Toward precision measurement of neutrinos

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- The present and the future
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The solar neutrino problem:

$\nu$ produced in Sun:

$\text{pp } \nu: \ p + p \rightarrow 2\ H + e^+ + \nu_e$

$\text{Boron } \nu: \ ^8\text{B} \rightarrow ^8\text{Be}^* + e^+ + \nu_e$

e tc. ... 

At low energy part of the spectrum about $\frac{2}{3} \nu_e$ flux detected

At high energy part of the spectrum about $\frac{1}{3} \nu_e$ flux detected

Missing of atmospheric $\nu_\mu$ also discovered
When flavor eigenstates are not the mass eigenstates ($\nu_1, \nu_2$),

\[
i \frac{d}{dx} \begin{pmatrix} \nu_e \\ \nu'_\mu \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m^2 \cos 2\theta & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & \Delta m^2 \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu'_\mu \end{pmatrix};
\]

\[
\begin{pmatrix} \nu_e \\ \nu'_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}.
\]

At distance $x$ one obtains

\[
|\nu_e(x)\rangle = \cos \theta e^{-iE_1x} |\nu_1\rangle + \sin \theta e^{-iE_2x} |\nu_2\rangle;
\]

\[
|\langle \nu_e | \nu_e(x) \rangle|^2 = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 x}{4E}.
\]

Averaging over phases or coherence lost (happened for $\nu_\odot$)

\[
P_{ee} = 1 - \frac{1}{2} \sin^2 2\theta = \frac{1}{2} (1 + \cos^2 2\theta).
\]

$P_{ee} \geq 1/2$ in vacuum oscillation.
Coherent forward scattering by medium modifies dispersion relation:

\[ E = E_k + V, \]

\( E_k \), the kinetic energy; \( V \), the potential energy. Examples include

- the optics: the case of electromagnetism;
- neutron optics: the case of strong interaction;
- MSW: the case of weak interaction, \( V_e = \sqrt{2} G_F N_e \):

\[
\begin{align*}
\frac{d}{dx} \begin{pmatrix} \nu_e \\ \nu'_\mu \end{pmatrix} &= \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta + V_e & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu'_\mu \end{pmatrix}; \\
\begin{pmatrix} \nu_e \\ \nu'_\mu \end{pmatrix} &= \begin{pmatrix} \cos \theta_m & \sin \theta_m \\ -\sin \theta_m & \cos \theta_m \end{pmatrix} \begin{pmatrix} \nu_{m1} \\ \nu_{m2} \end{pmatrix}.
\end{align*}
\]
LMA MSW solution:

\[ \Delta m^2 = (3.8 - 10) \times 10^{-5} \text{eV}^2, \]
\[ \tan^2 \theta = 0.32 - 0.47 \]

- \( L_{osc} \sim 200 \text{ km}. \)
- For \( E \gtrsim 7 \text{ MeV}, \)
  \[ P_{ee} = \frac{1}{2} (1 - \cos 2\theta) \approx 0.3; \]
- for \( E \lesssim 1 \text{ MeV}, \)
  \[ P_{ee} = \frac{1}{2} (1 + \cos^2 2\theta) \approx 0.6. \]
- \( 2E\nu_e/\Delta m_{21}^2 \lesssim 0.08 \text{ in Earth}; \)
  matter effect small in Earth.
The present and the future

The present status
Oscillation of $\nu$ confirmed, LMA MSW established

- **Solar $\nu$ experiment (Homestake, Super-K, SNO, etc.),** plus long baseline $\nu$ experiment (Kamland):
  \[ \Delta m^2_{21} \approx 0.8 \times 10^{-4} \text{ eV}^2, \quad \tan^2 \theta_{12} \approx 0.4 \]

- **Atmospheric $\nu$ experiments (Super-K, SNO, etc.):**
  \[ |\Delta m^2_{32}| \approx 3 \times 10^{-3} \text{ eV}^2, \quad \tan^2 \theta_{23} \approx 1.0 \]

- **Reactor $\nu$ experiment (Chooz, etc.):**
  \[ \sin^2 2\theta_{13} \lesssim 0.1 \]
In the future we will measure

- $\theta_{13}$ (reactor $\nu$, long baseline exp)
- mass hierarchy (long baseline exp)
- CP violation in neutrinos (long baseline exp)
- absolute mass scale
- the Earth matter effect (long baseline, solar $\nu$ exp)
- nature of neutrino mass
- magnetic moment of neutrino, etc.

Earth matter effect is important to most measurements
Earth matter effect

PReliminary Earth Model (PREM)

Matter profile very complicated

Earth matter density has many layers and changes
- sharply between two layers
- slowly in a layer:
  density height

\[ h = \left( \frac{dV}{dx} \right)^{-1} \sim R_{\text{Earth}} \]

Earth matter effect is a challenge to future precision measurement
Quest to understand the effect of the complex Earth matter in neutrino oscillation
Earth matter effect

Oscillation length tells how fast $\nu$ oscillates

$$L_{21} = \frac{4\pi E}{\Delta m^2_{21}}, \quad L_{32} = \frac{4\pi E}{|\Delta m^2_{32}|}$$

Note that available measurements tell us

- for $E \sim 10$ MeV (solar, supernova neutrinos)
  $$L_{21} \sim \text{hundreds km}, \quad L_{32} \sim \text{few km},$$
  $$L_{21}, L_{32} \ll h$$

- for $E \gtrsim 500$ MeV
  $$L_{21} \gtrsim h$$

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Earth matter effect

An important lesson:
oscillation well approximated by $1 - 2$ plus $2 - 3$ oscillation
i.e., oscillation in matter and in vacuum if $\theta_{13}$ small

$$H = \frac{1}{2E} U^* \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} V_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Rotation in $2 - 3$ sector does not change the potential term

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Crucial to compare $L_{21}$ and $h$ to understand matter effect;
$L_{31}$ not crucial when $\theta_{13}$ small
Low energy $\nu$ conversion in the Earth

For $E \lesssim 30$ MeV, $L_{21} \ll h$ (solar and supernova neutrinos)

$\nu_e$ survival well reduced to $1 - 2$ oscillation

$$i \frac{d}{dx} \begin{pmatrix} \psi_{1m} \\ \psi_{2m} \end{pmatrix} = \begin{pmatrix} -\frac{\Delta(x)}{4E} & -i\dot{\theta}_m(x) \\ i\dot{\theta}_m(x) & \frac{\Delta(x)}{4E} \end{pmatrix} \begin{pmatrix} \psi_{1m} \\ \psi_{2m} \end{pmatrix},$$

$(\psi_{m1}, \psi_{m2})^T$, neutrino mass state in matter.

The survival probability of $\nu_\odot$ on the Earth

$$P_{ee} = \frac{1}{2} \left( 1 + \cos 2\theta_m(x_0) \cos 2\theta_{12} - \cos 2\theta_m(x_0) f_{\text{reg}} \right).$$

$$f_{\text{reg}} = P(\nu_2 \to \nu_e) - \sin^2 \theta_{12}, \text{ the regeneration by the Earth}$$
Adiabatic perturbation theory
(de Holanda, Liao, Smirnov, 2004)

Search for the solution of the following form

\[
\begin{pmatrix}
\psi_1m(x) \\
\psi_2m(x)
\end{pmatrix}
\begin{pmatrix}
e^{i\Phi(x)} & c(x)e^{-i\Phi(x)} \\
-c^*(x)e^{i\Phi(x)} & e^{-i\Phi(x)}
\end{pmatrix}
\begin{pmatrix}
\psi_1m(x_0) \\
\psi_2m(x_0)
\end{pmatrix}
\]

\[\Phi(x) = \frac{1}{4E} \int_{x_0}^x dx' \Delta(x').\]

where \(|c(x)| \ll 1\) is supposed to hold (adiabatic perturbation)

We get

\[c(x) = -\int_{x_0}^x dx' \frac{d\theta m(x')}{dx'} \exp \left[ -i \int_x^{x'} dx'' \frac{\Delta(x'')}{2E} \right].\]
Low energy $\nu$ conversion in the Earth

$x$ regeneration in the Earth

$$f_{\text{reg}} = \frac{2E \sin^2 2\theta}{\Delta m^2} \sin \Phi_0 \sum \Delta V_i \sin \Phi_i,$$

$$\Phi_i = \int_{-L_i/2}^{L_i/2} dx \frac{\Delta(x)}{4E}.$$

- Leading contributions are from potential jumps between layers
- Analytic and numerical computations perfectly agree
- Oscillatory pattern is well understood using the adiabatic perturbation theory
Low energy $\nu$ conversion in the Earth

$\nu$ regeneration in the Earth

Properties:
- Averaging over energy tremendously simplifies the oscillation pattern
- Neutrinos of horizontal direction still give complicated oscillation pattern in Earth regeneration

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Low energy $\nu$ conversion in the Earth

Extension to asymmetric matter profile

$$f_{\text{reg}} = -\frac{E \sin^2 2\theta_{12}}{\Delta m^2_{21}} \sum_{i=0}^{k} \delta V_i \cos 2\phi_i, \quad \phi_i = \int_{x_i}^{x_k} dx \frac{\Delta(x)}{4E}, \quad i = 0, \cdots, k$$

Main uncertainties in the Earth matter effects are from contributions of small structures close to the detectors.
High energy $\nu$ conversion in the Earth

For $E \gtrsim 0.5$ GeV, $L_{21} \gtrsim h$, $L_{31} > \text{or} < h$

$\nu$ oscillation seems very complicated.

It turns out that
1) we can have a perturbation theory which perfectly describes the oscillation pattern
2) neutrino oscillation can be substantially simplified in the interested energy range
High energy $\nu$ conversion in the Earth

$\nu_\mu \rightarrow \nu_e$ conversion vs. $L$

A nice example:

For $E > 10$ GeV ($L_{21}, L_{31} > h$), neutrinos
  - can not see the structure of the Earth very well
  - see baseline dependent average potential in Earth

Earth matter effect is well described by a formulation using the baseline dependent average potential
High energy $\nu$ conversion in the Earth

Liao, 2008; Liao, 2008

A baseline dependent perturbation theory

\[ H = \bar{H} + \delta H, \quad \bar{H} = H_0 + \bar{V} \]

\[ \delta H = \delta V = V(x) - \bar{V}, \quad \bar{V} = \frac{1}{L} \int_0^L dx \ V(x) \]

$\bar{V}$ depends on baseline

The transition matrix is found

\[ M(L) = \bar{U}_m e^{-i \frac{A}{2E} L} (1 - iC) \bar{U}_m^\dagger \]

\[ C = \int_0^L dx \ e^{i \frac{A}{2E} L} \bar{U}_m^\dagger \delta V(x) \bar{U}_m e^{-i \frac{A}{2E} L} \]

This is a perturbation expanded using $\delta V$ around the baseline dependent average potential $\bar{V}$
High energy $\nu$ conversion in the Earth

So

$$C_{jj} = \int_0^L dx (\bar{U}_m^\dagger \delta V(x) \bar{U}_m)_{jj} = 0,$$

$$C_{jk} = \int_0^L dx \ e^{i \frac{\Delta j - \Delta k}{2E}} (\bar{U}_m^\dagger \delta V(x) \bar{U}_m)_{jk}, \ j \neq k$$

$|C_{jk}| \ll 1$ needed as a good perturbation approximation

$C_{jk}$ suppressed by

1) $\delta V_e / \bar{V}_e \lesssim 0.3$
2) small $\Delta m^2_{21} / (4E \bar{V}_e)$ and $\sin \theta_{13}$
High energy $\nu$ conversion in the Earth

$\nu_\mu \rightarrow \nu_e$ conversion vs. $E$

For $L \lesssim 6000$ km, the Earth matter effect is very well described by the baseline dependent average potential, the only parameter for a fixed baseline.

Plus 1st order correction, the theory perfectly describes the oscillation of high energy $\nu$ in the Earth.
The CP violating effect in Earth matter is very well described by this formulation of $\nu$ oscillation.
High energy $\nu$ conversion in the Earth

**Left, $\nu_\mu \rightarrow \nu_\tau$ vs. L; right, $\nu_e \rightarrow \nu_\tau$ vs. L**

$\nu_\mu - \nu_\tau$ is mainly vacuum oscillation, as can be seen

$\nu_e - \nu_\tau$ oscillation is very well described by the perturbation theory
Conclusions

- We found two nice theories which perfectly describe neutrino oscillation of interested energy range in Earth matter.

- Earth matter effect of interested energy range is well understood in these theories.

- For $E \lesssim 30$ MeV, the adiabatic perturbation theory says that main contributions are from potential jumps between layers of the Earth matter and averaging over energy can tremendously simplify the oscillation pattern.
Conclusions

- For $E \gtrsim 500$ MeV, expanding the potential around the baseline dependent average potential gives a perturbation theory.

- This perturbation theory perfectly describes the probability and CP violating effect of oscillation of high energy neutrinos.

- Ambiguities of Earth matter effect in measuring neutrino parameters in future experiments, in particular in measuring $\delta_{CP}$, are properly addressed in the framework of this perturbation theory.