Eta_c2 and X(3872): A Lattice Study

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Outline

I. An overview of lattice QCD
   what, why, how LQCD

II. Present Status of LQCD

III. Lattice Study on eta_c2
   motivation-X(3872)
   charmonium mass spectra
   eta_c2 radiative decay to J/psi

IV. Summary and perspectives
I. Overview of lattice QCD

What, why, how LQCD
Some points you should remember-----

- LQCD is QCD formulated on a discrete Euclidean space-time grid.
- LQCD is in a formulism of Feynmann path integral quantization.
- LQCD is a theory from the first principle, it retains the fundamental character of QCD.
- The functional integrals are calculated through the numerical Monte Carlo simulation instead of the perturbative expansion.
- Therefore, LQCD is a non-perturbative method for solving QCD.
- The numerical simulation of LQCD becomes the third branch of high energy study parallel to the theoretical and experimental approach.
Quantum Chromodynamics

Gross-Wilczek-Politzer 1973

- Quantum field theory of quarks and gluon fields

\[ q_f(x) \quad \text{Quark field} \]
\[ A_\mu(x) \quad \text{Gluon field} \]

\[ L_{QCD} = \frac{1}{8\pi\alpha_s} \text{Tr}(F_{\mu\nu}F_{\mu\nu}) + \sum_f \bar{q}_f (\gamma_\mu (\partial_\mu - iA_\mu) + m_f) q_f \]

QCD lagrangian

\[ \langle O(A, \bar{\psi}, \psi) \rangle = \frac{1}{Z} \int dA dq d\psi O(A, q, \psi) e^{-\int d^4x L_{QCD}} \]

Physical quantities by Feynman path integral

- Knowing

1 coupling constant

\[ \alpha_s = \frac{g_s^2}{4\pi} \]

and

6 quark masses

\[ m_u, m_d, m_s, m_c, m_b, m_t \]

will allow full understanding of hadrons and their strong interactions

"fulfilling Yukawa’s dream of 1934 in a refined way"
I) What is LQCD?

Wick Rotation from Minkowski Space to Euclidean Space

A D-dimensional Minkowski field theory is connected to a D-dimensional Euclidean field theory through analytical continuation----Wick rotation:

\[ x_0 \equiv t \rightarrow -ix_4 \equiv -i\tau, \]
\[ p_0 \equiv E \rightarrow ip_4. \]

\[ x^2_E = \sum_{i=1}^{4} x_i^2 = x^2 - t^2 = -x^2_M, \]
\[ p^2_E = \sum_{i=1}^{4} p_i^2 = p^2 - E^2 = -p^2_M. \]

\[ e^{iS_M} \equiv e^{i\int dx^4_M L(x_M)} = e^{\int dx^4_E L(x_E)} \equiv e^{-S_E} \]
Path Integral Quantization in Euclidean Space

- The generating functional of QCD in the Euclidean Space

\[ Z = \int D A_\mu \, D \psi \, D \overline{\psi} \, e^{-S} \]

\[ S = \int d^4 x \left( \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \overline{\psi} M \psi \right) . \]

- Integrating out the fermion fields, we have,

\[ Z = \int D A_\mu \, \det M \, e^{\int d^4 x \left( -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} \right)} . \]

\[ S = S_{\text{gauge}} + S_{\text{quarks}} = \int d^4 x \left( \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \right) - \sum_i \log(\det M_i) \]

- The physical observables are obtained by calculating the expectation value of a field operator

\[ \langle \mathcal{O} \rangle = \frac{1}{Z} \int D A_\mu \, \mathcal{O} \, e^{-S} . \]
QCD on a Euclidean Space-time Grid

- Space-time discretization

\[ x^\mu \]

\[ A_\mu(x) \rightarrow U_\mu(x) = e^{i\text{ag}A_\mu(x)} \]

\[ \psi(x) \rightarrow \psi(x) \]

Lattice spacing \( a \)
The symmetry group of the continuum theory – Poincaré invariance – is reduced to a discrete group. On a hypercubic lattice rotations by only 90° are allowed so the continuous rotation group is replaced by the discrete hypercubic group [34]. Translations have to be by at least one lattice unit, so the allowed momenta are discrete

\[ k = \frac{2\pi n}{La} \quad n = 0, 1, \ldots L \]

or equivalently

\[ k = \pm \frac{2\pi n}{La} \quad n = 0, 1, \ldots L/2 . \]

On the lattice momentum is conserved modulo $2\pi$.

It is easily seen that, the largest momentum on the lattice is $\frac{\pi}{a}$.

The finite lattice spacing provides a natural UV cutoff.

Lattice QCD as a regularization scheme of QCD
The Local gauge symmetry

• Local gauge transformation:

\[
\begin{align*}
\psi(x) & \rightarrow V(x)\psi(x) \\
\bar{\psi}(x) & \rightarrow \bar{\psi}(x)V^\dagger(x) \\
U_\mu(x) & \rightarrow V(x)U_\mu(x)V^\dagger(x + \mu)
\end{align*}
\]

• There exist only two types of gauge invariant quantities:

\[
\begin{align*}
\text{Tr} \bar{\psi}(x) \ U_\mu(x) \ U_\nu(x + \mu) \ldots U_\rho(y - \rho) \ \psi(y) \\
\hat{W}_1^{1\times 1} = \text{Re} \ \text{Tr} \ (\bar{U}_\mu(x) \ U_\nu(x + \mu) \ U_\mu^\dagger(x + \nu) \ U_\nu^\dagger(x))
\end{align*}
\]
The simplest gauge action

- The action should be gauge invariant, so it is constructed by Wilson loops.

\[ W^{1 \times 1}_{\mu \nu} = U_{\mu}(x)U_{\nu}(x + \mu)U_{\mu}^{\dagger}(x + \nu)U_{\nu}^{\dagger}(x) \]

\[ = 1 + i a^2 g F_{\mu \nu} - \frac{a^4 g^2}{2} F_{\mu \nu} F^{\mu \nu} + O(a^6) + \ldots \]

\[ e^A e^B = e^{A+B+[A,B]/2+\ldots} \]

\[ \text{Re} \text{Tr}(1 - W^{1 \times 1}_{\mu \nu}) = \frac{a^4 g^2}{2} F_{\mu \nu} F^{\mu \nu} + \text{terms higher order in } a \]

\[ \frac{1}{g^2} \sum_x \sum_{\mu < \nu} \text{Re} \text{Tr}(1 - W^{1 \times 1}_{\mu \nu}) = \frac{a^4}{4} \sum_x \sum_{\mu, \nu} F_{\mu \nu} F^{\mu \nu} \rightarrow \frac{1}{4} \int d^4 x F_{\mu \nu} F^{\mu \nu} \]

\[ S_g = \frac{6}{g^2} \sum_x \sum_{\mu < \nu} \text{Re} \text{Tr} \frac{1}{3}(1 - W^{1 \times 1}_{\mu \nu}) \]
Naïve fermion action

- Replace the derivatives to differentiate

\[ \partial_x \phi(x) \rightarrow \Delta_x \phi(x) = \frac{1}{2a} (\phi(x + a) - \phi(x - a)) \]

\[ \bar{\psi} \slashed{D} \psi = \frac{1}{2a} \bar{\psi}(x) \sum_{\mu} \gamma_{\mu} [U_{\mu}(x) \psi(x + \mu\hat{\mu}) - U_{\mu}^\dagger(x - \mu\hat{\mu}) \psi(x - \mu\hat{\mu})] \]

\[ S^N = m_q \sum_x \bar{\psi}(x) \psi(x) \quad \gamma_{\mu} = \gamma_{\mu}^\dagger \]

\[ + \frac{1}{2a} \sum_x \bar{\psi}(x) \gamma_{\mu} [U_{\mu}(x) \psi(x + \mu\hat{\mu}) - U_{\mu}^\dagger(x - \mu\hat{\mu}) \psi(x - \mu\hat{\mu})] \]

\[ \equiv \sum_x \bar{\psi}(x) M_{xy}^N [U] \psi(y) \]

\[ M_{i,j}^N [U] = m_q \delta_{i,j} + \frac{1}{2a} \sum_{\mu} \left[ \gamma_{\mu} U_{i,\mu} \delta_{i,j-\mu} - \gamma_{\mu} U_{i,\mu}^\dagger \delta_{i,j+\mu} \right] \]

- Satisfying the chiral symmetry

\[ \gamma_5 M + M \gamma_5 = 0 \]
The properties and problems of naïve fermion action

“Fermion doubling”

\[ S^{-1}(p) = m_q + \frac{i}{a} \sum_{\mu} \gamma_\mu \sin p_\mu a \]

\[ S^{-1}(p, m = 0) = \frac{i}{a} \sum_{\mu} \gamma_\mu \sin p_\mu a \]

• Sixteen poles can be divided into two groups with different chiral charge.

• These doublers cancel the axial anomaly which exists in the continuum.

No-Go theorem on finite lattices

On a four-torus

• Locality, hermicity, correct low-momentum limit
• Chiral symmetry and free of fermion doubling cannot be satisfied simultaneously
Adding an additional dimension 5 term to the conventional action,

\[ \partial^2 \phi(x) \rightarrow \frac{1}{a^2} \left( \phi(x + a) + \phi(x - a) - 2\phi(x) \right) \]

\[ A^W = m_q \sum_x \bar{\psi}(x) \psi(x) \]

\[ + \frac{1}{2a} \sum_{x, \mu} \bar{\psi}(x) \gamma_\mu [U_\mu(x) \psi(x + \hat{\mu}) - U^\dagger_\mu(x - \hat{\mu}) \psi(x - \hat{\mu})] \]

\[ - \frac{r}{2a} \sum_{x, \mu} \bar{\psi}(x) [U_\mu(x) \psi(x + \hat{\mu}) - 2\psi(x) + U^\dagger_\mu(x - \hat{\mu}) \psi(x - \hat{\mu})] \]

\[ \equiv \sum_{x,y} \bar{\psi}_x^L M^W_{xy} \psi_y^L \]

\[ m_q a = \frac{1}{2 \kappa} - 4r \]

\[ M^W_{x,y}[U] a = \delta_{xy} - \kappa \sum_\mu \left[ (r - \gamma_\mu) U_{x, \mu} \delta_{x, y - \mu} + (r + \gamma_\mu) U^\dagger_{x - \mu, \mu} \delta_{x, y + \mu} \right] \]
**Properites of Wilson Fermion**

- **Free of fermion doubling**

\[ S_F(p) = M_W^{-1}(p) = \frac{a}{1 - 2\kappa \sum_\mu (r \cos p_\mu a - i\gamma_\mu \sin p_\mu a)} \]

The 15 extra states get heavy masses proportional to \(2r/a\) and decouple from the theory when \(a\) is small enough.

- **Discretization errors**

\[ e^{Ea} = \frac{(ma + r) \pm \sqrt{(1 + 2mar + m^2a^2)}}{1 + r} \]

Naïve fermion (\(r=0\)): \[ E = m + O(m^2a^2) \]

Wilson fermion (\(r=1\)): \[ Ea = \log(1+ma) \]
Chiral Fermions

- **Ginsburg-Wilson relation---chiral symmetry on lattice**

\[ \gamma_5 D + D \gamma_5 = a D \gamma_5 D \]

- **Chiral transformation in the continuum**

\[ \psi \rightarrow e^{i \theta \gamma_5} \psi ; \]
\[ \bar{\psi} \rightarrow \bar{\psi} e^{i \theta \gamma_5} \]

- **Chiral transformation on the lattice**

\[ \psi \rightarrow e^{i \theta \gamma_5 (1 - aD / 2)} \psi ; \]
\[ \bar{\psi} \rightarrow \bar{\psi} e^{i \theta \gamma_5 (1 - aD / 2)} \]

The lattice action is invariant if GW relation holds for D
Two types of fermion actions that satisfy GW relation:

- overlap fermion
- domain-wall fermion

- Free of fermion doubling + chiral
- But computation is much more expensive.
II) How is LQCD study performed?

**MC Simulation——Importance Sampling**

*Quenched and Unquenched*
Monte Carlo Simulation of Lattice QCD

The equivalences between a Euclidean field theory and Classical Statistical Mechanics.

<table>
<thead>
<tr>
<th>Euclidean Field Theory</th>
<th>Classical Statistical Mechanics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action</td>
<td>Hamiltonian</td>
</tr>
<tr>
<td>unit of action $\hbar$</td>
<td>units of energy $\beta = 1/kT$</td>
</tr>
<tr>
<td>Feynman weight for amplitudes $e^{-S/\hbar} = e^{-\int \mathcal{L} dt/\hbar}$</td>
<td>Boltzmann factor $e^{-\beta H}$</td>
</tr>
<tr>
<td>Vacuum to vacuum amplitude $\int \mathcal{D}\phi e^{-S/\hbar}$</td>
<td>Partition function $\sum_{\text{conf.}} e^{-\beta H}$</td>
</tr>
<tr>
<td>Vacuum energy</td>
<td>Free Energy</td>
</tr>
<tr>
<td>Vacuum expectation value $\langle 0</td>
<td>\mathcal{O}</td>
</tr>
<tr>
<td>Time ordered products</td>
<td>Ordinary products</td>
</tr>
<tr>
<td>Green’s functions $\langle 0</td>
<td>T[\mathcal{O}_1 \ldots \mathcal{O}_n]</td>
</tr>
<tr>
<td>Mass $M$</td>
<td>correlation length $\xi = 1/M$</td>
</tr>
<tr>
<td>Mass-gap</td>
<td>exponential decrease of correlation functions</td>
</tr>
<tr>
<td>Mass-less excitations</td>
<td>spin waves</td>
</tr>
<tr>
<td>Regularization: cutoff $\Lambda$</td>
<td>lattice spacing $a$</td>
</tr>
<tr>
<td>Renormalization: $\Lambda \to \infty$</td>
<td>continuum limit $a \to 0$</td>
</tr>
<tr>
<td>Changes in the vacuum</td>
<td>phase transitions</td>
</tr>
</tbody>
</table>

Due to the similarity, we can borrow the methods of statistical mechanics to study lattice QCD, such as Monte Carlo simulation.
MC Simulation—Importance Sampling

- Taking $e^{-S[U,\psi,\bar{\psi}]}$ as a probability distribution, an ensemble of configurations are generated from MC simulation. This is the procedure that eats the computation resources mostly.

- After the generation of configurations, the functional integral

$$\langle O(A_{\mu},\bar{\psi},\psi) \rangle = \frac{1}{Z} \int [DA_{\mu}D\bar{\psi}D\psi] \exp(-S(A_{\mu},\bar{\psi},\psi))O(A_{\mu},\bar{\psi},\psi)$$

becomes the much simpler arithmetic average:

$$\langle O \rangle_{MC} = \frac{1}{N} \sum_{i=1}^{N} O_i$$

- Generally speaking, the quantities that are most commonly calculated are Green's function, say, the vacuum expectation values of field operators defined at different space-time points.
On the lattice, \( M \) is a very large matrix, such that the calculation of its trace is very expensive in the MC simulation. A way out this difficulty is to take the approximation

\[
\text{det } M \left[ U \right] = \text{const}.
\]

Theoretically, this means that we set the sea quark mass to be infinitely large such that they decouple from the gauge field. In other words, we will ignore the vacuum polarization diagram, say, the effects of sea quarks.
III) What can LQCD do?

**Computation:** calculations of Green’s functions

**Data Analysis:** extract physical quantities
Extracting Physical Quantities from MC Simulation

Example: LQCD calculation of pion mass and pion decay constant

• Two point function:

\[ C(t) = \langle 0|T[\sum_x \mathcal{O}_f(x,t)\mathcal{O}_i(0)]|0\rangle \quad \mathcal{O}_f = \mathcal{O}_i = A_4 = \bar{\psi} \gamma_4 \gamma_5 \psi \]

Theoretically, we have the following relations

\[ C(t) = \langle 0|\sum_x \mathcal{O}_f(x,t)\mathcal{O}_i(0)|0\rangle = \sum_n \frac{\langle 0|\mathcal{O}_f|n\rangle\langle n|\mathcal{O}_i|0\rangle}{2E_n} e^{-E_n t}. \]

\[ C(t) = \langle 0|\sum_x \mathcal{O}_f(x,t)\mathcal{O}_i(0)|0\rangle \quad t \to \infty \quad \frac{\langle 0|\mathcal{O}_f|\pi\rangle\langle \pi|\mathcal{O}_i|0\rangle}{2M_\pi} e^{-M_\pi t}. \]

\[ \langle 0|A_4(p = 0)|\pi\rangle = M_\pi f_\pi. \]
The local interpolating field operators for mesons and baryons in Wilson-like theories. Projection to zero momentum states is obtained by summing over the points $x$ on a time slice. The $C$-parity is only relevant for flavor degenerate meson states. The $()$ in baryon operators denote spin trace. $C = \gamma_2 \gamma_4$ and the symmetry properties of flavor indices for nucleons are discussed in the text. The decuplet baryon operator is completely symmetric in flavor index.

<table>
<thead>
<tr>
<th>State</th>
<th>$I^G(J^{PC})$</th>
<th>Operator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scalar($\sigma$)</td>
<td>$1^- (0^{++})$ $1^- (0^{++})$</td>
<td>$\bar{u}(x)d(x)$</td>
</tr>
<tr>
<td>Pseudoscalar</td>
<td>$1^- (0^{--})$ $1^- (0^{--})$</td>
<td>$\bar{u}(x)\gamma_5 d(x)$</td>
</tr>
<tr>
<td>Vector</td>
<td>$1^+ (1^{--})$ $1^+ (1^{--})$</td>
<td>$\bar{u}(x)\gamma_i d(x)$</td>
</tr>
<tr>
<td>Axial ($a_1$)</td>
<td>$1^- (1^{++})$</td>
<td>$\bar{u}(x)\gamma_i \gamma_5 d(x)$</td>
</tr>
<tr>
<td>Tensor($b_1$)</td>
<td>$1^+ (1^{+-})$</td>
<td>$\bar{u}(x)\gamma_i \gamma_j d(x)$</td>
</tr>
<tr>
<td>Nucleon octet</td>
<td>$\frac{1}{2} (\frac{1}{2}^-)$ $\frac{1}{2} (\frac{1}{2}^-)$</td>
<td>$(u_d^T C d_b) \gamma_5 s_c \epsilon^{abc}$</td>
</tr>
<tr>
<td>Delta decuplet</td>
<td>$\frac{3}{2} (\frac{3}{2}^+)$</td>
<td>$(u_d^T C \gamma_5 d_b) s_c \epsilon^{abc}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(u_d^T C \gamma_i d_b) s_c \epsilon^{abc}$</td>
</tr>
</tbody>
</table>
1. Generate the gauge configurations by MC.
2. Calculate the quark propagators for each gauge configuration.
3. Calculate the two-point function.
4. Fit the data using the theoretical formula.

Numerically,

$$m(t)a = \ln \frac{C(t)}{C(t+1)}$$

The mass plateau
Three-point function:

\[ C_{3,\mu}(t_x, p, t_y; q) = \sum_{x, y} e^{-i(p_x + q_y)} \langle 0 | T [K^+ V_\mu D^0] | 0 \rangle \]

\[ = \sum_{x, y} e^{-i(p_x + q_y)} \langle 0 | \bar{s}(0)\gamma_5 u(0)\bar{c}(y)\gamma_\mu s(y)\bar{u}(x)\gamma_5 c(x) | 0 \rangle \]

\[ \sim \frac{\exp \left( E_K(p + q)t_y - E_D(p)(t_x - t_y) \right)}{4E_K(p + q)E_D(q)} \times \langle 0 | \bar{s}\gamma_5 u | K(p + q) \rangle \langle K(p + q) | \bar{c}\gamma_\mu s | D(p) \rangle \langle D(p) | \bar{u}\gamma_5 c | 0 \rangle \]

Form factors
III) How good is LQCD?

Less uncertainties:

Systematical uncertainties are well controlled
Control of systematical uncertainties

Sources of systematical uncertainties:

- Pion masses are higher than the physical value through chiral extrapolation-ChPT
- Finite lattice spacing: continuum extrapolation
- Finite volume effects. $m_\pi L \geq 3.5$
Going back to the Continuum

• Dimensionful quantities measured on the lattice are all in unit of lattice spacing $a$, since the only dimensionful parameter in LQCD actions is the lattice spacing, apart from quark masses.

• The continuum limit can be reach by extrapolating the measured values at several finite lattice spacings.

$$ Q_{\text{phy.}} = \lim_{a \to 0} Q_{\text{lat}}(a) $$

• LQCD has the correct continuum limit, continuum QCD. This is guaranteed by the renormalization group theory.

• The only tunable parameter in the action of LQCD is the bare coupling constant, $g$, which directly relates to the lattice spacing according to the RG equation,

$$ a = \frac{1}{\Lambda_L} f(g) \quad f(g) = \left( \beta_0 g^2 \right)^\frac{\beta_1}{2 \beta_2^2} e^{-\frac{1}{2 \beta_0 g^2}} $$
II. Present Status of LQCD

Large scale numerical computation on supercomputers

Large international LQCD collaborations
I) Why is LQCD Computational Intensive?

\[ Z = \int [D U] e^{-S_g(U) + \text{Tr} \ln M[U]} \]

\[ \text{Tr} \ln M[U] \sim \text{const.} \quad \Rightarrow \quad \text{Quenched Approximation} \]

\[ m_q^{\text{val}} \neq m_q^{\text{sea}} \quad \Rightarrow \quad \text{Partially Quenched} \]

\[ M_{\text{val}}[U] \neq M[U] \quad \Rightarrow \quad \text{Mixed Action} \]

Otherwise, a unitary theory of full QCD on the lattice

Observables: VEV of operators, such as Green's functions.

\[ \langle O \rangle = \int [D U] \mathcal{O}(U) e^{-S_g(U) + \text{Tr} \ln M[U]} \Rightarrow \frac{1}{N} \sum_i O_i \]

\[ \text{Monte Carlo simulation, importance sampling} \]
Lattice QCD as computation (I)

- 4-d simple cubic lattice
  - Lattice volume \( V = L_x \times L_y \times L_z \times L_t \)
  - Lattice spacing \( a \)

- Parallelization
  - Mapping space-time lattice to processor array
  - QCD is a local field theory; only nearest neighbor interactions
    → only nearest neighbor data communication needed

Highly parallelizable and scalable
Lattice QCD as computation (II)

- Quark fields, being anticommuting, needs a special trick

\[
\int \prod_n d\bar{q}_n dq_n e^{-\sum_n D_{nm}(U) q_n} = \det D(U) = \prod_n d\bar{\phi}_n d\phi_n e^{-\sum_n \bar{\phi}_n \left( \frac{1}{D(U)} \right)_{nm} \phi_m}
\]

Grassmann rep

Boson rep

- Need to invert the lattice Dirac operator $D(U)$

\[
\sum_m D_{nm}(U) x_m = \phi_n \Rightarrow x_n = \left( \frac{1}{D(U)} \right)_{nm} \phi_m
\]

Core calculation of QCD

- $D(U) \approx i \gamma \cdot (\partial - i g A(x)) + m_q$ complex 12Vx12V dim matrix

- Condition number $\sim 1/m_q$

With iterative solvers, # arithmetic ops rapidly increases when quark mass $m_q$ becomes small (up, down, strange)

Computationally very intensive
Lattice QCD over the years...

1973 QCD
1974 lattice QCD

Physics
1st spec calculation 1981
Hamber-Parisi Weingarten

Lattice size L
0.8fm
8^3x16
1.6fm
16^3x32
2.4fm
24^3x48
3.0fm
64^3x118
3.0fm
32^3x64

Algorithms
N_f = # sea quarks
N_f = 0 quenched
N_f = 2 u, d
N_f = 2 + 1 u, d, s

Machines
1st generation
10 Gflops
APE1
APE100

2nd generation
100 Gflops
QCDPAK

3rd generation
1 Tflops
QCDSP

4th generation
10 Tflops
100 Tflops

QCDOC
BlueGene/L, P
CP-PACS
PACS-CS
Development of supercomputers

Major project machines in Japan
- NWT
- CP-PACS
- Earth Simulator

Vector computers
- CRAY/CDC
- Hitachi/Fujitsu/NEC

Vector-parallel computers
- Fujitsu
- NEC
- CRAY

Parallel computers
- CRAY
- IBM
- Intel
- TMC
- other
- Fujitsu
- Hitachi
- NEC/Sun

Project machines
- Tsukuba
- Columbia
- APE
- GFT1 (IBM)
About 10 major cites scattered in USA, EU(UK, Germany, Italy etc), Japan

In total 500~600Tflops in peak speed (US300Tf, EU150Tf, Japan100Tf)

Data sharing through **ILDG (International Lattice Data Grid)**
II) Re-organization of LQCD Community

- **Large LQCD Collaborations generating dynamical configurations**

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>Gauge Type</th>
<th>Flavor Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>MILC</td>
<td>Symanzik improved</td>
<td>(2+1) flavor staggered fermion.</td>
</tr>
<tr>
<td>CP-PACS</td>
<td>RG improved</td>
<td>(2+1) flavor clover fermion</td>
</tr>
<tr>
<td>JLQCD</td>
<td>RG improved</td>
<td>(2+1) flavor overlap fermion</td>
</tr>
<tr>
<td>RBC&amp;UKQCD</td>
<td>DBW2 gauge</td>
<td>(2+1) flavor domain wall fermion</td>
</tr>
<tr>
<td>ETMC</td>
<td>improved gauge</td>
<td>(2+1) flavor twisted-mass fermion</td>
</tr>
</tbody>
</table>

- **Smaller groups for physical projects based on these dynamical configurations**
Probes: valence quark propagators, etc.,

Dynamical configurations

Probe1, Probe2, .......

Expensive

Manpower intensive

Data analysis

Facilities

Detectors

Data analysis

Experiments
中国格点QCD合作组 (CLQCD)

研究人员：
马建平（ITP, CAS） 刘川（Peking Univ.）
刘玉斌（Nankai Univ.） 张剑波（Zhejiang Univ.）
陈莹（IHEP, CAS） 刘朝峰（IHEP, CAS）

研究生 + 博士后
CLQCD 计算资源

1. 目前可用计算资源：

   深腾7000 (中国科学院网络信息中心超级计算中心)
   
   120TFLOPs

   曙光5000 (上海超级计算中心)
   
   200 TFLOPs
2009中国HPC TOP100榜首：天河一号详解

来源：服务器在线

天河一号作为863“高效能计算机及网格服务环境”重大项目“千万亿次高效能计算机系统研制”课题成果，在性能上成为中国的第一个千万亿次超级计算机。在2009年9月，国防科技大学研制成功了我国首台千万亿次超级计算系统“天河一号”，其峰值性能达每秒1206万亿次双精度浮点运算。

6144个通用处理器；
5120个加速处理器
内存总容量98TB；
点对点通信带宽40Gbps；
共享磁盘总容量为1PB。

在国际上排一位


**CLQCD 的研究领域**

1. 与BEPCII/BESIII的物理实验密切结合
   - 轻强子性质
   - 奇特强子态（如胶球、混杂态等）
   - 粒夸克物理
     - Charmonium radiative decays (quenched and unquenched)
     - D meson masses and decay constants

2. 有限温度有限密度QCD
III. Lattice Study on eta_c2

- Motivation---X(3872) relevant
- System calibration
  
  charmonium spectrum

  \[ 2^{++} \rightarrow 1^{--} \text{ transition} \]

- Eta_c2 radiative decay to J/psi \( (2^{-+} \rightarrow 1^{--}) \)
- Charmonium radiative decays
1. Motivation → X(3872) relevant

Experimental facts:

- Belle, CDFIII, and D0 have found a charmonium-like narrow resonance
  \[ M = 3871.7 \text{ MeV} \]
  \[ \Gamma < 2.3 \text{ MeV} \]

- J^PC quantum number
  \[
  \begin{array}{c}
  1^{++} \\
  2^{--}
  \end{array}
  \]

\[
X \rightarrow \gamma J/\psi \quad C = +1 \quad \text{and} \quad X \rightarrow \rho^0 J/\psi \rightarrow (\pi^+ \pi^-) J/\psi \rightarrow P = 1
\]
• BaBar’s new analysis of the final state angular distribution of

\[ X(3872) \rightarrow \omega J/\psi \rightarrow \pi^0 \pi^+ \pi^- J/\psi \]

favors \( ^{2-+}_{2^{-+}} \) (PRD82,011101(2010)). While Belle’s new analysis is compatible with both the assignments (ArXiv:1109.1699(hep-ex)).

• Radiative decays of \( X(3872) \)

\[
\begin{align*}
\text{Br}(B^\pm &\rightarrow X(3872)K^\pm)\text{Br}(X(3872) \rightarrow J/\psi \gamma) \\
&= (2.8 \pm 0.8 \pm 0.1) \times 10^{-6} \quad \text{(BaBar)}, \\
\text{Br}(B^\pm &\rightarrow X(3872)K^\pm)\text{Br}(X(3872) \rightarrow J/\psi \gamma) \\
&= (1.78^{+0.48}_{-0.44} \pm 0.12) \times 10^{-6} \quad \text{(Belle)},
\end{align*}
\]

With the upper bound \( \text{Br}(B \rightarrow KX(3872)) < 3.2 \times 10^{-4} \)

\[ \text{Br}(X(3872) \rightarrow J/\psi \gamma) > 0.9\% \quad \text{(BaBar) or 0.6\% \quad (Belle).} \]

However,

\[
\frac{\text{Br}(X(3872) \rightarrow \gamma \psi')}{\text{Br}(X(3872) \rightarrow \gamma J/\psi)} = 3.4 \pm 1.4 \quad \text{(BaBar, PRL102,132001(2009))}
\]

\[
\frac{\text{Br}(X(3872) \rightarrow \gamma \psi')}{\text{Br}(X(3872) \rightarrow \gamma J/\psi)} < 2.1 \quad \text{(Belle, PRL107,091803(2011))}
\]
Theoretical concerns:

- If $1^{++}$, the charmonium assignment is possibly the $2P$ state $\chi_{c1}'$, but the quark model prediction of the mass is
  
  $$M(\chi_{c1}') \sim 3.925 \text{GeV}$$

- If $2^{-+}$, the charmonium assignment is possibly the $1D$ state $\eta_{c2}$ but the quark model prediction of the mass is
  
  $$M(\eta_{c2}) \sim 3.77 - 3.83 \text{GeV}$$

- If $1^{++}$, $E1$ radiative transition, larger partial width;

  \[
  \Gamma(2S+1L_J \rightarrow 2S'+1L_{J'}, \psi \rightarrow e^+ \pi^-) = \frac{4}{3} C_{\pi} \delta_{S,S'} e_c^2 \alpha \left| \langle \psi_f | r | \psi_i \rangle \right|^2 E_\gamma^3
  \]

  If $2^{-+}$, $M1$ radiative transition, smaller partial width;

  \[
  \Gamma(2S+1L_J \rightarrow 2S'+1L_{J'}, \psi \rightarrow e^+ \pi^-) = \frac{4}{3} \delta_{LL} \delta_{S,S' \pm 1} \frac{\alpha e_c^2}{m_c^2} \left| \langle \psi_f | r | \psi_i \rangle \right|^2 E_\gamma^3
  \]

Lattice study on this topic in this work.
2. System Calibration

- Lattice setup

\[ L^3 \times T = 8^3 \times 96 \quad \xi = a_s / a_t = 5 \]
\[ \beta = 2.4 \quad a_s = 0.222(2)\, fm \]

\[ L^3 \times T = 12^3 \times 144 \quad \xi = a_s / a_t = 5 \]
\[ \beta = 2.8 \quad a_s = 0.138(1)\, fm \]

Actions

- Tadpole improved Symanzik’s gauge action
- Tadpole improved Clover’s fermion action
• The ground state charmonium spectrum

<table>
<thead>
<tr>
<th>meson</th>
<th>$J^{PC}$</th>
<th>$M(2.4)$</th>
<th>$M(2.8)$</th>
<th>Expt.</th>
<th>QM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c(1^3S_0)$</td>
<td>0$^{-+}$</td>
<td>2.989(2)</td>
<td>3.007(3)</td>
<td>2.979</td>
<td>2.982</td>
</tr>
<tr>
<td>$J/\psi(3S_1)$</td>
<td>1$^{--}$</td>
<td>3.094(3)</td>
<td>3.094(3)</td>
<td>3.097</td>
<td>3.090</td>
</tr>
<tr>
<td>$h_c(1P_0)$</td>
<td>1$^{++}$</td>
<td>3.530(35)</td>
<td>3.513(14)</td>
<td>3.526</td>
<td>3.516</td>
</tr>
<tr>
<td>$\chi_{c0}(1P_0)$</td>
<td>0$^{++}$</td>
<td>3.472(34)</td>
<td>3.431(30)</td>
<td>3.415</td>
<td>3.424</td>
</tr>
<tr>
<td>$\chi_{c1}(2P_1)$</td>
<td>1$^{++}$</td>
<td>3.508(50)</td>
<td>3.499(25)</td>
<td>3.511</td>
<td>3.505</td>
</tr>
<tr>
<td>$\chi_{c2}(3P_2)$</td>
<td>2$^{++}$</td>
<td>3.552(17)</td>
<td>3.520(15)</td>
<td>3.556</td>
<td>3.556</td>
</tr>
<tr>
<td>$\psi''(3D_1)$</td>
<td>1$^{-+}$</td>
<td>-</td>
<td>-</td>
<td>3.770</td>
<td>3.785</td>
</tr>
<tr>
<td>$\eta_{c2}(1D_2)$</td>
<td>2$^{++}$</td>
<td>3.777(30)</td>
<td>3.789(28)</td>
<td>-</td>
<td>3.799</td>
</tr>
</tbody>
</table>

The results show good agreement with the experimental data. The systematic uncertainties are seemingly small.
3. Lattice study on charmonium radiative transitions

- Radiative decay width:
  \[ \Gamma(i \rightarrow \gamma f) = \int d\Omega \frac{1}{32\pi^2} \frac{|q|}{M_i^2} \frac{1}{2J_i + 1} \times \sum_{r_i, r_f, r_\gamma} |M_{r_i, r_f, r_\gamma}|^2, \]

- Transition amplitudes:
  \[ M_{r_i, r_f, r_\gamma} = \epsilon_\mu^* (\vec{q}, r_\gamma) \langle f(\vec{p}_f, r_f) | j_\text{em}^\mu (0) | i(\vec{p}_i, r_i) \rangle \]

- Multipole decomposition:
  \[ \langle f(\vec{p}_f, r_f) | j_\text{em}^\mu (0) | i(\vec{p}_i, r_i) \rangle = \sum_k \alpha_k^\mu (p_i, p_f) F_k (Q^2). \]

- Decay width expressed in terms of the form factors
  \[ \Gamma(i \rightarrow \gamma f) \propto \sum_k F_k^2 (0). \]

- So the major task is to calculate the matrix elements
• For this purpose, it is the three point function that will be calculated on lattice

\[ O_{m}^{f,i}(x) = \bar{c}(x) \Gamma_{m}^{f,i} c(x) \]

\[ j_{em}^{\mu}(x) = Q_{c} \bar{c}(x) \gamma^{\mu} c(x) \]

\[ \Gamma_{mn}^{(3)}(\vec{p}_{f}, \vec{q}, t, t') = \sum_{\vec{x}, \vec{y}} e^{-i\vec{p}_{f} \cdot \vec{x}} e^{i\vec{q} \cdot \vec{y}} \times \langle O_{m}^{f}(\vec{x}, t) j_{em}^{\mu}(\vec{y}, t') O_{n}^{i+}(\vec{0}, 0) \rangle \]

\[ \Gamma_{X, mn}^{(2)}(\vec{p}_{X}, t) = \sum_{\vec{a}} e^{i\vec{p}_{X} \cdot \vec{a}} \langle O_{m}^{X}(\vec{x}, t) O_{n}^{X+}(\vec{0}, 0) \rangle \]

\[ \rightarrow \frac{1}{2E_{X}} e^{-E_{X}t} \sum_{r_{X}} \langle 0 | O_{m}^{X} | X(\vec{p}_{X}, r_{X}) \rangle \times \langle X(\vec{p}_{X}, r_{X}) | O_{n}^{X+} | 0 \rangle \quad (t \rightarrow \infty). \]
1. Transition $\chi_{c2} \rightarrow \gamma J/\psi$

$$\Gamma(\chi_{c2} \rightarrow \gamma J/\psi) = \frac{16\alpha |\vec{k}|}{45 M_{\chi_{c2}}^2} (|E_1(0)|^2 + |M_2(0)|^2 + |E_3(0)|^2)$$

$$\langle V(\bar{p}V, \lambda V)|j^\mu(0)|T(\vec{p}r, \lambda T)\rangle = \alpha_1^\mu E_1(Q^2) A^\mu + \alpha_2^\mu M_2(Q^2) + \alpha_3^\mu E_3(Q^2) + \alpha_4^\mu C_1(Q^2) + \alpha_5^\mu C_2(Q^2)$$

Fit function

$$F_k(Q^2) = F_k(0)(1 + \lambda_k Q^2)e^{-\frac{Q^2}{16\pi^2}}$$
The lattice result is in good agreement with PDG data.

<table>
<thead>
<tr>
<th>β</th>
<th>$E_1$(GeV)</th>
<th>$M_2$(GeV)</th>
<th>$E_3$(GeV)</th>
<th>$\Gamma$(keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4</td>
<td>2.04(2)</td>
<td>$-0.218(4)$</td>
<td>0.014(3)</td>
<td>347 ± 20</td>
</tr>
<tr>
<td>2.8</td>
<td>2.08(2)</td>
<td>$-0.171(10)$</td>
<td>0.005(8)</td>
<td>352 ± 11</td>
</tr>
<tr>
<td>Cont.</td>
<td>2.11(2)</td>
<td>$-0.141(15)$</td>
<td>$-0.007(12)$</td>
<td>361 ± 9</td>
</tr>
</tbody>
</table>

PDG2010: $\Gamma = 384(38)\,keV$
2. Transition \( \eta_{c2} \rightarrow \gamma J/\psi \)

\[
\Gamma(\eta_{c2} \rightarrow \gamma J/\psi) = \frac{16\alpha |q|}{45M_{\eta_{c2}}^2}(|M_1(0)|^2 + |E_2(0)|^2 + |M_3(0)|^2).
\]

\[
\langle V(p_V, \lambda_V)|j^{\mu}(0)|T(p_T, \lambda_T)\rangle = i\alpha_1^{\mu} M_1(Q^2) + i\alpha_2^{\mu} E_2(Q^2) + i\alpha_3^{\mu} M_3(Q^2) - i\alpha_4^{\mu} C_2(Q^2)
\]
In this case, we adopt the following formulae to do the interpolation:

\[
\begin{align*}
M_1 &= |\vec{p}_V|(A_1(Q^2) + B_1(Q^2)\nu^2 + C_1(Q^2)\nu^4 + O(\nu^6)) \\
E_2 &= |\vec{p}_V|(A_2(Q^2) + B_2(Q^2)\nu^2 + C_2(Q^2)\nu^4 + O(\nu^6)) \\
M_3 &= |\vec{p}_V|(B_3(Q^2)\nu^2 + C_3(Q^2)\nu^4 + O(\nu^6)).
\end{align*}
\] (25)

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$M_1$(GeV)</th>
<th>$E_2$(GeV)</th>
<th>$M_3$(GeV)</th>
<th>$\Gamma$(keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4</td>
<td>0.133(13)</td>
<td>0.111(17)</td>
<td>-0.093(9)</td>
<td>4.4 ± 0.9</td>
</tr>
<tr>
<td>2.8</td>
<td>0.115(11)</td>
<td>-0.0007(14)</td>
<td>-0.117(9)</td>
<td>3.1 ± 0.6</td>
</tr>
<tr>
<td>Cont.</td>
<td>0.104(10)</td>
<td>-0.071(20)</td>
<td>-0.132(10)</td>
<td>3.8 ± 0.9</td>
</tr>
</tbody>
</table>

\[
|\vec{q}| = |\vec{p}_V| = 0.65\text{GeV} \quad \nu \sim 0.2 \quad \text{for} \quad J/\psi
\]
\[
|\vec{q}| = |\vec{p}_V| = 0.11\text{GeV} \quad \nu \sim 0.03 \quad \text{for} \quad \psi'
\]

So the partial width of eta_c2 to psi' should be suppressed by a kinematic factor

\[
(0.11/0.65)^3 \sim 1/200
\]


\[
\begin{align*}
M_1 &= 0.079(2)\text{GeV}, \\
E_2 &= -0.086(2)\text{GeV}, \\
M_3 &= -0.125(3)\text{GeV}, \\
\Gamma &= 3.54(12) \text{ keV}
\end{align*}
\]
Discussion

1. We carry out a quenched lattice study on the \eta_c2 mass and its radiative transition to \rm{J/psi}.
2. By the calculation of the ground state charmonium spectrum, we found the systematic uncertainties owing to the quenched approx. and finite lattice spacing are small.
3. The mass of 1D state \eta_c2 is determined to be 3.79(3) GeV.
4. The radative transition width of \eta_c2 to \rm{J/psi} is predicted to be 3.8(9)keV.
5. Our result is in agreement with LFQM result.
6. The radiative transition width of \eta_c2 to \psi' can be much smaller than to \rm{J/psi}, due to kinematic suppression of p-wave transition.
7. If BaBar's observation is confirmed, the 2-+ charmonium assignment of \rm{X(3872)} can be ruled out.
Thank You!