Electromagnetic field, flow vorticity, and anomalous transports in heavy-ion collisions

Xu-Guang Huang
Fudan University, Shanghai

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Outline

☐ Introduction

☐ Electromagnetic (EM) fields and vorticity in heavy-ion collisions

☐ EM-field induced anomalous transports (chiral magnetic effect, etc.)

☐ Vorticity induced anomalous transports (chiral vortical effect, etc.)

☐ summary
Introduction
Introduction

Phase diagram of quantum chromodynamics (QCD)

QCD confining scale ~ size of hadrons ~200 MeV
Introduction

Produce deconfined quark-gluon matter in Lab.: high-energy heavy-ion collisions
Introduction

High-energy heavy-ion collisions (HICs)

RHIC and LHC revealed intriguing properties of QGP

Perfect fluid

Jet quenching
Introduction

HICs generate not only hot QGP but also magnetic fields

- Imagine noncentral collision ⇒ Large B field in y direction. No (or small) E field.

- How strong? A crude estimate:
  - RHIC Au+Au collision, $Z = 79, \sqrt{s} = 200$ GeV ($\Rightarrow v_z \simeq 0.99995c$), impact parameter $b = 5$ fm
  - The B field at the colliding time, $t = 0$. Biot-Savart law

\[ eB_y \sim 2 \times \frac{\gamma e^2}{4\pi} Z v_z \left(\frac{2}{b}\right)^2 \approx 40m^2_\pi \sim 10^{19} \text{ Gauss} \]
**Introduction**

HICs generate not only hot QGP but also magnetic fields

**Comparison of magnetic fields**

<table>
<thead>
<tr>
<th>Description</th>
<th>Field Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Earth’s magnetic field</td>
<td>0.6 Gauss</td>
</tr>
<tr>
<td>A common, hand-held magnet</td>
<td>100 Gauss</td>
</tr>
<tr>
<td>The strongest steady magnetic fields achieved so far in the laboratory</td>
<td>$4.5 \times 10^6$ Gauss</td>
</tr>
<tr>
<td>The strongest man-made fields ever achieved, if only briefly</td>
<td>$10^7$ Gauss</td>
</tr>
<tr>
<td>Typical surface, polar magnetic fields of radio pulsars</td>
<td>$10^{13}$ Gauss</td>
</tr>
<tr>
<td>Surface field of Magnetars</td>
<td>$10^{15}$ Gauss</td>
</tr>
<tr>
<td><a href="http://solomon.as.utexas.edu/~duncan/magnetar.html">http://solomon.as.utexas.edu/~duncan/magnetar.html</a></td>
<td></td>
</tr>
</tbody>
</table>

Heavy ion collisions: the strongest magnetic field ever achieved in the laboratory

Off central Gold-Gold Collisions at 100 GeV per nucleon

$e_B(\tau=0) \sim 10^{19}$ Gauss
EM fields and vorticity
EM fields in heavy-ion collisions

Detailed study of the EM fields in HICs

- Very strong fields, $eB \sim 10^{18-20}$ G
- Event-by-event fluct.
- Event averaged $eB$:

$$e\langle B_y \rangle \propto \frac{\sqrt{s}}{2m_N} \frac{Z}{A^{2/3}} \frac{b}{2R_A} m_{\pi}^2, \text{ for } b < 2R_A$$

EM fields in heavy-ion collisions

Heavy-ion collisions generate EM fields

Fluctuating azimuthal direction

Strong in-plane E field in Cu + Au collisions

Charge dependence of $v_1$

 Bloczynski, XGH, Liao, Zhang, 2012

Hirono and Hirano, 2012

STAR 2015
EM fields in heavy-ion collisions

Heavy-ion collisions generate EM fields

Time evolution of the B field (insulating medium)

Well fitted by

$$\langle eB_y(t) \rangle \approx \frac{\langle eB_y(0) \rangle}{(1 + t^2/t_B^2)^{3/2}}$$

Life time of B field

$$t_B \approx R_A / (\gamma v_z) \approx \frac{2m_N}{\sqrt{s}} R_A$$

But QGP is ideally conducting: lattice QCD result

Open question: what is the time dependence of B field in HICs?

$$\sigma \approx (1/3)C_{EM}T$$
Vorticity in heavy-ion collisions

Heavy-ion collisions generate vorticity

Finite angular moment (AM)

Manifested as flow shear* 

Finite vorticity (local rotation)

\[ \vec{\omega} = \vec{\nabla} \times \vec{v} \]

\[ J_0 \sim A b \sqrt{s}/2 \]

\[ J(\vec{x}) \sim [x^2 - (x \cdot \vec{\omega})^2] \varepsilon(\vec{x}) \] is the moment of inertia density

\[ J_0 \] is about \(10^6\) for RHIC Au+Au @ 200 GeV, system volume is \(\sim\) fm\(^3\), very large AM density

*For low energy collision, the system after collision may be globally rotating
Vorticity in heavy-ion collisions

Heavy-ion collisions generate vorticity
The detailed simulations of vorticity came out very recently (Deng and XGH 2016, Jiang, Lin, Liao 2016)

- Vorticity of energy flow at RHIC at $b=10$ fm is $10^{22}$ Hz. (Fastest man-made rotation via laser light $\sim 10^7$ Hz (Arita et al. Nat. Comm. 2013))
- RHIC: Take $T \sim 300$ MeV, $T \times$ vorticity $\sim 10^4$ MeV$^2$ comparable to magnetic field $eB \sim 10^4$ MeV$^2$. But at LHC, $T \times$ vorticity is much smaller than $eB$.
- At $b<2R_A$, increase with $b$; then drops. Angular momentum of the overlapping region has a similar behavior.
Vorticity in heavy-ion collisions

Heavy-ion collisions generate vorticity
Vorticity at mid-rapidity has very nontrivial energy dependence

- Reason: higher energy: more AM carried by finite rapidity particles; mid-rapidity closer to Bjorken boost invariant; larger moment of inertia
- Indicates stronger vortical effect at lower energy

Liang and Wang 2005: global quark polarization wrt reaction plane due to spin-vorticity coupling
EM-field induced anomalous transports
What are the effects of EM fields and vorticity?
They can see the topological sector of QCD

Theta vacua

Instanton tunneling suppressed; but sphaleron transition easily happen when temperature is high

Cf. Leinweber
Chiral magnetic effect

B fields can monitor nontrivial topology of QCD

QED VVA triangle anomaly

Chiral magnetic effect (CME)

\[ J_V = \frac{N_c e}{2\pi^2} \mu_A B \]

QCD VVA triangle anomaly

- Parity-odd transport
- Time-reversal even, no dissipation
- Fixed by anomaly coefficient, universal

Kharzeev 2004, Kharzeev, Warringa, McLauren, Fukushima 2008 ... ...
Chiral magnetic effect

Phenomenology of CME in heavy-ion collisions: Event-by-event charge separation wrt. reaction plane

- Positive opposite-sign correlation, negative same-sign correlation
- Increase with centrality = \( B \) increases with centrality
- Nearly independent of collision energy = \( B \times t_B \sim \text{constant} \)
- Delta gamma dissipates for low energy = no chiral symmetry res.

The gamma correlator (Voloshin 2004)
Chiral magnetic wave

Same QED VVA triangle anomaly, but with V and A interchanged

Chiral separation effect (CSE)

Son and Zhitnitsky 2004,
Metlitski and Zhitnitsky 2004

CME + CSE give gapless wave modes: chiral magnetic wave
(Kharzeev and Yee 2010)
Chiral magnetic wave

Phenomenology of CMW in heavy-ion collisions: Elliptic flow splitting of charged pions (Burnier, Kharzeev, Liao, Yee 2011)

Intuitive picture

CMW ⇒ $v_2(\pi^-) \neq v_2(\pi^+)$; $v_2(\pi^-) - v_2(\pi^+) \approx rA_\pm$: linear approx. in net charge asymmetry $A_\pm = (N_+ - N_-)/(N_+ + N_-)$

STAR 2015

Au+Au 200 GeV: 30-40%
0.15 < $p_T$ < 0.5 GeV/c
Chiral electric separation effect (CESE)

Electric field induced anomalous transport

\[ J_A \approx 14.5163 \text{Tr}_f(QeQ_A) \frac{\mu V \mu_A}{T^2} \frac{e^2 T}{g^4 \ln(1/g)} E \]

- P-odd, C-odd, T-odd transport (may be dissipative)
- Non-universal (receive perturbative correction)

XGH and Liao 2013, Jiang, XGH, Liao 2015 ... ...
Chiral electric separation effect

- Collective modes by CESE, CME, and CSE.
- The complete electromagnetic response of a chiral matter:
  \[ j^\mu_V = \sigma E^\mu + \frac{e}{2\pi^2} \mu_A B^\mu, \]
  \[ j^\mu_A = \sigma_5 E^\mu + \frac{e}{2\pi^2} \mu_V B^\mu. \]

- Coupled evolution of vector and axial currents leads to several collective modes (XGH and Liao, PRL110(2013)232302):
  - If \( B = B\hat{z} \) and \( E = 0 \): two Chiral magnetic waves along \( B \)
    \[ \omega = \pm \sqrt{(v_\chi k_z)^2 - (e\sigma_0/2)^2 - i(e\sigma_0/2)} \]
  - If \( B = 0 \) and \( E = E\hat{z} + A\)-background: two Chiral electric waves
    \[ \omega = \pm \sqrt{(v_e k_z)^2 - (e\sigma_0/2)^2 - i(e\sigma_0/2)} \]
  - If \( B = 0 \) and \( E = E\hat{z} + V\)-background: one Vector density wave and one Axial density wave along E-field
    \[ \omega_V = v_\nu k_z - ie\sigma_0, \]
    \[ \omega_A = v_\alpha k_z \]

- These collective excitations transport chirality and charge, and leads to novel charge azimuthal distribution \( \Rightarrow \)
Chiral electric separation effect

- Possible implication: Recall that in-plane E-field in AuCu collisions.

- In-plane dipole due to usual Ohm conduction + out-of-plane dipole due to CME + quadrupole due to CESE and CME in Cu + Au collisions.

\[ f_1(q, \phi) \propto 1 + 2v_1^0 \cos(\phi - \psi_1) + 2q d_E \cos(\phi - \psi_E) + 2\chi q d_B \cos(\phi - \psi_B) \]
\[ + 2v_2^0 \cos[2(\phi - \psi_2)] + 2\chi q h_B \cos[2(\phi - \psi_c)] \] + higher harmonics
Vorticity induced anomalous transport
Chiral vortical effect

A modified “relativistic Larmor theorem”

\[ eB \sim 2\mu_V \omega \]

\[ J_V = \frac{N_c e}{2\pi^2} \mu_A B \quad \rightarrow \quad J_V = \frac{N_c \mu_V \mu_A}{2\pi^2} \omega \]

Chiral magnetic effect          Vector chiral vortical effect

This naïve mapping does not work for axial current. The calculation gives the vorticity induced axial current:

\[ J_A = N_c \left( \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2}{2\pi^2} \right) \omega \]

Axial chiral vortical effect

• Related to chiral anomaly and gravitational anomaly
• Universal (receive no perturbative correction*)

* \( T^2 \) term may have perturbative correction
Chiral vortical effect

Phenomenology of vector CVE in heavy-ion collisions: Event-by-event baryon separation wrt. reaction plane

- Positive opposite-sign correlation, negative same-sign correlation
- Increase with centrality = vorticity increases with centrality

Background effects: e.g., transverse momentum conservation, local baryon conser.
Chiral vortical wave

The vortical analogue of chiral magnetic wave

\[ \vec{J}_A = \left( \frac{T^2}{6} + \frac{\mu^2 + \mu_5^2}{2\pi^2} \right) \vec{\omega}, \quad \vec{J}_V = \frac{\mu \mu_5}{\pi^2} \vec{\omega} \]

- A new collective mode. To reveal its dispersion we use continuity eq.

\[ \partial_t n_{L,R} + \nabla \cdot \vec{J}_{L,R} = 0 \]

- Substitute CVE currents. Obtain Burgers wave equation which is linearized to normal wave equation

\[ \partial_t n_{L,R} = \pm \frac{\omega \alpha^2}{\pi^2} \partial_x (n_{L,R}^2) \quad \Rightarrow \quad \pm \frac{2\omega \alpha^2}{\pi^2} n_0 \partial_x (n_{L,R}) \]

\[ \alpha = \frac{\partial \mu}{\partial n} \sim \text{inverse baryon susceptibility} \]

Jiang, XGH, Liao 2015
Chiral vortical wave

Experimental implication: baryon charge quadrupole

- More baryon charges at the tips of the fireball, more antibaryon charges at the center
- Stronger in-plane radial expansion lets antibaryons get larger elliptic flow than baryons
**Chiral vortical wave**

**Lambda-anti-Lambda v\_2 splitting**

- Chemical potential shift of quark of flavor f (leading order in q):
  \[ \delta \mu_f \propto 2 q_f^\Omega \cos(2\phi_s) \]
  \[ q_f^\Omega = \left[ \int dx dy (\delta n_f^\Omega \cos(2\phi_s)) \right]/\left[ \int dx dy (\delta n_f) \right] \]
  \( \sim \) quadrupole moment

- Lambda (carries baryon charge but no electric charge: responses to CVW but not CMW)

\[ \delta \mu_\Lambda \propto 2(q^\mu_\Omega + q^d_\Omega + q^s_\Omega) \cos(2\phi_s) \]

\[ \Delta v_2 = v_2^\Lambda - v_2^\Lambda \propto |q_f^\Omega| A_\Lambda^A \]

\[ A_\Lambda^A = (N_\Lambda - N_\Lambda^\Lambda)/(N_\Lambda + N_\Lambda^\Lambda) \]
## Anomalous transports

<table>
<thead>
<tr>
<th></th>
<th>$E$</th>
<th>$B$</th>
<th>$\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_V$</td>
<td>$\sigma$</td>
<td>$\frac{N_C e}{2\pi^2 \mu_A}$</td>
<td>$\frac{N_C}{\pi^2} \mu_V \mu_A$</td>
</tr>
<tr>
<td></td>
<td>Ohm’s law</td>
<td>Chiral magnetic effect</td>
<td>Vector chiral vortical effect</td>
</tr>
<tr>
<td>$J_A$</td>
<td>$\propto \frac{\mu_V \mu_A}{T^2} \sigma$</td>
<td>$\frac{N_C e}{2\pi^2 \mu_V}$</td>
<td>$N_C \left( \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2}{2\pi^2} \right)$</td>
</tr>
<tr>
<td></td>
<td>Chiral electric separation effect</td>
<td>Chiral separation effect</td>
<td>Axial chiral vortical effect</td>
</tr>
</tbody>
</table>

Collective waves: chiral magnetic wave, chiral electric waves, chiral vortical wave, vector and axial density wave, ... ...
Discussions

The P-odd anomalous transports may happen also in:

- Weyl/Dirac semimetals: 3D materials with relativistic excitations
- A parallel E and B can induce CME

Weyl spin-orbit coupled Fermi gas under rotation

Institute of Physics CAS/Princeton/BNL 2015-2016......

XGH 2015
Discussions

The P-odd anomalous transports may happen also in:

- Neutrino gas in supernovae/protoneutron stars
  - Vilenkin 1979, Yamamoto 2015

There may also be other systems found to exhibit anomalous transports
Summary

- Heavy-ion collisions generate hot quark-gluon plasma and strong EM field and vorticity

- EM field and vorticity can monitor the topological sector of QCD via the anomalous transports

- There are several types of anomalous transports and the collective modes arising from them. They bring novel signals to heavy-ion collisions experiments

- Anomalous transports exist in other physical systems
Thank you!
Anomalous transport in cold atomic gases
Motivation

- The CME/CSE/CVE etc are masked by various backgrounds in HICs, it is hard to pin down and to explore their properties in HICs.
- Question: Is there any system that exhibits anomalous transport in a controllable way?
- Answer: Yes! One example is the Dirac or Weyl semimetal (Li, et al, 1412.6543 and many other recent experimental progresses).
- Here we propose another possibility: The cold atomic gases.
- Atomic gases experiments. $10^5 - 10^6$ atoms put in magnetic trap or optical trap, and cooled down to nano Kelvin by using laser cooling or evaporating cooling.

A lot of exciting low-temperature phenomena have been observed: superfluidity, Bose-Einstein condensation, BCS-BEC crossover, novel superfluid, polaron gases, ferromagnetism.
Spin-orbit coupled atomic gases

- In 2011, a new type of cold Bose gases generated in which the spin is coupled to the orbital motion of the atoms (Spielman et al 2011). The single-particle Hamiltonian (Rashba-Dresselhaus SOC):

\[ \mathcal{H} = \frac{p_x^2}{2m} - \lambda \sigma_y p_y \]

- In 2012, same type spin-orbit coupling (SOC) for Fermi gases produced in MIT (Zwierlein group 2012) and in Shanxi (Zhang group 2012).

- Other types of SOC also possible, e.g., the Weyl SOC: (Spielman et al 2012)

\[ \mathcal{H} = \frac{p_x^2}{2m} - \lambda \sigma \cdot p \]

- Now we show: there are CME and CSE in Weyl spin-orbit coupled Fermi gases.
Semiclassical equations of motion

- Consider the Weyl SOC, $\lambda \sigma \cdot p$, in single atom Hamiltonian

$$\mathcal{H} = \frac{p^2}{2m} - \lambda p \cdot \sigma$$

- Along $p$, the spin has two projection which defines two helicities (we will call them chiralities as well), right-hand (project along $p$) and left-hand (project along $-p$).

- Consider atoms in a harmonic trap and let them rotate.

$$\mathcal{H} = \frac{[p - A(x)]^2}{2m} - \lambda [p - A(x)] \cdot \sigma + A_0(x)$$

$A_0(x) = V(x) - (m/2)(\omega \times x)^2 - \mu$

$A(x) = m\omega \times x$

- Integrate out the spin degree of freedom and at $O(\hbar)$ level: the semiclassical EOM (Niu 1998-)

$$\sqrt{Gc}\dot{x} = \nabla_k \varepsilon_c + c\hbar E \times \Omega + c\hbar (\Omega \cdot \nabla_k \varepsilon_c) B,$$

$$\sqrt{Gc}\dot{A} = E + \nabla_k \varepsilon_c \times B + c\hbar (E \cdot B) \Omega$$

where $k = p - A$ is the kinetic momentum, $\sqrt{Gc} = 1 + c\hbar B \cdot \Omega$, $E = -\nabla V(x)$— effective $E$-field, $B = 2m\omega$—effective $B$-field, $\Omega$—Berry curvature. $c = \pm$ for right- or left-hand.
Chiral anomaly

- The kinetic equation reads (Son and Yamamoto 2012, Stephanov and Yin 2012, Gao, Wang, Pu, Chen, Wang 2012)

\[ \partial_t f_c + \dot{x} \cdot \nabla_x f_c + \dot{k} \cdot \nabla_k f_c = I[f_c] \]

- Direct calculation gives the \( U(1) \) chiral anomaly in current of chirality \( c \):

\[ \partial_t n_c + \nabla_x \cdot j_c = c(E \cdot B) \int \frac{d^3k}{(2\pi)^3} f_c \nabla_k \cdot \Omega = c f_c(k_0) \frac{W}{4\pi^2} E \cdot B \]

where \( W \) is the winding number of the Berry curvature.

- Write down the current \( j_c \) explicitly:

\[ j_c = \int \frac{d^3k}{(2\pi)^3} f_c \nabla_k \varepsilon_c + cE \times \int \frac{d^3k}{(2\pi)^3} \Omega f_c \]

\[ + cB \int \frac{d^3k}{(2\pi)^3} (\Omega \cdot \nabla_k \varepsilon_c) f_c. \]

- The third term is \( B \)-induced currents:

\[ j_c^{B-\text{ind}} = \chi_c B, \quad \chi_c = c \int \frac{d^3k}{(2\pi)^3} (\Omega \cdot \nabla_k \varepsilon_c) f_c \]
Chiral magnetic/separation effects

- The $B$-induced conductivity $\chi_c$ for Fermi gas (XGH, Sci.Rep. 6, 20601 (2016))

- If there is parity-odd domains in the Fermi gases $\Rightarrow$
  $\mu_R = \mu + \mu_A$, $\mu_L = \mu - \mu_A \Rightarrow$

  \[
  j_{V}^{B-\text{ind}} \equiv j_{R}^{B-\text{ind}} + j_{L}^{B-\text{ind}} = \frac{\mu A}{2\pi^2} B,
  \]

  \[
  j_{A}^{B-\text{ind}} \equiv j_{R}^{B-\text{ind}} - j_{L}^{B-\text{ind}} = \frac{\mu}{2\pi^2} B.
  \]

- These are exactly the chiral magnetic/separation effects!
- Question: how can produce parity-odd domins in Fermi gases?
Chiral dipole and mass quadrupole

- Very like what happen in QGP, the CMW exists in SOC atomic gases, which transport chirality and mass (XGH, Sci.Rep. 6, 20601 (2016)).

- Unlike in QGP, the presence of trap will finally stop these transport currents and system reaches a equilibrium configuration where appear a mass quadrupole and chiral dipole. The mass quadrupole may be tested by light absorption images technique.
Link the hottest to the coldest

- The similar thing happens also in Bose gases, e.g., the BIC

- The CME/CSE initiated in the study of the hottest matter, the QGP, can possibly be realized in the coldest matter, the cold atoms.
Backup

Angular momentum in overlapping region

Velocity profile
Backup

Spacial distribution

\[ \omega_2 (\text{of } \nu_1) \quad \omega_2 (\text{of } \nu_2) \quad \omega_1 (\text{of } \nu_1) \quad \omega_1 (\text{of } \nu_2) \]

\[ \langle \omega_y \rangle \quad (\text{fm}^{-1}) \]

\[ \text{Au+Au, } \sqrt{s} = 200\text{GeV} \quad b = 10\text{fm} \]

\[ \text{Pb+Pb, } \sqrt{s} = 2.76\text{TeV} \quad b = 10\text{fm} \]

Rapidity dependence

\[ \langle -\omega_y \rangle (\text{fm}^{-1}) \]

\[ \text{Au+Au} \quad \tau_0 = 0.4 \text{ fm} \]

\[ \text{200GeV} \quad \text{546GeV} \quad \text{900GeV} \quad \text{2760GeV} \]

based on energy flow
Chiral magnetic effect

- **Other sources of** \( \langle \cos(\phi_\alpha + \phi_\beta) \rangle \):
  - **Transverse momentum conservation** (Pratt et al 2011, Liao et al 2011):
    \[ \langle \cos(\phi_\alpha + \phi_\beta) \rangle \approx -v_2/N, \text{ charge independent.} \]

- **Local charge conservation** (Pratt and Schlichting, 2011):
  \[ \langle \cos(\phi_\alpha + \phi_\beta) \rangle \propto v_2/N \text{ for opposite sign } \langle \cos(\phi_\alpha + \phi_\beta) \rangle \approx 0 \text{ for same sign.} \]

- **Dipole fluctuation** (Teaney and Yan 2010), clustering correlation (Wang, 2010), ... ...
Chiral magnetic wave

Other sources of $v_2(\pi^\pm)$ difference.

- Quark transport and coalescence (Dunlop, Lisa, and Sorensen 2011; Campbell and Lisa 2013):
  Neutron rich $\Rightarrow v_2(d) > v_2(u)$ (assume transported quarks $v_2$ is larger than produced quarks) $\Rightarrow v_2(\pi^- = d\bar{u}) > v_2(\pi^+ = d\bar{u})$

- Hadronic mean-field potential (Xu, Chen, Ko, and Lin 2012):
  Works at low-energy collision. Neutron rich $\Rightarrow$ repulsive (attractive) potential for $\pi^-(\pi^+)\Rightarrow \pi^-(\pi^+)$ is push-forward (pull-back) in in-plane $\Rightarrow v_2(\pi^-) \gtrsim v_2(\pi^+)$

- Electric field effect (Deng and XGH, 2012; Stephanov and Yee, 2013):
  Electric field points outwards in $y$ direction $\Rightarrow$ positive (negative) charges move outwards (inwards) in $y$ direction $\Rightarrow$ charge quadrupole $\Rightarrow v_2(\pi^-) > v_2(\pi^+)$

- Local charge conservation and rapidity cut (Bzdak and Bozek 2013):
  $v_2(|\eta| \sim 1) < v_2(|\eta| \sim 0) \Rightarrow$ consider a positive-negative charge pair, if at $\eta = 1$ and the positive charge inside and negative charge outside the rapidity window then $A > 0$ and the corresponding $v_2$ is smaller than that when the pair is inside the rapidity window $\Rightarrow v_2(\pi^+)_A < v_2(\pi^+)_{A=0} \Rightarrow$ at small $A$
  $v_2(\pi^+)_A = v_2(\pi^+)_{A=0} - \#A$. Similarly, $v_2(\pi^-)_A = v_2(\pi^-)_{A=0} + \#A$

- Viscous hydrodynamics combined with finite isospin effect may also explain the data (Hatta, Monnai, Xiao 2015)

... ...
Chiral electric separation effect

- Signals for CESE in Cu + Au: \( \zeta_{\alpha \beta} = \langle \cos \left[ 2 \left( \phi_\alpha + \phi_\beta - 2 \psi_{\text{RP}} \right) \right] \rangle \) and \( \Psi_2^q \) (the event-plane for hadrons of charge \( q \)).

\[ \Delta \zeta = \zeta_{\text{opp}} - \zeta_{\text{same}} \] and \( \Delta \Psi = \langle |\Psi_2^+ - \Psi_2^-| \rangle \) sensitive to CESE, survive final interaction (Ma and XGH, PRC 91(2015)054901)

- Possible backgrounds for \( \Delta \zeta = \zeta_{\text{opp}} - \zeta_{\text{same}} \): local charge conservation, chiral magnetic wave. Need more studies.