DBI Inflation with Kinetic Coupling to Einstein Gravity

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Based on my recent paper: 1512.02887
Inflationary cosmology

Basic requirements:

- \( a(t) \propto e^{Ht} \)
- \( N = \int_{t_i}^{t_f} H \, dt \approx 60 \)
- generate proper fluctuations
- ....
DBI inflation

Motivated from string theory!

The action:

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} - \frac{1}{f(\phi)} \left( \sqrt{1 + \alpha \nabla_\mu \phi \nabla^\mu \phi} - 1 \right) - V(\phi) \right]$$

E. Silverstein, D. Tong, PRD 2004;
M. Alishahiha, E. Silverstein, D. Tong, PRD 2004;
X. Chen, PRD 2005;
X. Chen, JHEP 2005;
......
DBI inflation

DBI action can be generalized!

Mostly-discussed: higher-order corrections in the DBI field.

\[ S = S_{DBI} + \alpha^2 S^{(2)} + \alpha^3 S^{(3)} + \alpha^4 S^{(4)} + \ldots \]

C. P. Bachas, P. Bain, M. B. Green, JHEP 1999;
N. Wyllard, NPB, 2001;
A. Fotopoulos, JHEP, 2001;
N. Wyllard, JHEP, 2001;

There are also other generalizations, such as including other (gauge) fields.

D. Junghans, G. Shiu, JHEP 2015;

......
Alternatives of Gravity Theories

Arthur Stanley Eddington

\[ S = 2\kappa \int d^4x \sqrt{|R|} \]

Eddington, 1924

Schrodinger, 1950

\[ S = \int d^4x \sqrt{-ag_{\mu\nu} + bR_{\mu\nu} + cX_{\mu\nu}(R)} \]

S. Deser, G. W. Gibbons, CQG 1998

\[ S = \frac{2}{\kappa} \int d^4x \left[ \sqrt{|g_{\mu\nu} + \kappa R_{\mu\nu}(\Gamma)|} - \lambda \sqrt{g} \right] + S_M[g_{\mu\nu}, \Gamma, \Psi] \]

D. N. Vollick, PRD 2004, PRD 2005
M. Banados, P. G. Ferreira, PRL, 2010
Our model

\[
S = \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} - \frac{1}{f(\phi)}(\sqrt{D} - 1) - V(\phi) \right]
\]

where

\[
D = 1 - 2\alpha f(\phi)X + 2\beta f(\phi)\ddot{X}
X = -\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi
\dot{X} = -\frac{1}{2M^2} G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi
\]

The equation of motion:

\[
\frac{3\beta H^2 + \alpha M^2}{M^2 D^{3/2}} (\dddot{\phi} + 3HD\ddot{\phi}) - \frac{f_\phi}{2f^2} \left( \frac{3D - 1}{D^{3/2}} - 2 \right) + \frac{3\beta}{M^2} HHH \frac{D + 1}{\sqrt{D}} \phi + V_\phi = 0
\]

Energy density:

\[
\rho = f^{-1}(\phi)(\sqrt{D} - 1) + V(\phi) + \frac{\alpha \dot{\phi}^2}{\sqrt{D}} + \frac{6\beta H^2 \dot{\phi}^2}{M^2 \sqrt{D}}
\]

Pressure:

\[
p = -f^{-1}(\phi)(\sqrt{D} - 1) - V(\phi) - \frac{3\beta H^2 \dot{\phi}^2}{M^2 \sqrt{D}} - \left( \frac{\beta H \dot{\phi}^2}{M^2 \sqrt{D}} \right)_0
\]
1) Background

From the action we can see that, for $\beta \to 0$ where the model reduces to normal DBI inflation:

\[ \alpha f \dot{\phi}^2 \leq 1 \quad \sqrt{D} \geq 0 \]
Large non-Gaussianities (mildly disfavored by the data)

\[ \alpha f \dot{\phi}^2 \ll 1 \quad \sqrt{D} \sim 1 \]
Close to canonical scalar field slow roll inflation
1) Background

We are considering the last case but with $\beta \neq 0$

For homogeneous background, we have:

$$X = \frac{\dot{\phi}^2}{2}, \quad \ddot{X} = -\frac{H^2}{M^2} \frac{\dot{\phi}^2}{2} \quad \text{with} \quad H \gg M$$

For $V(\phi) = \frac{1}{2}m^2\phi^2$, the equation of motion is:

$$\ddot{\phi} + (3 - \epsilon - s)H \dot{\phi} - \left(\frac{mM}{H}\right)^2 \phi = 0 \quad \text{with} \quad m_{\text{eff}} = \frac{mM}{H} \ll m$$

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad s \equiv \frac{\dot{\phi}}{H \sqrt{D}}$$

Constraints on the mass of inflaton is released, and a heavier inflaton can drive proper inflation!
2) Perturbations

Several conditions from theory and observations:

✓ Stable (no ghost nor gradient instability) for scalar and tensor perturbations.

✓ Near Scale-invariance for scalar perturbations

✓ Amplitude for scalar perturbations

✓ Small tensor-to-scalar ratio
2) Perturbations

2.1 Tensor Perturbations

Perturbed metric:
\[ ds^2 = -N^2 dt^2 + a^2(t)e^{2\xi \delta_{ij} + 2\gamma_{ij}}(dx^i + N^i dt)(dx^j + N^j dt) \]

Making use of
\[ R = (3)R + K_{ij}K^{ij} - K^2 \quad X = \dot{\phi} / 2N \quad \ddot{X} = -\frac{1}{4M^2 N^2} \phi^2 \frac{\dot{\phi}^2}{(3)R - K_{ij}K^{ij} + K^2} \]

One can get the perturbative action:
\[ S_2^T = \frac{1}{8\kappa^2} \int d^4 x a^3 \left[ F_T \dot{\gamma}_{ij}^2 - G_T \left( \nabla \gamma_{ij} \right)^2 \right] \]

Instability requires \( F_T > 0 \) \( G_T > 0 \)

which means \( \kappa^2 |\beta| \phi^2 < 2M^2 \sqrt{D} \)
From the action, we obtain the equation of motion:

$$\gamma''_{ij} - c_T^2 \nabla^2 \gamma_{ij} + \left( \frac{a^2 F_T}{a^2 F_T} \right)' \gamma'_{ij} = 0$$

with the solution:

$$\gamma_{ij} = \text{const.}, \quad \int \frac{dt}{a^3(t) F_T}$$

The tensor power spectrum is:

$$P_T = \frac{k^3}{2\pi^2} \left| \gamma_{ij} \right|^2 = \frac{2H^2}{G_T c_T \pi^2}$$

With the spectral index:

$$n_T \equiv \frac{d \ln P_T}{d \ln k} = \frac{\kappa^2 \beta \dot{\phi}^2}{\kappa^2 \beta \dot{\phi}^2 - 2M^2 \sqrt{D}} (2t - s) - 2\epsilon - s_T \quad s_T \equiv \frac{\dot{c}_T}{Hc_T} \quad t \equiv \frac{\ddot{\phi}}{H \dot{\phi}}$$

• Nearly scale-invariant tensor spectrum;
• Corrections on spectral index being order of slow-roll parameters.
2.2 Scalar perturbation:

First of all, from the Hamilton/momentum constraint equations:

\[
\frac{\delta S}{\delta N} = 0 \quad \frac{\delta S}{\delta N_i} = 0 \quad N = 1 + A 
\]

One obtain the two equations:

\[-[12 H^2 + \frac{2\kappa^2}{f(\phi)\sqrt{D}} (1 - \frac{9\beta H^2}{M^2} f(\phi)\dot{\phi}^2) - \frac{2\kappa^2}{f(\phi)D^{3/2}} (1 + \frac{3\beta H^2}{M^2} f(\phi)\dot{\phi}^2)^2]A + 12 H [1 - \frac{\kappa^2 \beta \dot{\phi}^2}{M^2 \sqrt{D}} - \frac{\kappa^2 \beta \dot{\phi}^2}{2 M^2 D^{3/2}} (1 + \frac{3\beta H^2}{M^2} f(\phi)\dot{\phi}^2)]\dot{\xi} - 4 a^{-2} [1 - \frac{\kappa^2 \beta \dot{\phi}^2}{2 M^2 D^{3/2}} (1 + \frac{3\beta H^2}{M^2} f(\phi)\dot{\phi}^2)] \partial^2 \xi = 0 \]

\[-4 a^{-2} H [1 - \frac{\kappa^2 \beta \dot{\phi}^2}{M^2 \sqrt{D}} - \frac{\kappa^2 \beta \dot{\phi}^2}{2 M^2 D^{3/2}} (1 + \frac{3\beta H^2}{M^2} f(\phi)\dot{\phi}^2)] \partial^2 \psi = 0 \]

\[ [2(1 - \frac{\kappa^2 \beta \dot{\phi}^2}{M^2 \sqrt{D}})H - \frac{\kappa^2 \beta \dot{\phi}^2}{M^2 D^{3/2}} (1 + \frac{3\beta H^2}{M^2} f(\phi)\dot{\phi}^2)]A - (2 - \frac{\kappa^2 \beta \dot{\phi}^2}{M^2 \sqrt{D}}) \dot{\xi} + \frac{\kappa^2 \beta^2 H f(\phi)\dot{\phi}^4}{M^4 D^{3/2}} (3H\dot{\xi} - a^{-2} \partial^2 \xi - a^{-2} H \partial^2 \psi) = 0 \]
In the usual case (e.g. the single scalar field discussed in X. Chen, et al, JCAP 2006, or Galileon field, etc ), one could have

\[ A \sim \dot{\xi} \quad \partial^2 (\xi + H\psi) \sim \dot{\xi} \]

which can lead to the quadratic second order perturbation action.

However, in our model, \( A \) and \( \partial^2 (\xi + H\psi) \) will also have some nontrivial dependences on \( \partial^2 \xi \)!

The reason is that since there is \( G_{\mu\nu} \) term inside the square-root, the action will be nonlinear to the extrinsic curvature \( K_{ij} K^{ij} - K^2 \)

\[ S \sim \Upsilon (K_{ij} K^{ij} - K^2) \Rightarrow \frac{\delta S}{\delta N_i} \sim \Upsilon \nabla_i (K_{ij} K^{ij} - K^2) + (K_{ij} K^{ij} - K^2) \nabla_i \Upsilon = 0 \]

\[ HA - \dot{\xi} \]
In the slow-roll limit, one gets the relations below:

\[
A \approx \frac{\dot{\xi}}{H} - \frac{4x_\beta y}{a^2 H^2} \partial^2 \xi \\
\partial^2 \psi \approx \frac{a^2(1 - D)}{2 D^2 y} \dot{\xi} - \frac{1}{H} \partial^2 \xi
\]

Therefore the scalar perturbation action is:

\[
S_2^c = \frac{1}{2\kappa^2} \int d^4 x a^3 \left[ 6 \frac{x_\beta}{D} \dot{\xi}^2 - \frac{2\varepsilon}{a^2} (\partial \xi)^2 + \frac{16 x_\beta y}{a^4 H^2} (\partial^2 \xi)^2 \right]
\]

Ghost-free: \( x_\beta > 0 \)

Free of gradient instability at large-k: \( y < 0 \)
The equation of motion can be written as:

\[ u'' + \omega^2 k^2 u - \frac{z''}{z} u = 0 \]

where

\[ u = z \xi \quad z = a \sqrt{\frac{3x_\beta}{D}} \quad \omega^2 = \frac{\varepsilon D}{3x_\beta} k^2 \left( 1 + 24 \frac{x_\beta^5 |y|}{\varepsilon^2 D^2 (c_s k)^2} \right) \quad c_s^2 = \frac{\varepsilon D}{3x_\beta} \]

From above we can see that there is a pivot scale: \( k_c = aH \sqrt{\frac{\varepsilon D}{8x_\beta^4 |y|}} \)
Small-k (large scale) solution:

$$\xi = \text{const.}, \quad \frac{1}{3} \int \frac{Ddt}{a^3(t)x_\beta}$$

Which can lead to power spectrum:

$$P^{(i)}_S \equiv \frac{k^3}{2\pi^2} |\xi|^2 \approx \frac{H^2}{8\pi^2} \sqrt{\frac{3x_\beta}{\varepsilon^3D}}$$

The spectral index:

$$n^{(i)}_S \equiv 1 + \frac{d \ln P^{(i)}_S}{d \ln k} \approx 1 + 2\varepsilon - \frac{3}{2} \eta + \frac{3}{4} s \quad \eta \equiv \frac{\dot{\varepsilon}}{H\varepsilon}$$

And the tensor/scalar ratio:

$$r^{(i)} \equiv \frac{P_T}{P^{(i)}_S} \approx 16\varepsilon \sqrt{\frac{\varepsilon D}{3x_\beta}}$$
For large $k$, at the inside-horizon region $k^4$ term will take part in. However, the correction term in our case is of the form

$$k^2 \left( \frac{c_s k}{aH} \right)^2$$

It has been proved (Lu, Piao, IJMPD, 2010) that the perturbations with dispersion relation

$$\omega^2 \sim k^2 \left( \frac{c_s k}{aH} \right)^n, n \in \mathbb{N}$$

lead to scale-invariant spectrum.

Large-$k$ (small scale) solution:

$$\xi = H \sqrt{\frac{D}{6x_{\beta} \omega^3}}, \quad H \sqrt{\frac{\omega^3 D}{6x_{\beta}}} |\eta|^3$$

Power spectrum:

$$P_s^{(s)} = \frac{H^2}{8\pi^2} \sqrt{\frac{3x_{\beta}}{\epsilon^3 D}} \left[ 1 - 36 \frac{x_{\beta}^5 |y|}{\epsilon^2 D^2} \left( \frac{c_s k}{aH} \right)^2 \right]$$
Since the amplitude has a deficit in the small-scales in the power spectrum, one can expect a red-tilt of the spectrum!

This is an effect additional to the spectral tilt, which is due to the correction term rather than the parameter running!

Normal case: \( P_S \sim \frac{H^2}{\epsilon} \)

\( H \) and \( \epsilon \) is not an exact constant \( \Rightarrow n_S = 1 + \frac{d \ln P_S}{d \ln k} \neq 1 \) \( \Rightarrow \) Spectrum gets tilted
The spectral index:

\[ n_s^{(s)} = 1 + 2\varepsilon - \frac{3}{2}\eta + \iota - \frac{3}{4}s + \Delta n, \]

\[ \Delta n = \frac{36x^5|y|}{36x^5|y| - \varepsilon^2 D^2}(5\varepsilon_x + \varepsilon_y - 2\eta - 2s) \]

The tensor/scalar ratio:

\[ r^{(s)} \equiv \frac{P_T}{P_S^{(s)}} \approx 16\varepsilon \sqrt{\frac{\varepsilon D}{3x_\beta}} \left[ 1 + 36\frac{x^5|y|}{\varepsilon^2 D^2} \left( \frac{c_s k}{aH} \right)^2 \right] \]

\[ \varepsilon_x \equiv \frac{x_\beta}{Hx_\beta} \]

\[ \varepsilon_y \equiv \frac{\dot{y}}{Hy} \]
3) \textbf{Constraints on the parameters}

The characteristic parameters/functions that has to be constrained:

\[ x_\beta \equiv \frac{\kappa^2 \beta \phi^2}{2M^2 \sqrt{D}} \to \beta \]

\[ y \equiv \frac{f(\phi)M_p^2 H^2}{\sqrt{D}} \to f(\phi) \]

1) Making use of the tensor/scalar ratio:

\[ x_\beta \approx 8.53 \times 10^{-3} \times \left( \frac{\epsilon}{0.01} \right)^3 \left( \frac{0.1}{r} \right)^2 \]

Therefore \( x_\beta \) will be of order \( 10^{-2} \sim 10^{-3} \) for \( \epsilon \sim 0.01 \) and \( r \sim 0.1 \) (same order as the slow-roll parameters).

2) From the small-scale power spectrum:

\[ y \approx \frac{\Delta P_s}{P_s} \left( \frac{\epsilon}{x_\beta} \right) \left( \frac{0.01}{\epsilon} \right)^3 \times 10^6 \]

If future observations can distinguish different reasons for the spectrum tilt and determine the latter, \( y \) can be constrained. A step-like variation tends to support our model.
Conclusions

We proposed a new inflation model which:

✓ Inspired from DBI inflation, Galileon as well as Born-Infeld gravity theories;
✓ Can relax the mass of inflaton;
✓ Can give rise to nearly scale-invariant tensor/scalar power spectrum;
✓ Can give rise to a red-tilt due to the large-k corrections;
✓ Can be tested for now/in the future.
Thanks!
Happy New Year!