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Scalar type interaction in

$\tau^- \to \pi^- \pi^0 \nu_\tau$ decay

Dao-Neng Gao

ICTS USTC
Oct. 25 2004

Outline

- Introduction and Motivation

- Scalar type interaction and angular distribution asymmetry in $\tau^- \to \pi^- \pi^0 \nu_\tau$

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Introduction and Motivation

* τ decays provide an ideal tool both to test the electroweak sector and the strong sector in the standard Model

$$m_\tau \doteq 1777 \text{ MeV}$$

the heaviest lepton observed so far

The lightest hadron

$$m_\pi = 135 \text{ MeV} > \begin{cases} m_\mu = 105.6 \text{ MeV} \\ m_e = 0.511 \text{ MeV} \end{cases}$$

So among the leptons. Only τ lepton can decay into the final states containing hadrons (quark bound states)

$\Rightarrow$ τ semileptonic decays involve both weak and strong interactions !

Note, for hadrons in the final state

- NO heavy flavour hadrons $\boxed{m_{D^0} \doteq 1864 \text{ MeV}}$
- No baryons, since

$m_p = 938$ MeV proton lightest baryon

$m_\tau < 2 m_p$

$\Uparrow$

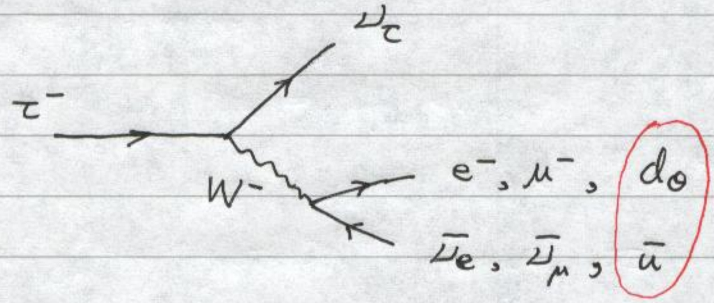
$\boxed{\text{Baryon Number Conservation}}$

So, Within the Standard model, the τ lepton decays via the W coupling to the Charged Current.

$$\mathcal{L}_{cc} = \frac{g}{2\sqrt{2}} W_\mu^+ \left\{ \sum_\ell \bar{\nu}_\ell \gamma^\mu (1-\gamma_5) \ell + \bar{u} \gamma^\mu (1-\gamma_5) d_\theta \right\} + h.c.$$

$d_\theta = \cos\theta_c \, d + \sin\theta_c \, s$ $\qquad$ θ_c: Cabbibo angle

With Feynman diagram shown as $\qquad \frac{g^2}{8m_W^2} = \frac{G_F}{\sqrt{2}}$. G_F Fermi Coupling constant



three colors

If final masses and gluonic corrections are neglected, these five modes will give equal contributions to the τ decay width.

Hence, the branching ratios for these different modes are expected to be approximately:

$$B_\ell \equiv B_r(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell) \simeq \frac{1}{5} = 20\% \qquad (\ell = e, \mu)$$

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} \simeq N_c = 3$$

$$\Gamma(\tau^- \to \nu_\tau \underbrace{d_\theta \bar{u}}_{\text{including all Color numbers.}})$$

Compared with the experimental data (NOT the newest one):

$$\boxed{\begin{aligned} B_r(\tau^- \to \nu_\tau e^- \bar{\nu}_e) &= (17.786 \pm 0.072)\% \\ B_r(\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu) &= (17.317 \pm 0.078)\% \\ R_\tau &= 3.649 \pm 0.014 \end{aligned}}$$

The agreement is good.

The measured τ hadronic width provides the evidence for the color degree of freedom

- The leptonic decays $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$ $(\ell = e, \mu)$ are theoretically well understood. pure electroweak interaction
- $\tau^- \to \nu_\tau + \text{hadrons}$ both electroweak and strong interactions

$$\Downarrow$$

$$\begin{aligned} \tau^- \to\ & \nu_\tau \pi^- \\ & \nu_\tau K^- \\ & \nu_\tau \pi^- \pi^0 \\ & \nu_\tau \pi^- K^0 \\ & \vdots \\ & \nu_\tau (n\pi) \end{aligned}$$

(If the phase space is large enough)

The corresponding key matrix element

$$\langle \text{hadrons} |\, \bar{d}_\theta \gamma^\mu (1-\gamma_5) u \,| 0 \rangle$$

* $\tau^- \to \pi^-\pi^0\nu_\tau$ decay and CVC relation

Now the corresponding hadronic matrix element is

$$\cos\theta_c \cdot \langle \pi^-\pi^0 | \bar{d}\gamma^\mu u | 0 \rangle$$

Only Vector Current contributes

As we know, for Low Energy QCD, Chiral $SU(2)_L \times SU(2)_R$ Symmetry is spontaneously broken to $SU(2)_V$ with pions identified as Goldstone particles.

$\Rightarrow$ Conservation of the vector Current (CVC) still holds

Thus one can use one form factor to parameterize

$$\langle \pi^-\pi^0 | \bar{d}\gamma^\mu u | 0 \rangle = \sqrt{2}\, F_V(s)\, (P_1 - P_2)^\mu$$

$$\langle \pi^+\pi^- | \tfrac{1}{\sqrt{2}}(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d) | 0 \rangle = F_V(s)\, (P_1 - P_2)^\mu$$

when F_V is the pion form factor $s = (P_1+P_2)^2$. P_1, P_2 pions' momenta

We will get a CVC relation for $\Gamma(\tau^- \to \pi^-\pi^0\nu_\tau)$ and $\sigma(e^+e^- \to \pi^+\pi^-)$:

$$\frac{d\Gamma(\tau^- \to \pi^-\pi^0\nu_\tau)}{ds} = \frac{G_F m_\tau^5}{192\pi^3} \frac{3\cos^2\theta_c}{2\pi\alpha^2 m_\tau^2}\, s \left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \sigma_{e^+e^- \to \pi^+\pi^-}(s)$$

CVC relation

At low energy, the dominant Contribution to $F_V(s)$ is from the $\rho(770)$ resonance

ρ: 1^- Vector meson

$m_\rho = 770$ MeV

From the present data. ~~the prese~~ the CVC relation works very well for $4m_\pi^2 \lesssim s \lesssim 0.8\ \mathrm{GeV}^2$

But there exists the discrepancy for high s.

* CVC relation is exact only in the limit of isospin symmetry $m_u = m_d$

* After correcting for possible SU(2) breaking sources such as $\rho^0-\omega$ mixing, the masses and widths of the charged and neutral ρ mesons.

There is still a discrepancy at percent level for high s

it seems a problem ?

How to understand this discrepancy ?

Other possible contributions :

$\sqrt{s} \simeq 1\ \mathrm{GeV}$

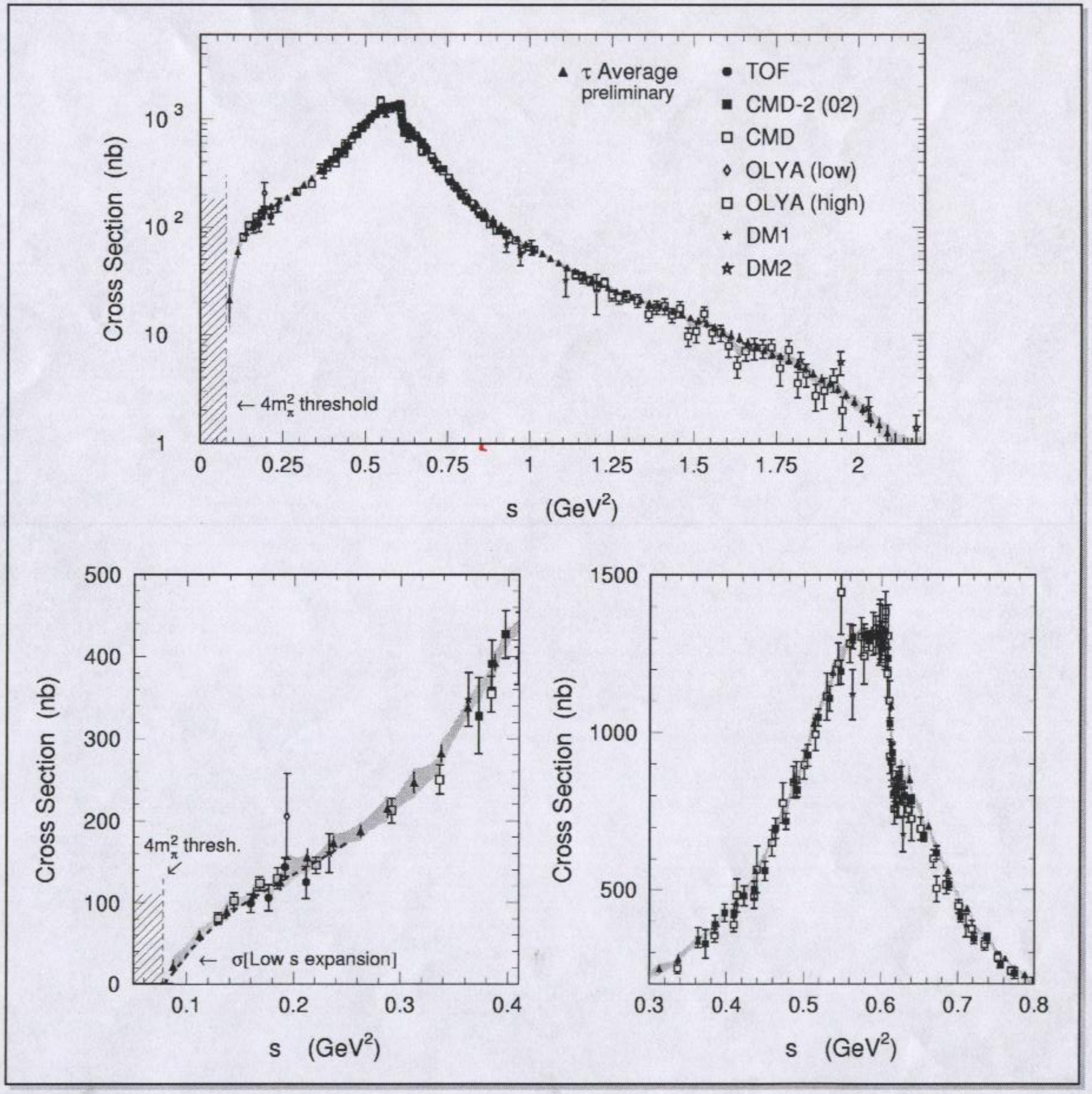
$\tau^- \to \eta\pi^-\nu_\tau$

$\eta-\pi^0$ mixing $m_u - m_d \neq 0$

may through $a_0^0(980)$ resonance $m_a = 980\ \mathrm{MeV}$

No $(\pi\pi)$ scalar resonance observed in $\tau^- \to \pi^-\pi^0\nu_\tau$ $\sqrt{s} \lesssim 1\ \mathrm{GeV}$

Comparing $e^+e^- \rightarrow \pi^+\pi^-$ and $\tau \rightarrow \pi^-\pi^0\nu_\tau$



Correct τ data for missing ρ-ω mixing (taken from BW fit) and all other SU(2)-breaking sources

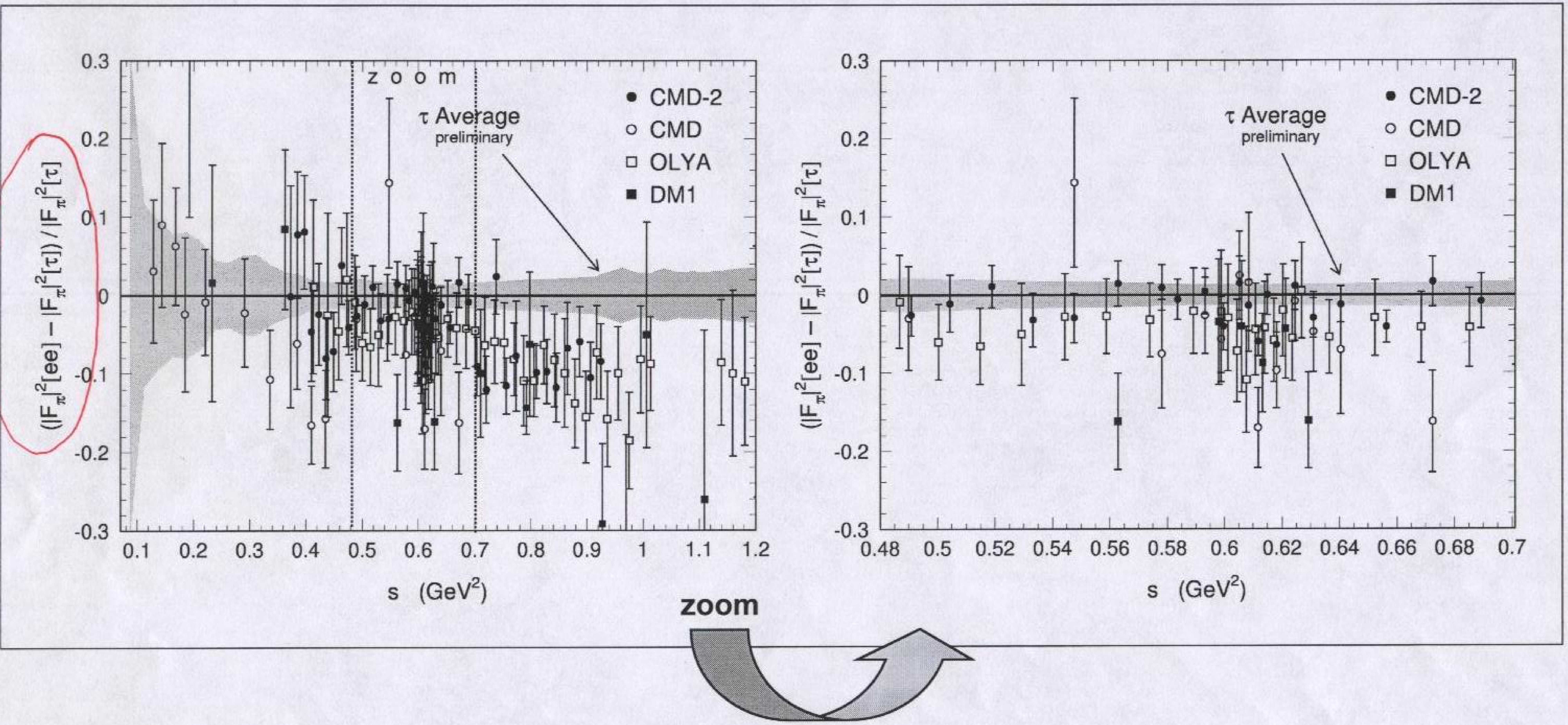
Remarkable agreement

<u>But:</u> not good enough...

...

The Problem

Relative difference between τ and e^+e^- data:



Scalar type interaction and angular distribution asymmetry in $\tau^- \to \pi^- \pi^0 \nu_\tau$

* Scalar type interaction

Experimentally . No Scalar $(\pi\pi)$ resonance at $\sqrt{s} \sim 1$ GeV

But might exist high mass ~~res~~ scalar particle which can contribute to this process. and this high mass suppression can be ~~compensel~~ Compensated by other formalism.

W. M. Morse. hep-ph/0410062

The decay $\tau^- \to \pi^- \pi^0 \nu_\tau$ can proceed through either W^- exchange or H^- exchange.

SM. dominated by $\rho(770)$ at low Energy

New high mass Scalar particle Such as charged higgs with large $\tan\beta$

$$\Rightarrow \quad \psi_{\pi\pi\nu}^2 = (\psi_W + \psi_H)^2 = \psi_W^2 + 2\psi_W\psi_H + \psi_H^2$$

$$\simeq \psi_W^2 + 2\psi_W\psi_H$$

This term should be important

$$\psi_{\pi\pi\nu}^2 - \psi_W^2 \qquad 2\psi_W\psi_H$$

The large Charged Higgs mass can be compensated by <u>large $\tan\beta$</u>

If ψ_H is not very strongly Suppressed. thus R might be enhanced

up to 1% Level. for $\sqrt{s} \sim 1$ GeV. from the interference

between ψ_W and ψ_H.

unfortunately there is a very big Suppressed factor which

is missed in the above analysis

$$\tau^- \longrightarrow \pi^- \quad \pi^0 \quad \nu_\tau$$

$$P \qquad\quad P_1 \quad P_2 \quad P_3$$

In general One Can write its invariant amplitude as

$$\mathcal{M} = G_F \cos\theta_C \left[F_V(s) (P_1 - P_2)_\mu \, \bar{u}(P_3) \gamma^\mu (1-\gamma_5) u(p) \right.$$
$$\left. + F_S \, m_\tau \, \bar{u}(P_3) (1+\gamma_5) u(p) \right]$$

here only assuming Left-handed neutrinos

$$\Rightarrow \frac{d\Gamma(\tau^- \to \nu_\tau \pi^- \pi^0)}{ds} = \frac{\cos^2\theta_C}{2m_\tau^2} \frac{G_F^2 m_\tau^5}{192\pi^3} \sqrt{1 - \frac{4m_\pi^2}{s}} \left(1 - \frac{s}{m_\tau^2}\right)^2$$
$$\times \left\{ |F_V(s)|^2 \left(1 + \frac{2s}{m_\tau^2}\right)\left(1 - \frac{4m_\pi^2}{s}\right) + 3|F_S|^2 - 6 \, \mathrm{Re}[F_V F_S^*] \frac{m_{\pi^-}^2 - m_{\pi^0}^2}{s} \right\}$$

Since F_V is dominant, F_S should be very small.

thus in general $|F_S|^2$ is not as important as

$\mathrm{Re}[F_V F_S^*]$ term. $\qquad F_S \sim O(0.01)$ (at most)

Note that

(i) Scalar Contribution to $\sigma(e^+e^- \to \pi^+\pi^-) / \frac{d\Gamma}{ds}(\tau^- \to \pi^-\pi^0 \nu_\tau$

Will in general be proportional to $\left(\frac{m_e}{m_\tau}\right)^2$

Thus Scalar Contribution to $\sigma(e^+e^- \to \pi^+\pi^-)$ Obviously Vanishingly Small

(ii) One can find the interference between Vector and Scalar Contributions has a factor

$$\frac{m_{\pi^-}^2 - m_{\pi^0}^2}{s} \longrightarrow \frac{m_{\pi^-}^2 - m_{\pi^0}^2}{(1\,\text{Gev})^2}$$

for $\sqrt{s} \sim 1$ Gev

$$\Downarrow$$

$$= \frac{2 \times 0.135 \times 0.00459}{1} \sim 10^{-3}$$

EM

$m_u \neq m_d$ $\Big\}$ $m_{\pi^\pm} - m_{\pi^0} = 4.59$ Mev

$m_\pi \simeq 135$ Mev

It is easy to understand this point since in the limit of isospin symmetry ($m_u = m_d$. $e = 0$ no EM), there is no interference between Vector and Scalar Contribution.

$\pi^-\pi^0$	0^{++}	1^{--}
	S	V

* The angular distribution asymmetry

It is difficult to see the significant effects due to the Scalar contribution by measuring the $d\Gamma(\tau^- \to \pi^-\pi^0\nu_\tau)/ds$ spectrum distribution.

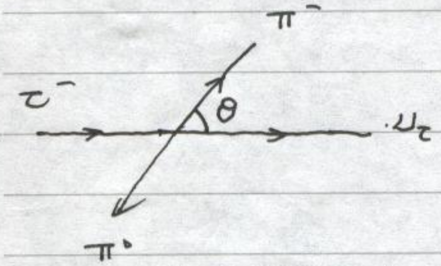
however. there might exist an observable which can show the interference between the Scalar and Vector Contributions

$\Rightarrow$

$$\frac{d\Gamma}{ds\, d\cos\theta} = \frac{3\cos^2\theta_c}{4m_\tau^2}\frac{G_F^2 m_\tau^5}{192\pi^3}\sqrt{1-\frac{4m_\pi^2}{s}}\left(1-\frac{s}{m_\tau^2}\right)^2\left\{|F_V(s)|^2\left[\frac{s}{m_\tau^2}+\left(1-\frac{s}{m_\tau^2}\right)\cos^2\theta\right]\right.$$

$$\left.\cdot\left(1-\frac{4m_\pi^2}{s}\right)+|F_S|^2+\underbrace{2\,\mathrm{Re}(F_V F_S^*)\sqrt{1-\frac{4m_\pi^2}{s}}\cos\theta}\right\}$$

Now we have neglected terms proportional to $(m_{\pi^-}^2 - m_{\pi^0}^2)$



$(\pi^-\pi^0)$ Center of mass frame

θ: angle between π^- three-momentum and τ^- three-momentum in $(\pi^-\pi^0)$ CM frame

This term proportional to $\cos\theta$, which vanishes after integrating over the full phase space, will give an angular distribution asymmetry:

$$0<\theta<\frac{\pi}{2} \qquad\qquad \frac{\pi}{2}<\theta<\pi$$

$$\text{Asymmetry} = \frac{\int_0^1 \frac{d\Gamma}{ds\, d\cos\theta}\, d\cos\theta - \int_{-1}^0 \frac{d\Gamma}{ds\, d\cos\theta}\, d\cos\theta}{\int_0^1 \frac{d\Gamma}{ds\, d\cos\theta}\, d\cos\theta + \int_{-1}^0 \frac{d\Gamma}{ds\, d\cos\theta}\, d\cos\theta}$$

$$= \frac{\frac{3\cos^2\theta_c}{2m_\tau^2}\frac{G_F^2 m_\tau^5}{192\pi^3}\left(1-\frac{4m_\pi^2}{s}\right)\left(1-\frac{s}{m_\tau^2}\right)^2 \mathrm{Re}(F_V F_S^*)}{d\Gamma/ds}$$

One can also define the integrated asymmetry by integrating over s in the above equation.

As we know. F_S in the SM is an isospin symmetry breaking effect. for instance $\tau^- \to a_0 \nu_\tau$

$$\hookrightarrow \eta\pi^- \to \pi^0\pi^-$$

$\eta-\pi^0$ mixing

Experimentally no $(\pi\pi)$ scalar resonance observed in low-energy region in this decay

large mass scalar particle contribution will generally be suppressed by the inverse of this large mass squared.

Therefore. in general. ~~the~~ It seems that one cannot expect the scalar effects in this decay both for the spectrum $d\Gamma/ds$ and for $\mathcal{A}$

Charged Higgs Contribution from ~~the~~ two-Higgs-doublet Model

In the Standard Model. Only One Higgs doublet

$SU(2)_L$ doublet

A single physical neutral Higgs is left over

after spontaneous symmetry breaking ?

Experimentally not observed yet

$\Downarrow$

Why two-Higgs-doublet Model ?

(i) The standard model is not a complete theory.

The two-Higgs-doublet Model is an kind of the Simplest extension of the Standard Model with one extra scalar doublet.

(ii) The minimal Supersymmetric Standard Model requires at least two-Higgs-doublet. One higgs-doublet is not enough. (up and down quark mass. anomaly Cancellation)

So in general. we can introduce two scalar doublets.

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ (\phi_i^0 + i\chi_i^0)/\sqrt{2} \end{pmatrix} \qquad i = 1, 2.$$

By imposing CP symmetry and requring to spontaneously break

$$V(\Phi_1,\Phi_2) = \lambda_1\left(\Phi_1^+\Phi_1 - \frac{v_1^2}{2}\right)^2 + \lambda_2\left(\Phi_2^+\Phi_2 - \frac{v_2^2}{2}\right)^2$$
$$+ \lambda_3\left[\left(\Phi_1^+\Phi_1 - v_1^2/2\right) + \left(\Phi_2^+\Phi_2 - v_2^2/2\right)\right]^2$$
$$+ \lambda_4\left[(\Phi_1^+\Phi_1)(\Phi_2^+\Phi_2) - (\Phi_1^+\Phi_2)(\Phi_2^+\Phi_1)\right]$$
$$+ \lambda_5\left[\mathrm{Re}(\Phi_1^+\Phi_2) - \frac{v_1 v_2}{2}\right]^2 + \lambda_6\left[\mathrm{Im}(\Phi_1^+\Phi_2)\right]^2$$

λ_i, $i=1\cdots 6$ Real

Hermiticity

$\lambda_i \geq 0$. it is obvious that the minimal of the potential is at

$$\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ v_1/\sqrt{2} \end{pmatrix}, \quad \langle\Phi_2\rangle = \begin{pmatrix} 0 \\ v_2/\sqrt{2} \end{pmatrix}.$$

which will break $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$.

Since now there are two scalar doublets. the Yukuwa Coupling of the scalars and fermions are of several ways. In order to aviod flavor changing neutral Higgs interactions, A necessary and sufficient Condition is that each quark of a given Charge must receive its mass from at most one Higgs doublet.:

(i) type I model: Couple all the fermions to a single Higgs doublet;

(ii) type II model: The two $SU(2)_L$ scalar doublets are Coupled ~~to~~ separately to up- and down-type quarks

$\Rightarrow$

$$\mathcal{L}_Y = -Y_d \bar{Q}_L \Phi_1 d_R - Y_u \bar{Q}_L \Phi_2^c u_R - Y_\ell \bar{L} \Phi_1 \ell_R + h.c.$$

where $\Phi_2^c = i\tau_2 \Phi_2^*$, Y_f. $f = u, d, \ell$. Yukawa coupling matrix

Same as in the SM. $W^\pm$ and Z^0 Bosons get masses after SSB:

$$m_W^2 = \frac{1}{2} g^2 v^2. \qquad v^2 = v_1^2 + v_2^2$$

$$\tan\beta = v_2 / v_1$$

Also $\Longrightarrow$ three unphysical Goldstone Bosons $G^\pm$. G^0, and five physical Higgs particles: two Charged Higgs $H^\pm$ and three neutral ones h^0, H^0, A^0, which can be obtained by diagonalizing the Higgs mass matrices:

$$G^\pm = \cos\beta\, \phi_1^\pm + \sin\beta\, \phi_2^\pm, \qquad H^\pm = -\sin\beta\, \phi_1^\pm + \cos\beta\, \phi_2^\pm,$$

$$G^0 = \cos\beta\, \chi_1^0 + \sin\beta\, \chi_2^0, \qquad A^0 = -\sin\beta\, \chi_1^0 + \cos\beta\, \chi_2^0,$$

$$H^0 = \cos\alpha\, \phi_1^0 + \sin\alpha\, \phi_2^0, \qquad h^0 = -\sin\alpha\, \phi_1^0 + \cos\alpha\, \phi_2^0.$$

Now we will get the interaction lagrangian guiding the physical Higgses couplings to fermions. Only for charged Higgs:

$$\mathcal{L}_{H^\pm f\bar{f}} = -\frac{g\tan\beta}{2\sqrt{2}\, m_W} m_\ell\, H^+ \bar{\nu}_\ell (1+\gamma_5)\ell^-$$

$$- \frac{g\tan\beta}{2\sqrt{2}\, m_W} m_d\, H^+ \bar{u}\, (1+\gamma_5) d \cdot \cos\theta_c$$

$\Rightarrow$ Charged Higgs contribution to $\tau^- \to \pi^- \pi^0 \nu_\tau$ decay:

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \cos\theta_c \frac{m_\tau}{m_{H^\pm}^2} \tan^2\beta \; \bar{\nu}_\tau (1+\gamma_5) \tau^-$$

$$\cdot \underbrace{\left[m_d \bar{d} (1-\gamma_5) u + \frac{m_u}{\tan^2\beta} \bar{d} (1+\gamma_5) u \right]}_{\text{Same order}} + \text{h.c.}$$

$\tan\beta = \dfrac{v_2}{v_1}$ if $\tan\beta \sim 1$

thus obviously the scalar contribution from the above $\mathcal{L}$ will be strongly suppressed by $\frac{1}{m_{H^\pm}^2}$ for $m_{H^\pm} \sim$ several hundred GeV

But for large $\tan\beta$. $\tan\beta \simeq 30 \sim 50$. This contribution might receive large enhancement.

In large $\tan\beta$ limit. One has (We can neglect the second term in [ ] of $\mathcal{L}$)

$$F_S = \frac{m_d \tan^2\beta}{m_{H^\pm}^2} \frac{m_\pi^2}{m_u + m_d}$$

$$\simeq 0.01 \, (\text{GeV})^2 \frac{\tan^2\beta}{m_{H^\pm}^2}$$

Experimentally $m_{H^\pm} > 78.6$ GeV no evidence for $m_{H^\pm}$ at present

or $\dfrac{\tan\beta}{m_{H^\pm}} < 0.5 \text{ GeV}^{-1}$ (PDG 2004)

$\Rightarrow$ F_S can be up to 10^{-3} In 2HDM with large $\tan\beta$

without conflicts ~~to~~ to other processes prediction

At low Energy. $s < 1.2\ \mathrm{GeV}^2$ $F_V(s)$ is dominant.

by $\rho(770)$ Resonance

$$F_V(s) = \frac{m_\rho^2}{m_\rho^2 - s - i m_\rho \Gamma_\rho(s)}$$

$$\Gamma_\rho(s) = \frac{m_\rho s}{96\pi f_\pi^2} \left\{ \left(\sqrt{1-\frac{4m_\pi^2}{s}}\right)^3 \theta(s-4m_\pi^2) + \frac{1}{2}\left(\sqrt{1-\frac{4m_K^2}{s}}\right)^3 \theta(s-4m_K^2) \right\}$$

Vector Resonances have to be included into the theory since $\sqrt{s} \sim 1$ GeV, where Chiral perturbation theory cannot be reliable.

Note: Asymmetry $\sim \mathrm{Re}(F_V F_S^*)$

$$\mathrm{Re}\, F_V = \frac{m_\rho^2 (m_\rho^2 - s)}{(m_\rho^2-s)^2 + m_\rho^2 \Gamma_\rho^2(s)}$$

$$\mathrm{Re}\, F_V (m_\rho^2 = s) = 0$$

Conclusions and Remarks

- Scalar type interactions Cannot explain the discrepancy between the data from $\sigma(e^- e^- \to \pi^+ \pi^-)$ and $d\Gamma/ds\,(\tau^- \to \pi^- \pi^0 \nu_\tau)$ If it Really exist.

- Scalar type interactions will lead to possible ~~si~~ interesting effects in the angular distribution asymmetry in $\tau^- \to \pi^- \pi^0 \nu_\tau$

- In 2HDM, large $\tan\beta$ and present experimental Constraints Can permit this asymmetry up to 10^{-3}. Thus the measurement of the asymmetry might ~~either~~ gives ~~the~~ signal of charged Higgs, or impose bound on it.

- ~~tensor~~ type interactions: $\bar{d}_R \sigma^{\mu\nu} u_L \, \bar{\nu}_L \sigma_{\mu\nu} l_R$, $\bar{d}_L \sigma^{\mu\nu} u_R \, \bar{\nu}_L \sigma_{\mu\nu} l_R$?

CP Violation: $\dfrac{\Gamma(\tau^- \to \pi^- \pi^0 \nu_\tau) - \Gamma(\tau^+ \to \pi^+ \pi^0 \nu_\tau)}{\Gamma(\tau^- \to \pi^- \pi^0 \nu_\tau) + \Gamma(\tau^+ \to \pi^+ \pi^0 \nu_\tau)}$?

Thank you !

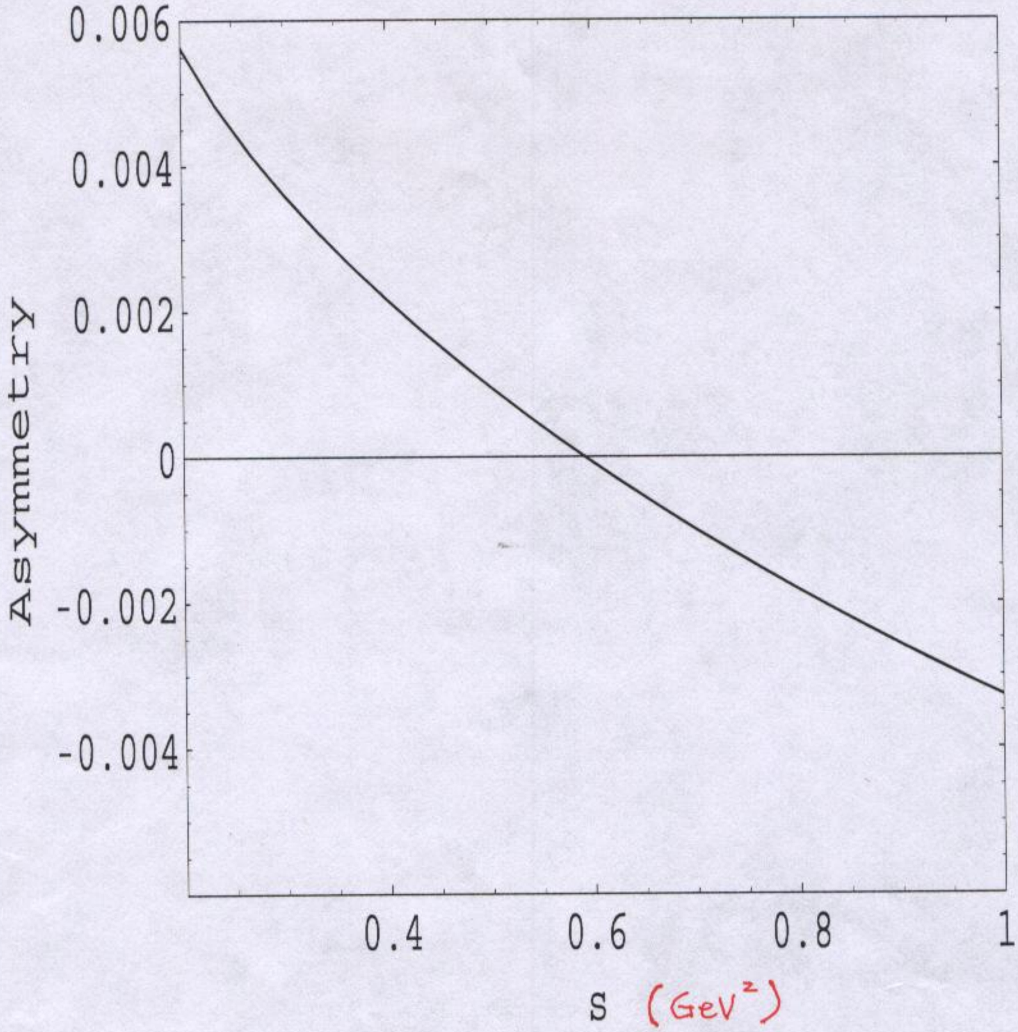


Figure 1: