e

If the number of e-foldings of inflation has an upper bound, according to the following relation,

$$a_{i.b.} = [e^{N_e} \frac{T_{reh}}{T_{tod}}]^{-1}$$

Then the scale factor when inflation begins cannot be set as small as we want to. By T. Banks and W. Fishler's Estimation[hep-th/0307459], $N_{e(u.b.)} = 65$, so when inflation begins, the scale factor cannot be smaller than 10^{-56}

Physics Implication of $N_{e(u.b.)}$ (1)

Requiring the universe inflate from a more little scale factor could get the large scale perturbation modes falling inside the horizon when inflation begins.

$$k^{-1} << [aH]^{-1}_{i.b.}$$

Number of e-foldings of inflation has an upper bound implies that we cannot set the scale factor as small as we want to. As long as the scale of perturbations is sufficiently large,

$$k^{-1} \not< [aH]^{-1}_{i.b.}$$

Such perturbations lie outside the horizon.

Physics Implication of $N_{e(u.b.)}$ (2)

The upper bound on the number of e-foldings of inflation implies the upper bound on the comoving horizon of universe as we trace back to the origin of it.

$$(aH)^{-1}_{u.b.} = e^{N_{u.b.}} \frac{T_{reh}}{T_{tod}} \frac{\sqrt{3}M_{pl}}{\Lambda_{infl}}$$

perturbations at scales greater than this scale lie outside the horizon of universe regardless how early we trace back to the origin of universe.

Physics Implication of $N_{e(u.b.)}$ (3)

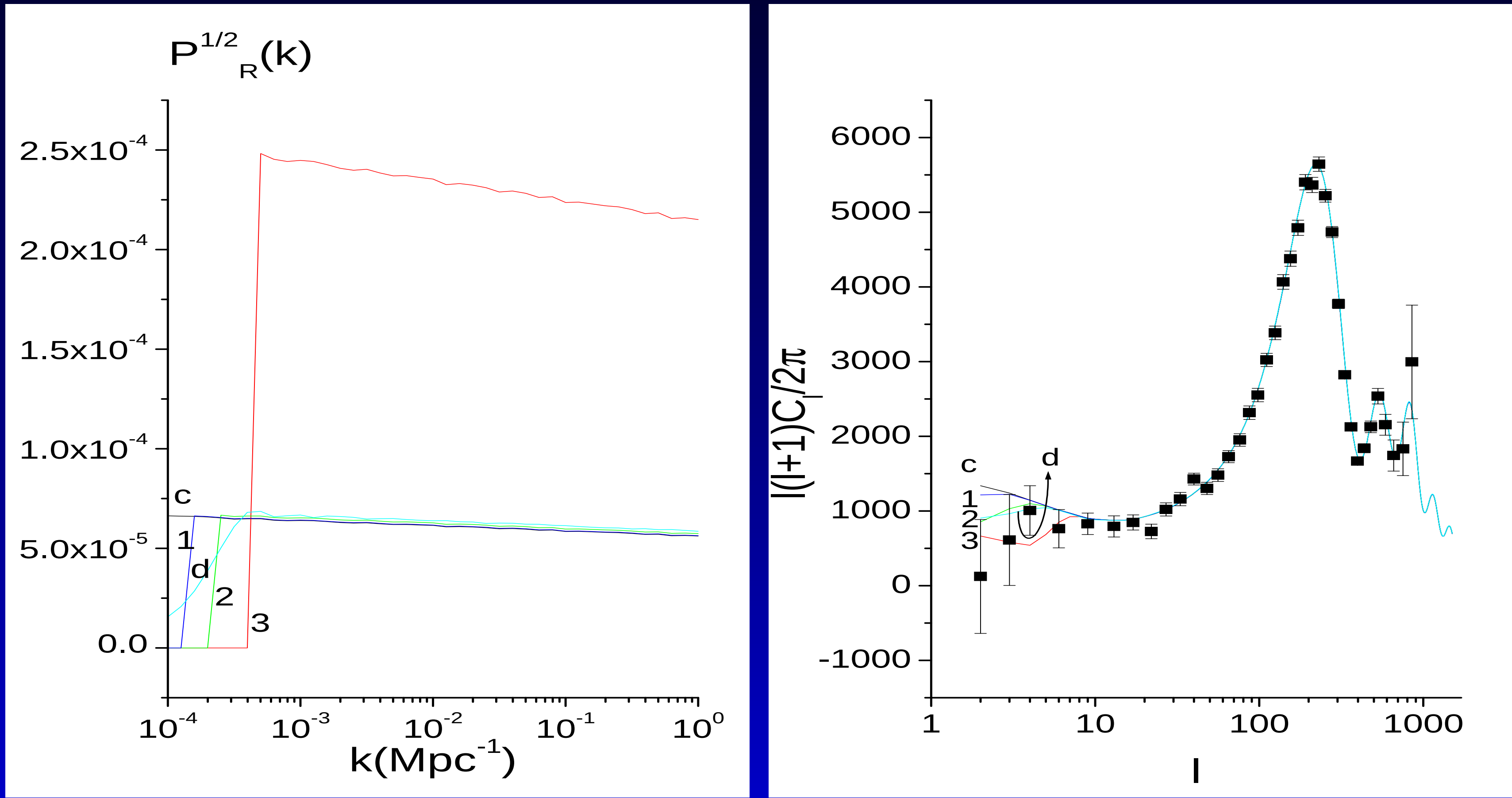
We suppose that such perturbations either do not exist, or they do not affects the large scale structure formation of the universe. So they have no contributions to CMBA multi-poles.

$$C_l = 4\pi \int_{k_c}^{\infty} dk P_{\mathcal{R}}(k) \Theta_l^2(t,k) \frac{2l+1}{4\pi}$$

$$k_c^{-1} = (aH)_{u.b.}^{-1} = e^{N_{u.b.}} \frac{T_{reh}}{T_{tod}} \frac{\sqrt{3} M_{pl}}{\Lambda_{infl}}$$

$N_{u.b.} = 65$, $T_{reh} = 10^{15} Gev$, $T_{tod} = 10^{-4} ev$, $\Lambda_{infl} = 8 \times 10^{-3} M_{pl}$ yields $k_c = (7082 Mpc)^{-1}$

Physics Implication of $N_{e(u.b.)}$ (4)



$k^{-1}_{c1,2,3} = 7082,\ 4225,\ 2113\ Mpc$ respectively.

Conclusions

1. In numerical calculating the PPSP produced during inflation, we must integrate the perturbation mode equations beginning from a smaller scale factor, which means we require the universe inflate more number of e-foldings.

2. If the number of e-foldings of inflation has an upper bound, then perturbations beyond the comoving horizon when inflation begins maynot contribute to CMBA.

Thanks!