Entanglement and chaos in 2D CFTs and beyond CFTs

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Starting Point

- Counting problems
  1. Counting the Dimension of Hilbert space,
     e.g. Partition Function, Correlation Function, Central Charge of field theory, Black Hole entropy, ...
  2. Many Physical observables can be obtained by Correlation Functions,
     e.g. Partition function(0pt), condensation(1pt), conductivity(2pt), S-matrix(n-pt)...
  3. Counting the effective degree freedom of subset of Hilbert space,
     e.g. Entanglement entropy, Rényi entropy (OTOC)...

- S.H, Tokiro Numasawa, Tadashi Takayanagi, Kento Watanabe, Phys. Rev. D90, 041701(R) (2014).(Rational CFTs)
- Wuzhong Guo, S.H. JHEP **1504**, 099 (2015).(With Defect)
- Bin Chen, Wuzhong Guo, S.H., Jie-Qiang Wu, JHEP **1510**, 173 (2015). (Descendent Excitation)
- Wu-Zhong Guo, S.H, Zhuxi Luo, JHEP **1805**, 154 (2018). (Anyon interpretation)
- L. Apolo, S. H, W. Song, J. Xu and J. Zheng, JHEP **1904**, 009 (2019). (Warped CFT)
- S.H., Phys.Rev. D99, 026005(2019). (2D Quantum gravity)
- S.H., Hongfei Shu, Entanglement in $T\bar{T}/T\bar{J}$ deformed CFTs, 1907.12603.

Outline

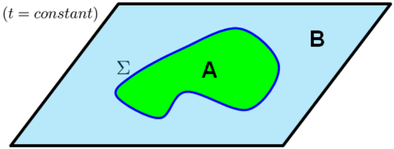
- Entanglement Entropy (EE).
- Setup in 2D CFTs.
- EE v.s. Quantum dimension in rational CFT.
- EE in Liouville field theory.
- EE and OTOC in $T\bar{J}/T\bar{T}$ deformed CFTs. (If the time is available.)
- Summary.

Basics of EE

- EE is a useful quantum information quantity to measure the degrees of freedom in quantum many body systems.
  1. Using EE to detect the central charge (the coefficient of logarithmic divergent term in Even dimesion)[C. Holzhey, F. Larsen and F. Wilczek, 94][P. Calabrese and J. L. Cardy, 04][S. Ryu and T. Takayanagi,06][...].
  2. Detecting the topological degrees of freedom of topological field theories (finite piece of EE)[A. Kitaev and J. Preskill,05][M. Levin and X.G.Wen,05].
  3. Measuring the degrees of freedom of local operators (Quantum dimension).[S. He, T. Numasawa, T. Takayanagi and K. Watanabe,14][P. Capta, M. Nozaki and T. Takayanagi, 14][M. Nozaki,14][Wu-Zhong Guo, S. He,15][Wu-Zhong Guo, S. He, Zhuxi Luo,18][L. Apolo, S. He, W. Song, J. Xu and J. Zheng, 18]....
- ...

Basics of EE

- Consider bi-partite system (A and B) and use entropy as measure of correlations between subsystems



- Integrate out degrees of freedom in outside region (B). Remaining dof are described by a density matrix ρ_A.

$$S_A = -\text{Tr}_A \rho_A \log \rho_A \tag{1}$$

Replica to calculate EE in QFT

- How to calculate EE in quantum system.
- A basic method of calculating EE in QFTs is so called the replica method.

$$S_A = -\frac{\partial \mathrm{Tr}(\rho_A)^n}{\partial n}|_{n=1} = \lim_{n\to 1} S_A^n$$

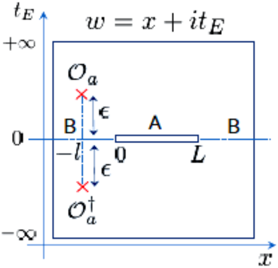
- Other approaches:
  1. AdS/CFT (Well Studied).[S. Ryu and T. Takayanagi, 06]
  2. String theory approach initiated by[L. Susskind, 93]. Exactly realied in string theory by [S.H, Tokiro Numasawa, Tadashi Takayanagi, Kento Watanabe, 15] [E. Witten,19].

Motivation I: Time evolution of EE

- In Chaotic system, the late time behavior of physical quantities are very sensitive to the early time input.
- Out-of-time order correlation function (OTOC) can diagnose the chaotic behavior of many body systems
  1. The chaotic behavior characterized by: Lyapnov behavior, scrambling and Ruelle resonance. [A. Larkin and Y. Ovchinnikov,1969],[A. Kitaev,15]
  2. In integrable CFTs such as RCFT, cannot see such chaotic behavior[E. Perlmutter,16],[Y. Gu and X. L. Qi,16].
  3. Holographic dual CFTs show maximal chaotical signals, Lyapnov, OTOC, ETH. [E. Perlmutter,16],[J. L. Karczmarek, J. M. Maldacena and A. Strominger,16],[J. M. Maldacena, D. Stanford,16].
  4. The essential differences between integrable CFTs and chaotic CFTs seem to be captured by the Maximal chaotic signals (time evolutions of REE=OTOC)..
- In this talk, we will focus on the time evolution of EE and OTOC in CFTs and TT/TJ deformations.

Our Setup

- In this talk, we setup in 1+1 dimension space time



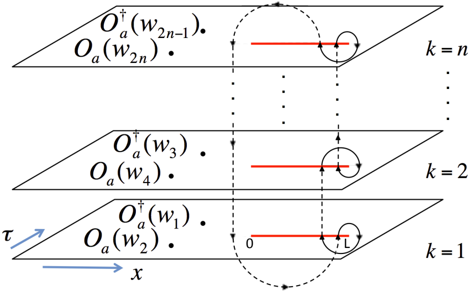
$w_i, \bar{w}_i$ can be expressed by

$$w_1 = i(\epsilon - it) - l, \quad w_2 = -i(\epsilon + it) - l, \tag{2}$$

$$\bar{w}_1 = -i(\epsilon - it) - l, \quad \bar{w}_2 = i(\epsilon + it) - l. \tag{3}$$

Replica

- Where (w_{2k+1}, w_{2k+2}) for $k = 1, 2, ..., n-1$ are $n-1$ replicas of (w_1, w_2) in the k-th sheet of Σ_n. We just glue all sheets with proper boundary conditions to construct Σ_n.



EE for Excited State

- Where REE for $|\Psi(t)\rangle = e^{-itH-\epsilon H} O(-l)|0\rangle$,

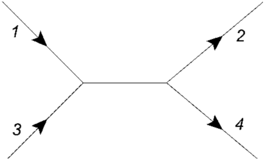
$$S^{(n)}[|\Psi(t)\rangle] = \frac{1}{1-n}\log\left[\frac{\int d\phi O^{+}(x_1)O(x_2)...O^{+}(x_{2n-1})O(x_{2n})e^{-S}}{(\int d\phi O^{+}(x_1)O(x_2)e^{-S})^n}\right] \quad (4)$$

- The excess of EE $\Delta S_A^{(n)} = S^{(n)}[|\Psi(t)\rangle] - S^{(n)}[|0\rangle]$

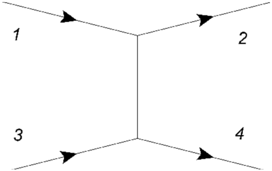
$$\begin{aligned}\Delta S_A^{(n)} &= \frac{1}{1-n}\left[\log\frac{\left\langle \mathcal{O}_a^{\dagger}(w_l,\bar{w}_1)\mathcal{O}_a(w_2,\bar{w}_2)..\mathcal{O}_a(w_{2n},\bar{w}_{2n})\right\rangle_{\Sigma_n}}{\left\langle \mathcal{O}_a^{\dagger}(w_l,\bar{w}_1)\mathcal{O}_a(w_2,\bar{w}_2)\right\rangle_{\Sigma_1}^{n}} - \log(1)\right] \\ &= \frac{1}{1-n}\left[\log R_A^{(n)}\right] \quad (5)\end{aligned}$$

EE for Excited State

- We are interested in two different time evolution regions (Early time and Late time) [S.H, Tokiro Numasawa, Tadashi Takayanagi, Kento Watanabe, 15]
- $(z, \bar{z}) \to (0, 0) \equiv t < l$ and $t > L$ (Earlier time)



- $(z, \bar{z}) \to (1, 0) \equiv L > t \gg l$ (Late time)



2D Ising Model

- The unitary minimal models are numbered by an integer m=3,4..., and describe the universality class of the multicritical Ginzburg- Landau model:

$$\mathcal{L} \sim (\partial\phi)^2 + \lambda\phi^{2m-2} \tag{6}$$

  For $m = 3$, the Ising model is in the same universality class.
- The central charge of the model is

$$c = 1 - \frac{6}{m(m-1)}. \tag{7}$$

- All Virasoro primaries are scalar $O_{r,s}$ $1 \leq s \leq r \leq m-1$ whose dimension is

$$\Delta_{r,s} = \frac{(r+m(r-s))^2 - 1}{4m(m+1)} \tag{8}$$

EE in Ising model

- We consider primary operator $O_{2,2}$ in Ising model whose conformal dimension is

$$\Delta_{2,2} = \frac{3}{4m(m+1)}\Big|_{m=3} = \frac{1}{16} \tag{9}$$

called spin operator.

- For Ising model, the Green function of spin operator can be expressed by

$$G(z,\bar{z}) = \frac{1}{\sqrt{2}}\sqrt{\sqrt{\frac{|z|}{|1-z|}} + \frac{1}{\sqrt{|z||1-z|}} + \sqrt{\frac{|1-z|}{|z|}}}. \tag{10}$$

Using this explicit expression, one can take late time limit $(z,\bar{z}) \to (1,0)$ to obtain

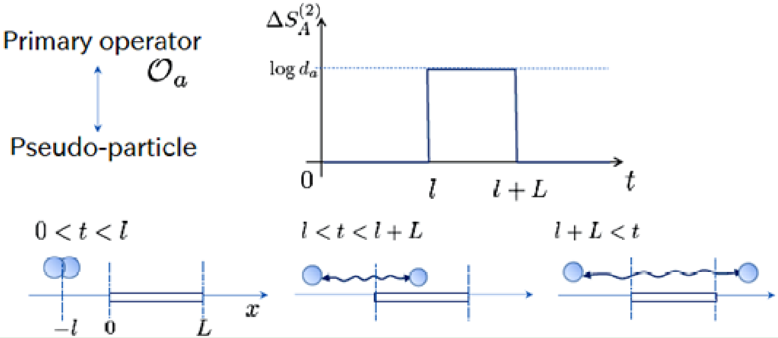
$$\Delta S_A^{(2)} = \log\sqrt{2}. \tag{11}$$

EE in Ising model

- Through very very highly nontrivial calculation, we can show that $\Delta S_A^{(2)} = \Delta S_A^{(3)} = \Delta S_A^{(4)} = ... = \log\sqrt{2}$.
- $\Delta S_A^{(2)} = \Delta S_A^{(3)} = \Delta S_A^{(4)} = ... = \log\sqrt{2}$ [Wuzhong Guo, S.H.(2015)(with comformal defects), Bin Chen, Wuzhong Guo, S.H., Jie-Qiang Wu,(2015).(For descendent states),[Wu-Zhong Guo, S. He, Zhuxi Luo,18](Associated with anyons),[L. Apolo, S. He, W. Song, J. Xu and J. Zheng, 18]]
- So it is nature to ask What is the meaning of $\sqrt{2}$.
- An: The $\sqrt{2}$ is exact quantum dimension of spin operator σ in Ising model.

Memory effect of EE in Ising model

Causality argument



What is quantum dimension

- Here we just list the standard alternative definition of quantum dimension in Minimal model.

Quantum Dimension d_a ? ?

2D CFT

d_a → Maximal eigenvalue of N_{ab}^{c}

$\mathcal{O}_a \cdot \mathcal{O}_b = \sum_c N_{ab}^{c}\, \mathcal{O}_c$: Fusion rule

of the primary fields in $\overbrace{\mathcal{O}_a \cdots \mathcal{O}_a}^{N} = \sum_c (N_a \cdots N_a)_a^c \, \mathcal{O}_c \quad \sim (d_a)^N$

$\sum_c (N_a \cdots N_a)_a^c$ → M_N $(N \to \infty)$

$$\longrightarrow \log d_a = \lim_{N\to\infty} \frac{\log M_N}{N}$$

⇨ Quantum Dimension $d_a =$ " The effective d.o.f. of $\mathcal{O}_a$ "

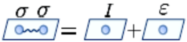
What is quantum dimension

- Especially in Ising model, one can easily work out quantum dimension of spin operator σ.

Quantum Dimension $d_a =$ " The effective d.o.f. of $\mathcal{O}_a$ "

Ising model

$$\mathcal{O}_a = \{I, \sigma, \varepsilon\} \qquad \begin{cases} \varepsilon \cdot \varepsilon = I \\ \sigma \cdot \sigma = I + \varepsilon \\ \varepsilon \cdot \sigma = \sigma \end{cases}$$



$$\sigma^{2N} = (I+\varepsilon)^N = 2^{N-1}I + 2^{N-1}\varepsilon \longrightarrow \log d_\sigma = \lim_{N\to\infty} \frac{\log 2^N}{2N} = \log\sqrt{2}$$

$$\longrightarrow d_\sigma = \sqrt{2} \qquad \text{Similarly} \quad d_I = d_\varepsilon = 1$$

- Comment: In the Ising model, the identity I, the spin σ and the energy operator ψ.
  1. $\Delta S_A^{(n)}$ is always vanishing for I and ϵ, due to quantum dimension 1.
  2. $\Delta S_A^{(n)} = \log\sqrt{2}$ for any n as $d_\sigma = \sqrt{2}$

How about EE in irrational CFTs

- In irrational CFTs, the spectrum Δ will be continous V.S. discrete rational number in rational CFTs .
- Infinity dimensional representation of Viasoro symmetry V.S. finite dimensional representation.
- Integral boostrap equation V.S. Algebraic boostrap equation.
- Large C limit (Holographic potentially) V.S. No large C count part.
- ...

REE in LFT or SLFT

- Observed BTZ entropy (Higher spin BH) = Log of quantum dimension of primary operator in LFT (Toda).[L. McGough and H. Verlinde(2013)].
- Variation of REE with entangled pair ingoing BTZ = Log of quantum dimension of primary operator in LFT.[S. Jackson, L. McGough and H. Verlinde(2014)].
- LFT can be reformulated by 3d Gravity with boundary (AdS/CFT like correspondence??)[H. L. Verlinde(1990)].
- To study associated aspects of 2D quantum gravity from quantum information point of view (REE).

Liouville field theory

- The Liouville field theory action

$$S_L = \frac{1}{4\pi}\int d^2\xi\sqrt{g}\left[\partial_a\phi\partial_b\phi g^{ab} + QR\phi + 4\pi\mu e^{2b\phi}\right], \qquad (12)$$

- where $Q = b + \frac{1}{b}$. The conformal dimension of corresponding primary operator $V_\alpha = e^{2\alpha\phi}$ is

$$\Delta(e^{2\alpha\phi}) = \bar{\Delta}(e^{2\alpha\phi}) = \alpha(Q-\alpha), \qquad (13)$$

where $\alpha \in (0, Q)\bigcup Q/2 + ip$.

- Two point Green function.

$$\langle V_{\bar{\alpha}}(x_1)V_\alpha(x_2)\rangle = \frac{\delta(0)}{(x_{12}\bar{x}_{12})^{\Delta_{\alpha_1}}}. \qquad (14)$$

2nd REE in LFT

- We mainly focus on 2nd REE in early time or late time limit

$$R^{(2)}_{EE} = \lim_{(z,\bar{z})\to(0,0),\text{or } (z,\bar{z})\to(1,0)} \frac{\langle V_{\bar{\alpha}} V_{\alpha} V_{\bar{\alpha}} V_{\alpha} \rangle_{\Sigma_2}}{\langle V_{\bar{\alpha}} V_{\alpha} \rangle^2_{\Sigma_1}} \tag{15}$$

Then

$$S^{(2)}_{EE}\left[V_{\alpha}|0\rangle\right] = -\log(R^{(2)}_{EE}) \tag{16}$$

2nd REE in LFT

- S-channel, Early time

$$\begin{aligned}\langle V_{\bar{\alpha}} V_{\alpha} V_{\bar{\alpha}} V_{\alpha} \rangle &= \frac{1}{2} |z_{13}|^{-4\Delta} |z_{24}|^{-4\Delta} \\ & \int_{-\infty}^{\infty} \frac{dp}{2\pi} C(\bar{\alpha}, \alpha, \frac{Q}{2} + ip) C(\bar{\alpha}, \alpha, \frac{Q}{2} - ip) \\ & F_{s\bar{1}2\bar{3}4}(\Delta_i, \Delta_p, z) F_{s\bar{1}2\bar{3}4}(\Delta_i, \Delta_p, \bar{z}). \end{aligned} \tag{17}$$

- For $\alpha = Q/2 + iP$, $(z, \bar{z}) \to (0, 0)$

$$\begin{aligned} S_{EE}^{(2)}[V_{\alpha}|0\rangle] &= -\log \lim_{(z,\bar{z}) \to (0,0)} \frac{\langle V_{\bar{\alpha}} V_{\alpha} V_{\bar{\alpha}} V_{\alpha} \rangle_{\Sigma_2}}{\langle V_{\bar{\alpha}} V_{\alpha} \rangle_{\Sigma_1}^2} \\ &= -\log 0. \end{aligned} \tag{18}$$

[S. H, Phys.Rev. D99, 026005(2019)]

2nd REE in LFT

- T-channle, Late time. Boostrap equation

$$
\begin{aligned}
&\langle V_{\bar{\alpha}} V_{\alpha} V_{\bar{\alpha}} V_{\alpha} \rangle \\
= \ &\int_{-\infty}^{\infty} \frac{dp}{2\pi} C(\bar{\alpha}, \alpha, \frac{0}{2} + ip) C(\bar{\alpha}, \alpha, \frac{0}{2} - ip) \\
&\int d\alpha_t F_{s\bar{1}2\bar{3}4}(\Delta_i, \Delta_p, \bar{z}) F_{t\bar{1}2\bar{3}4}(\Delta_i, \Delta_p, 1-z) F^{L}_{st} \begin{bmatrix} \bar{\alpha} & \alpha \\ \alpha & \bar{\alpha} \end{bmatrix}
\end{aligned} \tag{19}
$$

- For $\alpha = Q/2 + iP$, $(z, \bar{z}) \to (1, 0)$

$$
S^{(2)}_{EE} \left[V_{\alpha} |0\rangle \right] \simeq -\log 0. \tag{20}
$$

Comments

- For $V_\alpha|0\rangle, \alpha = Q/2 + iP$

$$\begin{aligned} \Delta S_A^{(n)}\left[V_\alpha|0\rangle, 1|0\rangle\right](t=0) &= \text{Divergent} \\ \Delta S_A^{(n)}\left[V_\alpha|0\rangle, 1|0\rangle\right](t=\infty) &= \text{Divergent} \end{aligned} \tag{21}$$

- How to resolve the divergence? Why $\Delta S_A^{(n)}\left[V_\alpha|0\rangle, 1|0\rangle\right]$ are divergent?

Comments

- Why $\Delta S_A^{(n)}\left[V_\alpha|0\rangle, |0\rangle\right]$ are divergent?
- Very intuitive interpretation to the divergence.

$$\langle 0|\nabla\phi|0\rangle = \langle 0|2e^{b\phi}|0\rangle = 0$$

No translation invariant vacuum due to the positive definite of exponential.

- Choose proper reference state to redefine [S. H, Phys.Rev. D99, 026005(2019)]

$$\Delta S_A^{(n)}\left[V_\alpha|0\rangle, V_{\alpha_r}|0\rangle\right](t) = S_A^{(n)}[V_\alpha(t)]|0\rangle(t) - S_A^{(n)}[V_{\alpha_r}(t)|0\rangle](t)$$

Where $V_{\alpha_r}|0\rangle$ is reference state but not vacuum state as in RCFTs.

Final Results in LFT

- The difference between Early and Late time

$$\begin{aligned}\Delta S^{(2)}_{EE} &= S^{(2)}_{EE}\left[V_{\alpha}|0\rangle\right](t\to\infty) - S^{(2)}_{EE}\left[V_{\alpha_r}|0\rangle\right](t\to 0) \\ &= -\log\left(\frac{F^{L}_{Q/2,Q/2}\begin{bmatrix}\bar{\alpha}\,\alpha\\ \alpha\,\bar{\alpha}\end{bmatrix}}{F^{L}_{Q/2,Q/2}\begin{bmatrix}\bar{\alpha}_r\,\alpha_r\\ \alpha_r\,\bar{\alpha}_r\end{bmatrix}}\right)\Bigg|_{p\to 0}, \\ &\alpha,\alpha_r\in\{Q/2+ip\}, p\in\mathbb{R}. \end{aligned} \tag{22}$$

[S. H, Phys.Rev. D99, 026005(2019)]

EE in deformed CFTs

- The deformed operator

$$T\overline{T}\left(z, z'\right) = T_{zz}(z)T_{\overline{z}\overline{z}}\left(z'\right) - T_{z\bar{z}}(z)T_{z\bar{z}}\left(z'\right) \tag{23}$$

- The following is true very generally in a reasonably well behaved 2d QFT which has a local conserved stress tensor.

$$\langle T\overline{T}\rangle = \langle T_{zz}\rangle \langle T_{\overline{z}\overline{z}}\rangle - \langle T_{z\bar{z}}\rangle^2 \tag{24}$$

EE in deformed CFTs

- The deformation is

$$\mathcal{L}^{(\lambda+\delta\lambda)} = \mathcal{L}^{(\lambda)} + \delta\lambda T\overline{T} \tag{25}$$

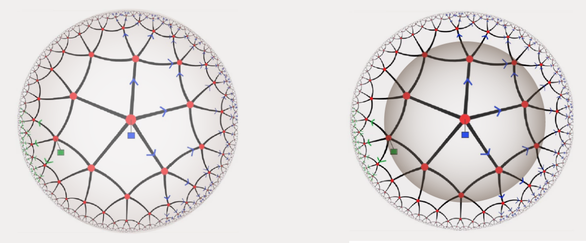
$$\frac{dS(\lambda)}{d\lambda} = \int d^2x T\overline{T}(x) \tag{26}$$

- Why we should care about the deformation?

EE in deformed CFTs

- The spectrum of the deformed theory can be solved exactly and non-perturbatively.[Smirnov-Zamolodchikov; Cavaglia-Negro-Szecsenyi-Tateo]
- Deforming an integrable QFT by this operator preserves integrability. [F. A. Smirnov and A. B. Zamolodchikov,16]
- Deforming by $T\bar{T}$ = Finite cutoff in terms of AdS/CFT.

[McGough-Mezei-Verlinde,18]



$$Z_{\text{AdS}} = Z_{\text{CFT}}$$

$$Z_{\text{cutoff-AdS}} \stackrel{?}{=} Z_{T\bar{T}}$$

EE in deformed CFTs

- Since the excess of EE $\Delta S_A^{(n)} = S^{(n)}[|\Psi(t)\rangle] - S^{(n)}[|0\rangle]$

$$\Delta S_A^{(n)} = \frac{1}{1-n}\left[\log \frac{\left\langle \mathcal{O}_a^{\dagger}(w_l,\bar{w}_1)\mathcal{O}_a(w_2,\bar{w}_2)..\mathcal{O}_a(w_{2n},\bar{w}_{2n})\right\rangle_{\Sigma_n}}{\left\langle \mathcal{O}_a^{\dagger}(w_l,\bar{w}_1)\mathcal{O}_a(w_2,\bar{w}_2)\right\rangle_{\Sigma_1}^{n}}\right] \quad (27)$$

- Up to first order, the conformal symmetry is still hold in Deformed Theory.

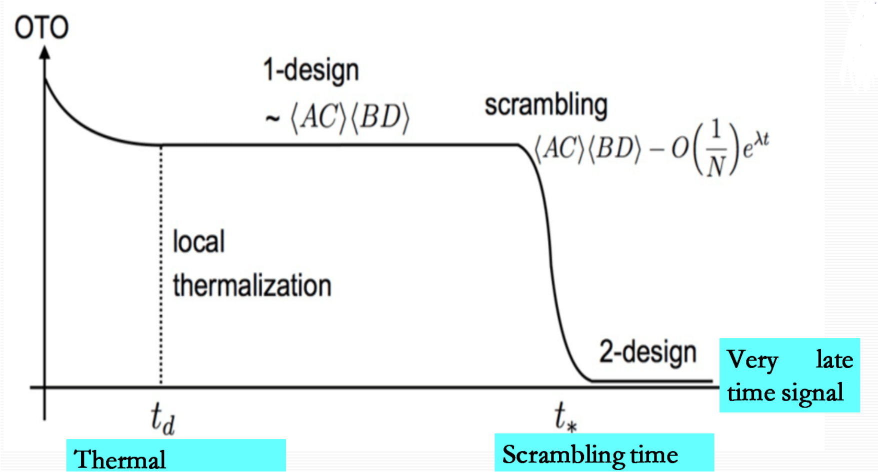
EE in deformed CFTs

- Focus on the EE of deformation perturbatively up to first order.

$$
\begin{aligned}
&\Delta S^{(2)}_{A,0} + \Delta S^{(2)}_{A,\lambda} \\
&= -\Big\{ \log\Big(\prod_{i=1}^{4} |\frac{dw_i}{dz_i}|^{-2h_a} \frac{\langle \mathcal{O}^{\dagger}_a(z_1,\bar{z}_1)\mathcal{O}_a(z_2,\bar{z}_2)\mathcal{O}^{\dagger}_a(z_3,\bar{z}_3)\mathcal{O}_a(z_4,\bar{z}_4)\rangle_{\Sigma_1}}{\big(\langle \mathcal{O}^{\dagger}_a(w_1,\bar{w}_1)\mathcal{O}_a(w_2,\bar{w}_2)\rangle_{\Sigma_1}\big)^2}\Big) \\
&+ \lambda \int d^2 z \frac{|z^2-1|^4}{4L^2|z|^2} \frac{\langle \big(T(z)+\frac{c}{8z^2}\big)\big(T(\bar{z})+\frac{c}{8\bar{z}^2}\big)\mathcal{O}^{\dagger}_a(z_1,\bar{z}_1)\mathcal{O}_a(z_2,\bar{z}_2)\mathcal{O}^{\dagger}_a(z_3,\bar{z}_3)\mathcal{O}_a(z_4,\bar{z}_4)\rangle_{\Sigma_1}}{\langle \mathcal{O}^{\dagger}_a(z_1,\bar{z}_1)\mathcal{O}_a(z_2,\bar{z}_2)\mathcal{O}^{\dagger}_a(z_3,\bar{z}_3)\mathcal{O}_a(z_4,\bar{z}_4)\rangle_{\Sigma_1}} \\
&- 2\lambda \int d^2 w \frac{\langle T\bar{T}(w,\bar{w})\mathcal{O}^{\dagger}_a(w_1,\bar{w}_1)\mathcal{O}_a(w_2,\bar{w}_2)\rangle_{\Sigma_1}}{\langle \mathcal{O}^{\dagger}_a(w_1,\bar{w}_1)\mathcal{O}_a(w_2,\bar{w}_2)\rangle_{\Sigma_1}} \Big\} + \mathcal{O}(\lambda^2).
\end{aligned}
$$

[S.H., Hongfei Shu, 1907.12603.]

OTOC in deformed CFTs



OTOC in deformed CFTs

OTOC in TT-deformed CFTs

$$\frac{\langle W(t)VW(t)V\rangle_\beta}{\langle W(t)W(t)\rangle_\beta\langle VV\rangle_\beta}$$

Put the excitations on the thermal CFTs (Cylinder)

$$\begin{aligned}
&\frac{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)\rangle_\beta\langle V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}\\
&\times\Big(1-\lambda(\frac{2\pi}{\beta})^2\int d^2z_b|z_b|^2\frac{\langle\big(T(z_b)-\frac{c}{24z^2}\big)\big(\bar{T}(\bar{z}_b)-\frac{c}{24\bar{z}^2}\big)W(z_1,\bar{z}_1)W(z_2,\bar{z}_2)\rangle}{\langle W(z_1,\bar{z}_1)W(z_2,\bar{z}_2)\rangle}\\
&-\lambda(\frac{2\pi}{\beta})^2\int d^2z_c|z_c|^2\frac{\langle\big(T(z_c)-\frac{c}{24z^2}\big)\big(\bar{T}(\bar{z}_c)-\frac{c}{24\bar{z}^2}\big)V(z_3,\bar{z}_3)V(z_4,\bar{z}_4)\rangle}{\langle V(z_3,\bar{z}_3)V(z_4,\bar{z}_4)\rangle}\\
&+\lambda(\frac{2\pi}{\beta})^2\int d^2z_a|z_a|^2\frac{\langle\big(T(z_a)-\frac{c}{24z^2}\big)\big(\bar{T}(\bar{z}_a)-\frac{c}{24\bar{z}^2}\big)W(z_1,\bar{z}_1)W(z_2,\bar{z}_2)V(z_3,\bar{z}_3)V(z_4,\bar{z}_4)\rangle}{\langle W(z_1,\bar{z}_1)W(z_2,\bar{z}_2)V(z_3,\bar{z}_3)V(z_4,\bar{z}_4)\rangle}+\mathcal{O}(\lambda^2)\Big)
\end{aligned}$$

[S.H., Hongfei Shu, 1907.12603.]

OTOC in deformed CFTs

Late time of OTOC

S. He, Hongfei Shu [1907.12603]

$$\frac{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)\rangle_\beta\langle V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}$$

$$\xrightarrow{T\bar{T}} \frac{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)\rangle_\beta\langle V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}\left\{1-\lambda C_1(x)+\lambda C_2(x)e^{-\frac{2\pi t}{\beta}}+\cdots\right\},$$

The choices of the sign of λ do not affect the late time behavior exp[-2πβt] in above equation.

D. J. Gross, J. Kruthoff, A. Rolph and E. Shaghoulian,19

Comments and Summary

1. The time evolutions of the n-th-REE ($n \geq 2$) for local excitations in Rational CFTs, 2D quantum gravity, TT/TJ deformed theory.
2. OTOC confirm that the TT/TJ deformation preserve the maximal chaotic behavior in terms of quantum information prespective.

Future Directions

1. It is natural to ask how about the generic CFT, e.g. Liouvile Theory with c<1, large c CFTs, Logrithmic CFTs, Non-diagonal CFTs...
2. Conformal defects (ZZ, FZZT)in LFT V.S. the black hole horizon or not?
3. Modularity in 4-point correlation function of the deformed CFTs. [Working in progress.]

Thanks for your attention!
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