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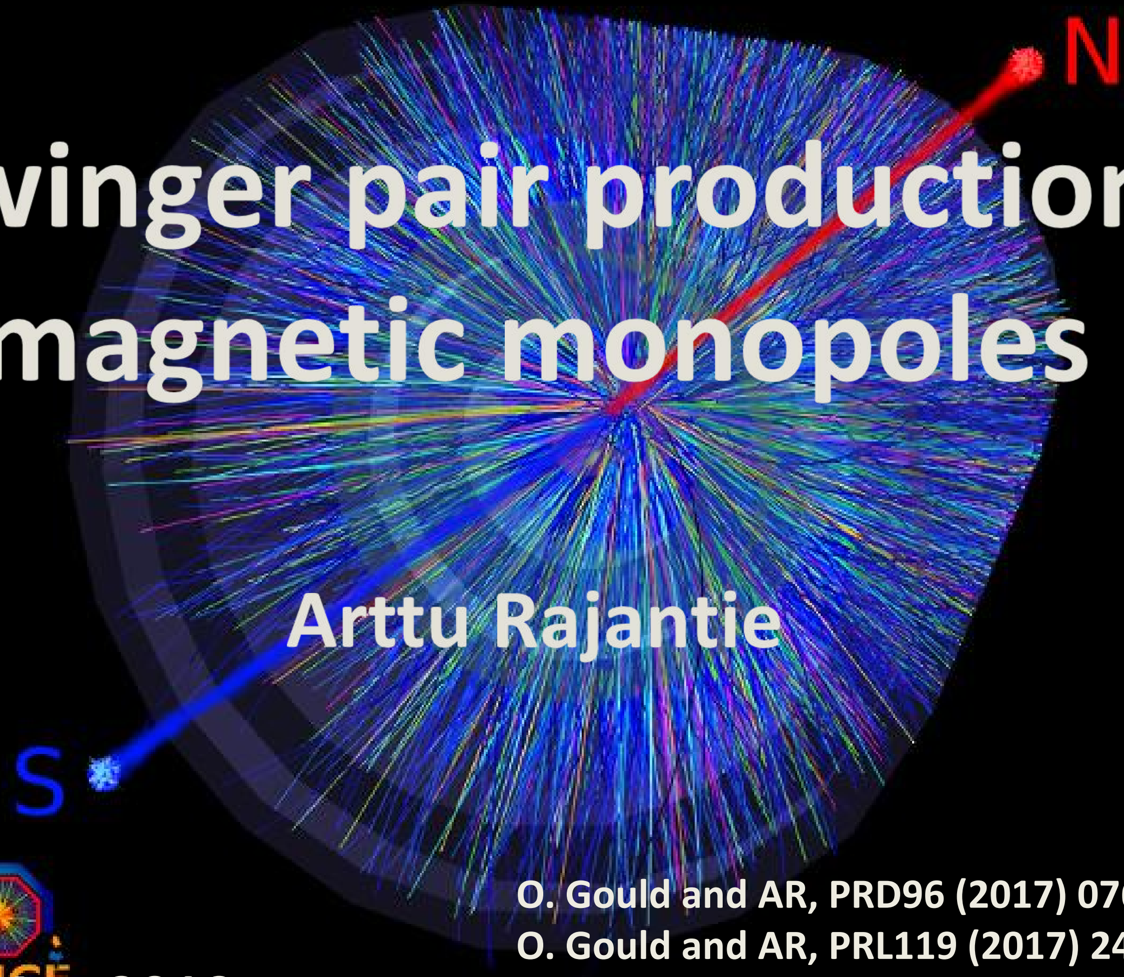
Schwinger pair production of magnetic monopoles



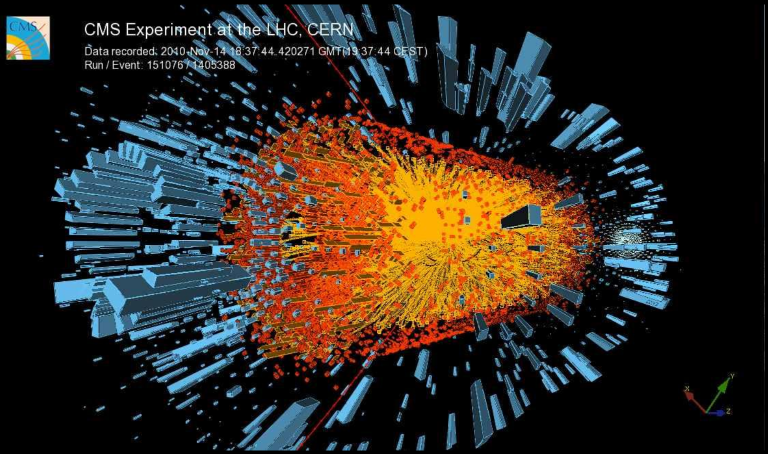
Arttu Rajantie

USTC, 30 May 2019

O. Gould and AR, PRD96 (2017) 076002
O. Gould and AR, PRL119 (2017) 241601
O. Gould, AR and C. Xie, PRD98 (2018) 056022
O. Gould, D.L-J. Ho and A. Rajantie, arXiv:1902.04388

Outline

- Magnetic monopoles
- Schwinger pair production at zero temperature
- Schwinger pair production at non-zero temperature
- Schwinger pair production from time-dependent fields
- Monopole mass bounds from heavy ion collisions and neutron stars



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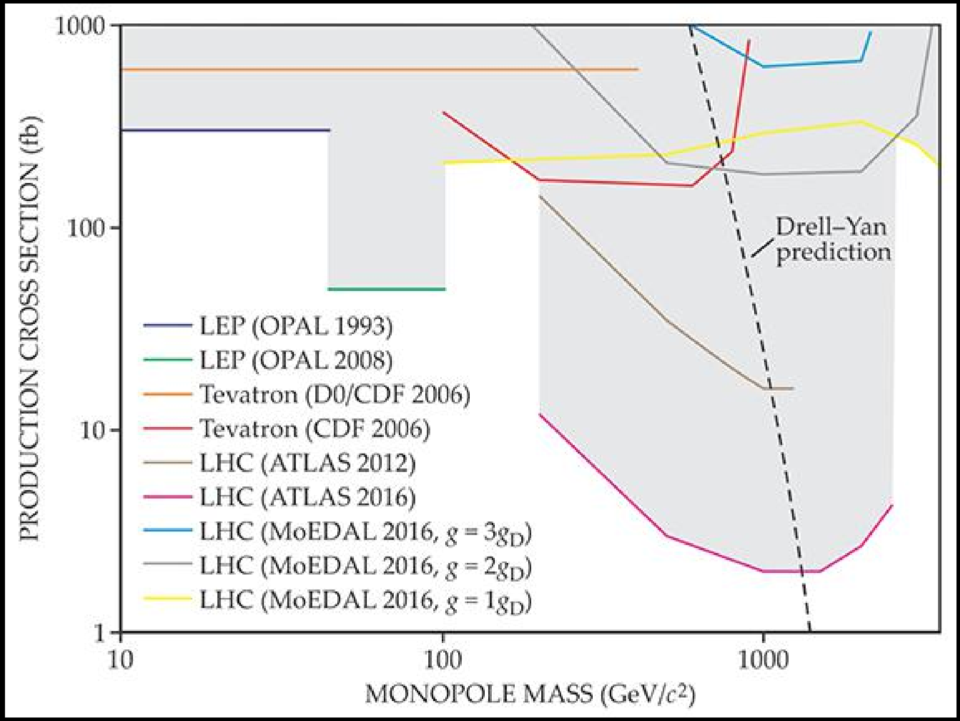
Monopole Searches in Colliders



AR, Physics Today 2016

Monopole Searches in Colliders

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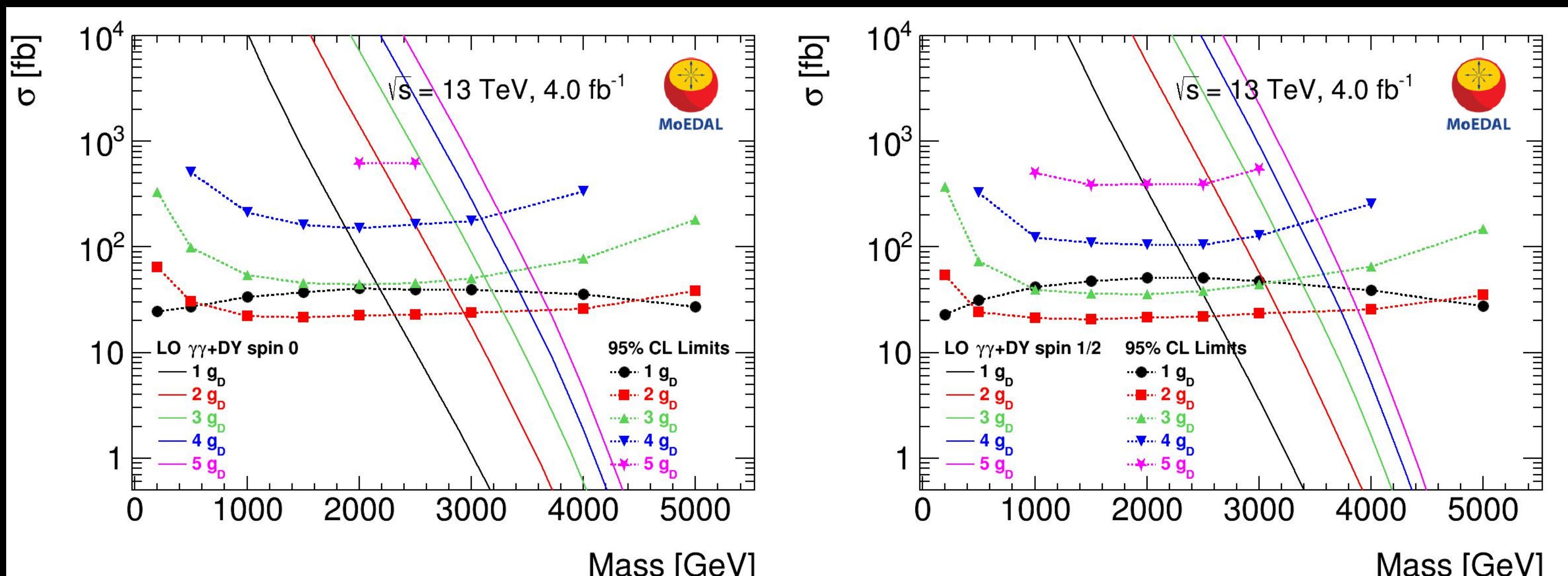
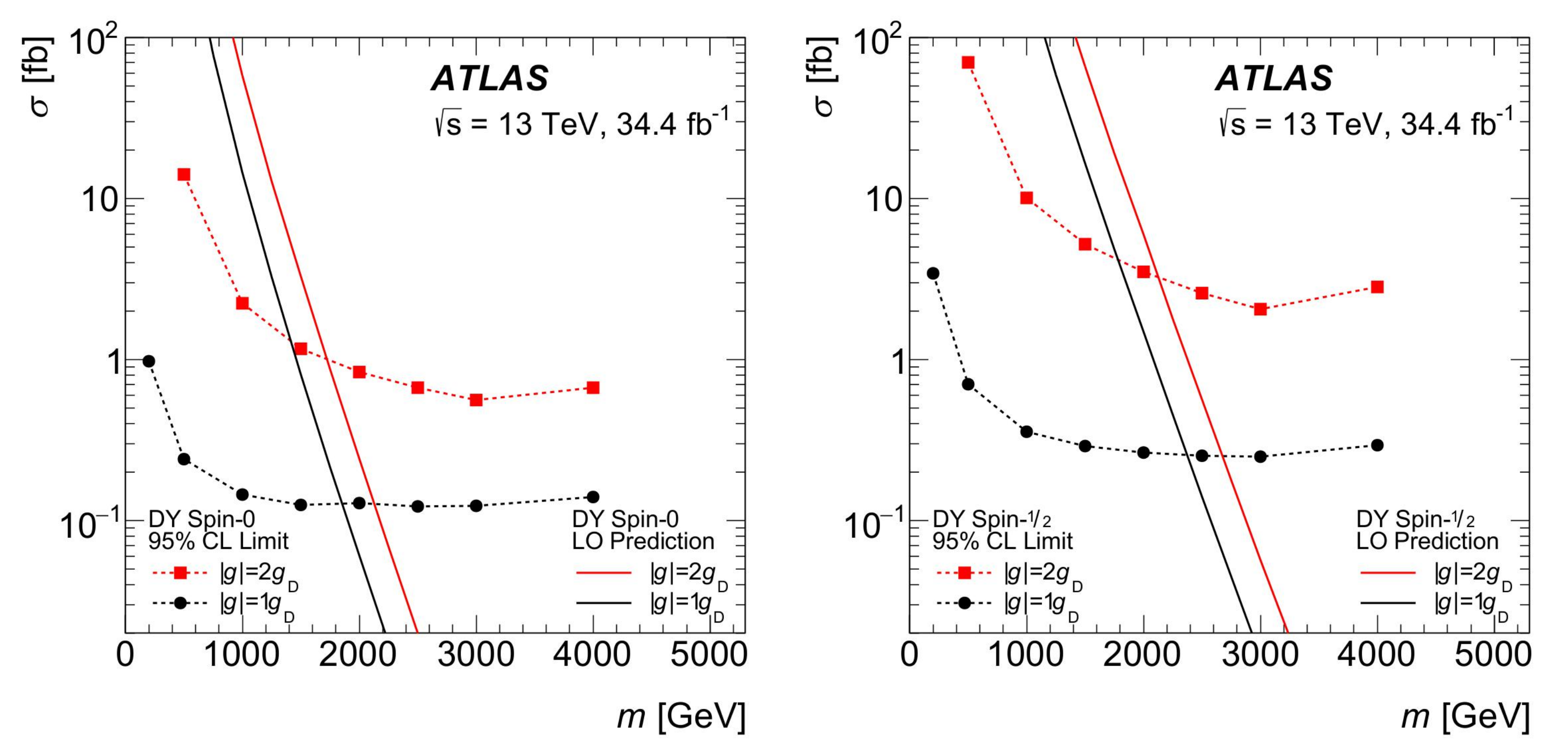


MoEDAL Collaboration, 2019

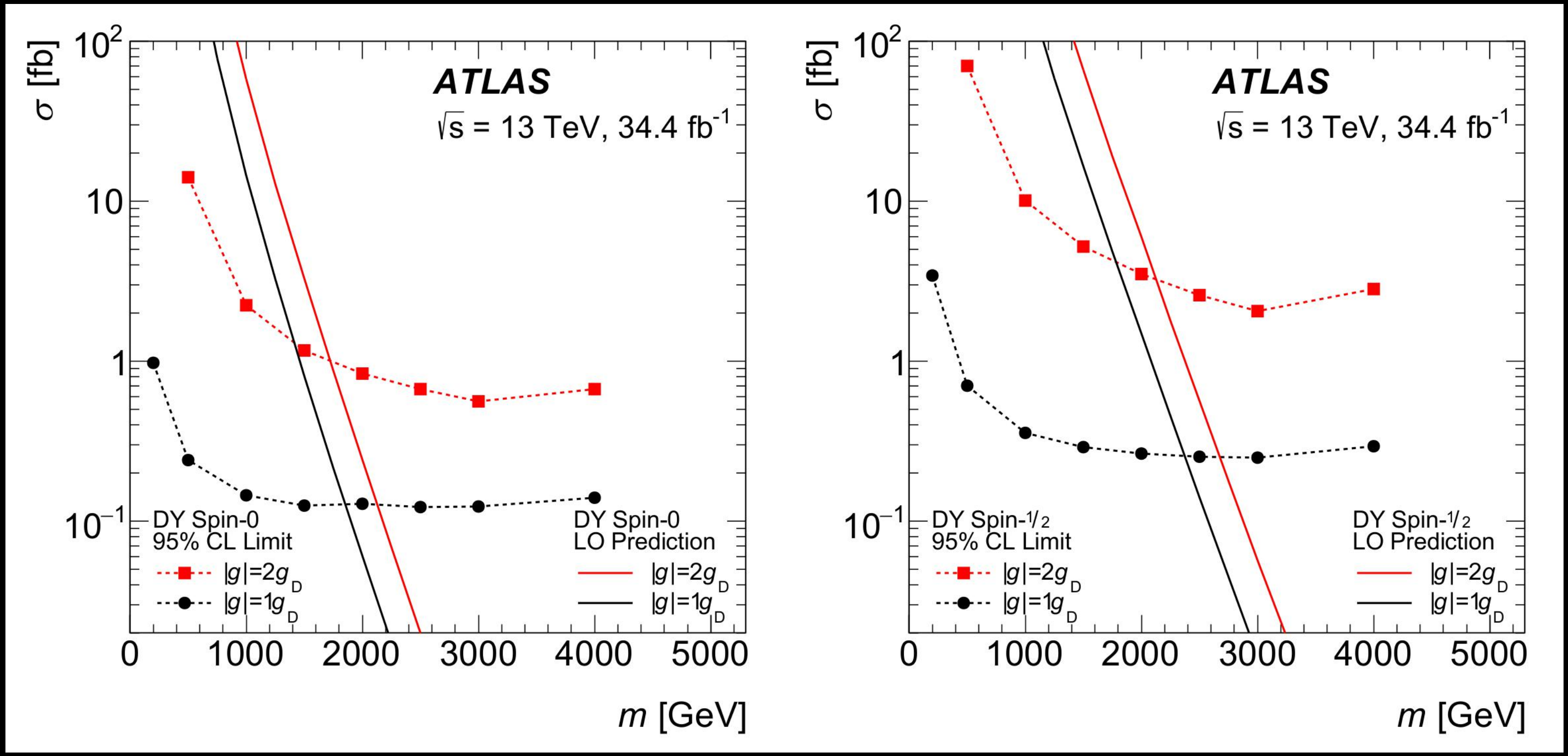
Monopole Searches in Colliders



ATLAS Collaboration, arXiv:1905.10130

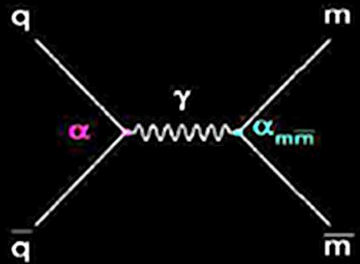
- Upper bounds on production cross section
- To obtain a bound on the monopole mass, one needs to calculate the cross section from theory

Monopole Searches in Colliders



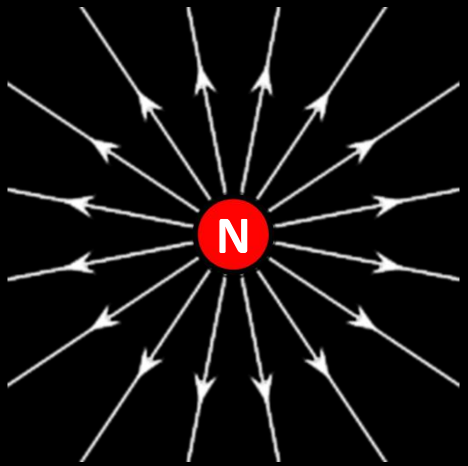
ATLAS Collaboration, arXiv:1905.10130

- Searches usually assume tree-level Drell-Yan cross section with $e \rightarrow g = 2\pi/e$ (EM duality)
- But $g \approx 20.7 \gg 1 \Rightarrow$ Not reliable!



Two Types of Monopoles

- Solitonic monopoles (’t Hooft 1974, Polyakov 1974)
  - Smooth, semiclassical solutions in a renormalised, weakly coupled theory
- Elementary monopoles (e.g. Dirac 1931, Schwinger 1966, Zwanziger 1971)
  - New field in the Lagrangian



't Hooft-Polyakov Monopole

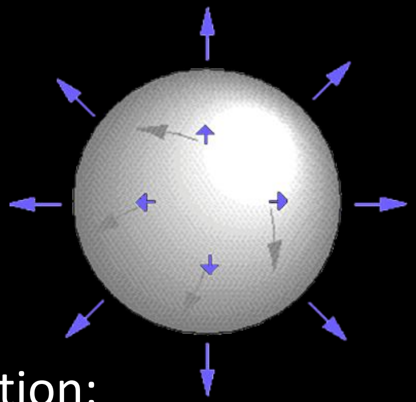
- Magnetic monopole in weakly coupled, renormalisable quantum field theory
- Georgi-Glashow model: SU(2)+adjoint Higgs

$$\mathcal{L} = -\mathrm{Tr} F^{\mu\nu} F_{\mu\nu} + \mathrm{Tr}[D_\mu, \Phi][D^\mu, \Phi] - m^2 \mathrm{Tr}\Phi^2 - \lambda \mathrm{Tr}\Phi^4$$

- $\Phi \neq 0 \Rightarrow$ Symmetry breaking $SU(2) \rightarrow U(1)$
- Electrodynamics with magnetic field given by

$$B_i = \frac{1}{2}\epsilon_{ijk}\mathrm{Tr}\hat{\Phi}\left(F_{jk} - \frac{i}{2e}[D_j, \hat{\Phi}][D_k, \hat{\Phi}]\right)$$

't Hooft-Polyakov Monopole



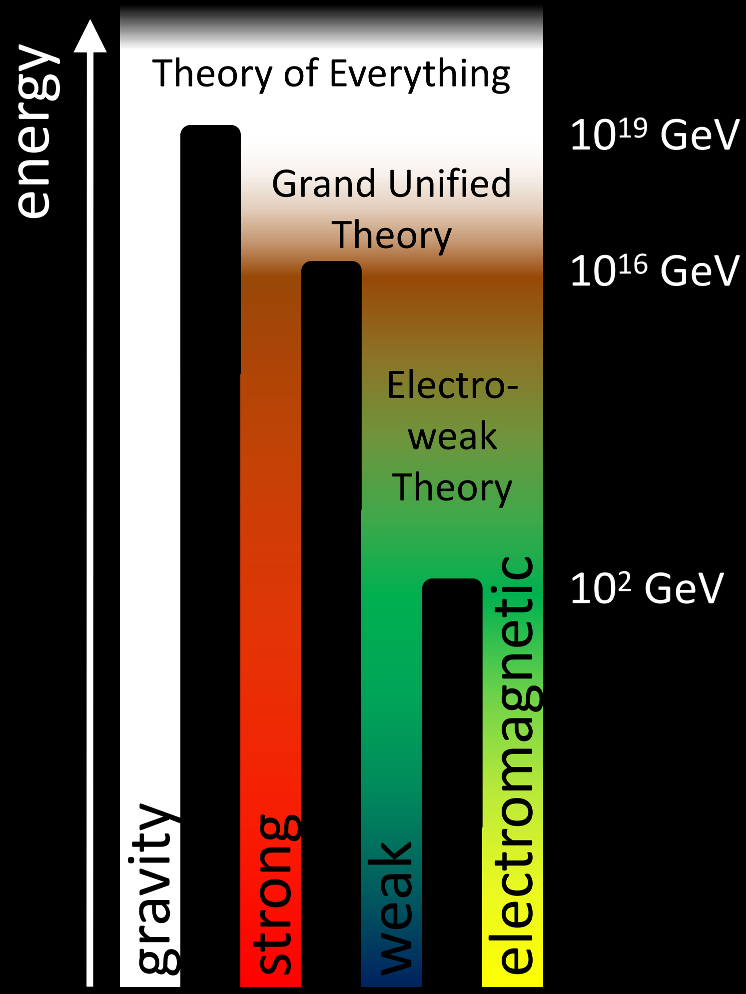
- Smooth "hedgehog" solution:

$$\Phi^a \propto x_a, \quad A_i^a \propto \epsilon_{iaj} x_j$$

- Magnetic charge $g = \int d\vec{S} \cdot \vec{B} = 2g_D = 4\pi/e$
- Calculable properties:
  - Finite mass $M \approx 4\pi v/e \sim m/e^2 \gg m$
  - Non-zero size $R \approx 1/m \gg 1/M$

GUT Monopoles

- Grand Unified Theory (GUT): Electroweak & strong forces unified above $\sim 10^{16}\mathrm{GeV}$
  → 'tHooft-Polyakov monopoles of mass $\sim 10^{17}\mathrm{GeV}$
- Lower mass in some models, maybe even TeV-scale?



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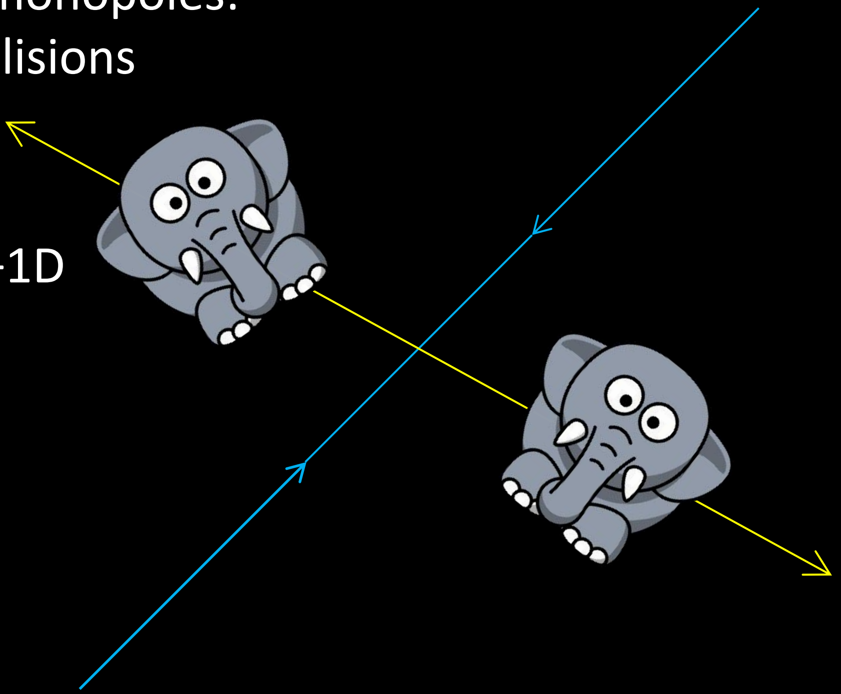
Production Amplitude

- Semiclassical argument for solitonic monopoles: pair production from two-particle collisions suppressed by $\sim e^{-4/\alpha} \sim 10^{-238}$ (Witten, Drukier&Nussinov)
- Confirmed numerically for kinks in 1+1D (Demidov&Levkov 2011)
- Production of solitonic monopoles may be practically impossible in two-particle collisions



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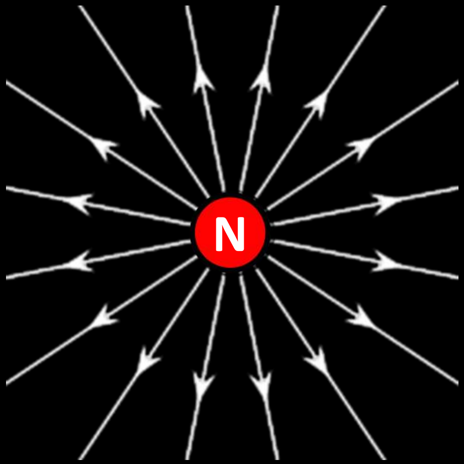
Elementary Monopoles: Classical

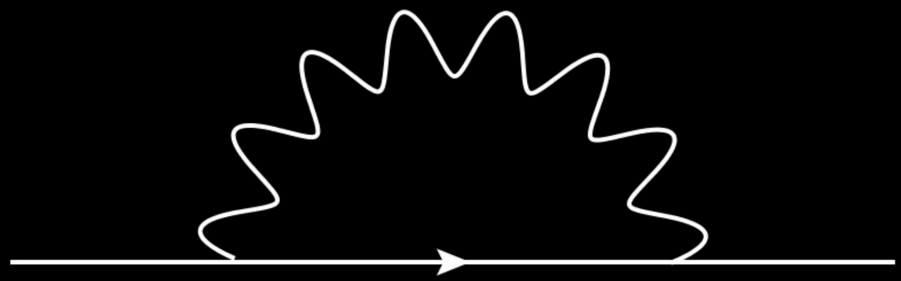


- Point particle with magnetic charge
- Magnetic Coulomb field: $\vec{B} = \frac{g}{4\pi}\frac{\vec{r}}{r^3}$
- Divergent energy:

$$E = \frac{1}{2}\int d^3x\vec{B}^2 \sim g^2\Lambda \sim \frac{\Lambda}{e^2}$$

- Cutoff-scale mass?

Elementary Monopoles: Quantum



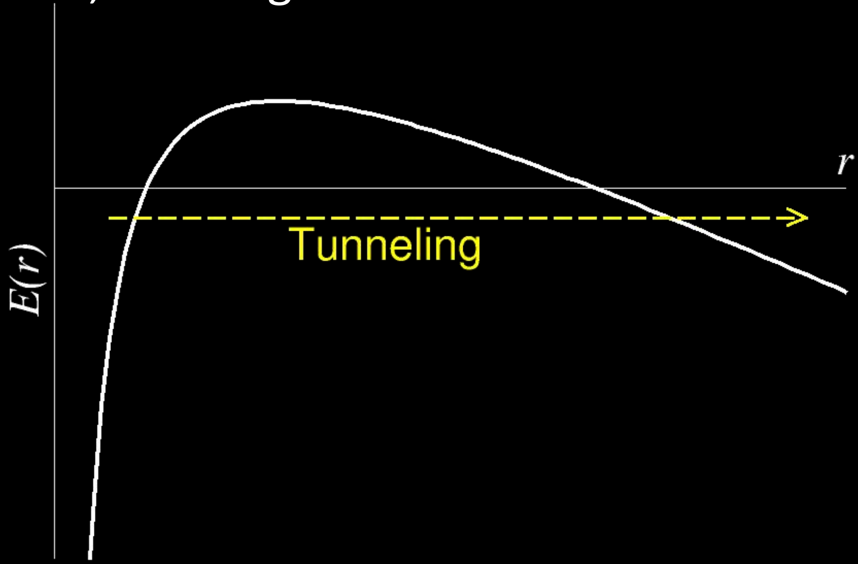
- Chiral symmetry: loop correction $= -\frac{e^2}{2\pi^2} m \log\frac{\Lambda}{m} \ll e^2\Lambda$
- Renormalisation:
  Mass = (divergent) bare mass + (divergent) loops $< \infty$
- Similarly, the mass of an elementary monopole is a free parameter
- Also, effective size $R \sim g^2/4\pi M$ (Goebel 1970, Goldhaber 1983)
- No non-perturbative QFT results for production cross section

Schwinger Pair Production

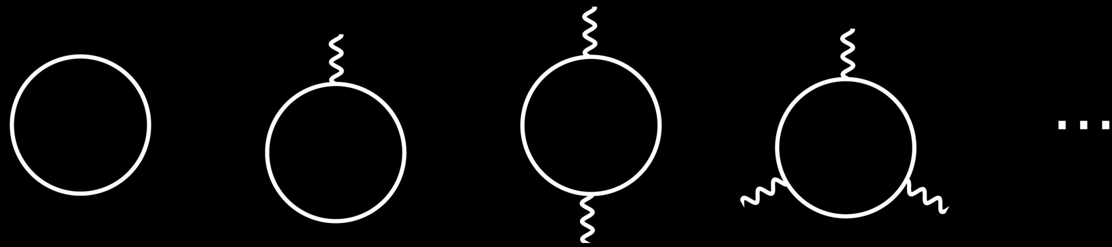
- Energy of a charged particle-antiparticle pair in uniform electric field $\vec{E}$:

$$E(\vec{r}) = 2m - \frac{e^2}{4\pi r} - e\vec{E}\cdot\vec{r}$$

- Sauter 1931, Heisenberg&Euler 1936, Schwinger 1951:
  Tunneling through
  potential barrier
  $\Rightarrow$ pair production from vacuum



Weak Coupling



- Schwinger 1951:
  Pair production rate per unit time per unit volume for $e \ll 1$:

$$\Gamma = \frac{e^2|\vec{E}|^2}{8\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n^2}e^{-\frac{\pi m^2 n}{e|\vec{E}|}}$$

- Field strength needed to overcome suppression:

$$|\vec{E}| \gg \frac{m^2}{e} \sim 10^{18}\ \mathrm{V/m}$$

- A few orders of magnitude higher than the most powerful lasers

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Arbitrary Coupling

- More generally $\Gamma \sim \exp\left(-S_{\text{inst}}\right)$, where S_{inst} is the instanton action
- Action $S[x] = mL[x] - gBA[x] + g^2V[x]$,

$$L[x] = \left(\int d\tau \dot{x}^\mu \dot{x}_\mu\right)^{1/2}$$
$$A[x] = \int d\tau\, x_3 \dot{x}^4$$
$$V[x] = \frac{1}{2}\int_0^1 d\tau\, d\tau'\, \dot{x}^\mu(\tau)\dot{x}^\nu(\tau')G_{\mu\nu}\left(x(\tau), x(\tau')\right)$$

- Prefactor from functional determinant
- Affleck, Alvarez & Manton 1981:
  - Circular instanton with radius $r = m/e|\vec{E}|$
  - Action $S_{\text{inst}} = \frac{\pi m^2}{e|\vec{E}|} - \frac{e^2}{4}$

$$\Gamma = \frac{e^2|\vec{E}|^2}{8\pi^3} e^{-\frac{\pi m^2}{e|\vec{E}|}+\frac{e^2}{4}}$$

- Arbitrary coupling but weak field $e|\vec{E}| \ll m^2$

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Worldline Calculation

- Weak field $\epsilon = e|\vec{E}|/m^2 \ll 1$
  $\Rightarrow$ Saddle point approximation in s integration

$$\Gamma = -\frac{2}{\mathcal{V}}\sqrt{2\pi\epsilon}\,\mathrm{Im}\int \frac{\mathcal{D}x^\mu}{L[x]^{1/2}}\, e^{-\frac{\tilde{S}[x]}{\epsilon}}$$

where

$$\tilde{S}[x] = L[x] - A[x] + \kappa V[x]$$

$$L[x] = \left(\int d\tau \dot{x}^\mu \dot{x}_\mu\right)^{1/2}$$

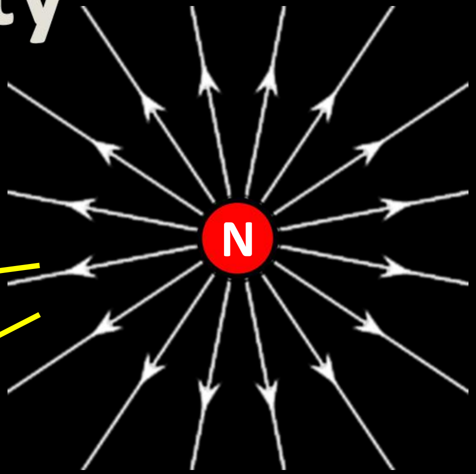
$$A[x] = \int d\tau\, x_3 \dot{x}^4$$

$$V[x] = \frac{1}{2}\int_0^1 d\tau\, d\tau'\, \dot{x}^\mu(\tau)\dot{x}^\nu(\tau')G_{\mu\nu}(x(\tau), x(\tau'))$$

and $\kappa = \frac{e^3|\vec{E}|}{m^2} = e^2\epsilon$

Electric-Magnetic Duality

$$\vec{\nabla}\cdot\vec{E} = \rho_{\mathrm{E}}$$
$$\vec{\nabla}\cdot\vec{B} = \rho_{\mathrm{M}}$$
$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} + \vec{j}_{\mathrm{M}}$$
$$\vec{\nabla}\times\vec{B} = -\frac{\partial\vec{B}}{\partial t} + \vec{j}_{\mathrm{E}}$$



- Symmetry $(\vec{E}, \rho_{\mathrm{E}}, \vec{j}_{\mathrm{E}}) \leftrightarrow (\vec{B}, \rho_{\mathrm{M}}, \vec{j}_{\mathrm{M}})$
- Dual Schwinger process:
  Production of magnetic monopoles by strong magnetic field

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Monopoles from Schwinger Process

- Calculation does not require weak coupling: $g \gg 1$ is not a problem
- Largely independent of microscopic structure of monopoles (elementary or solitonic)
- Pair production rate (constant field, $T = 0$): (Gould&AR, PRD2017)

$$\Gamma = \frac{g^2 |\vec{B}|^2}{8\pi^3} e^{-\frac{\pi M^2}{g|\vec{B}|} + \frac{g^2}{4}}$$

- Pair production needs a strong magnetic field $|\vec{B}| \gtrsim M^2/g$
- Strongest magnetic fields in lab $|\vec{B}| \sim 100\ \mathrm{T} \sim 10^{-13}\ \mathrm{GeV}^2$ $\Rightarrow M \gtrsim 1\ \mathrm{keV}$

Magnetars

- Neutron stars with very strong magnetic fields, up to $B \sim 10^{11}\ \mathrm{T} \approx 10^{-4}\ \mathrm{GeV}^2$
- Low temperature $T \approx 10^{-6} \ldots 10^{-8}\ \mathrm{GeV}$
- Monopole pair production would make the field decay
- Bound
  $M \gtrsim 0.3\ \mathrm{GeV},\ g = g_D$
  $M \gtrsim 0.7\ \mathrm{GeV},\ g = 2g_D$
  (Gould&AR, PRL2017)

Heavy Ion Collisions

(Video: Brookhaven National Laboratory)

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Heavy Ion Collisions at SPS

- Monopole search at CERN Super Proton Synchrotron (He, PRL 1997)
  - $160A\mathrm{GeV}$ fixed target Pb beam, $\sqrt{s_{NN}} \approx 17\ \mathrm{GeV}$
  - No monopoles $\Rightarrow$ Production cross section bound $$\sigma_{MM} < 1.9\ \mathrm{nb}$$
  - Rest frame field $B \approx 0.0097\ \mathrm{GeV}^2$, temperature $T \approx 0.185\ \mathrm{GeV}$
- High temperature
  - Instantons with periodic Euclidean time

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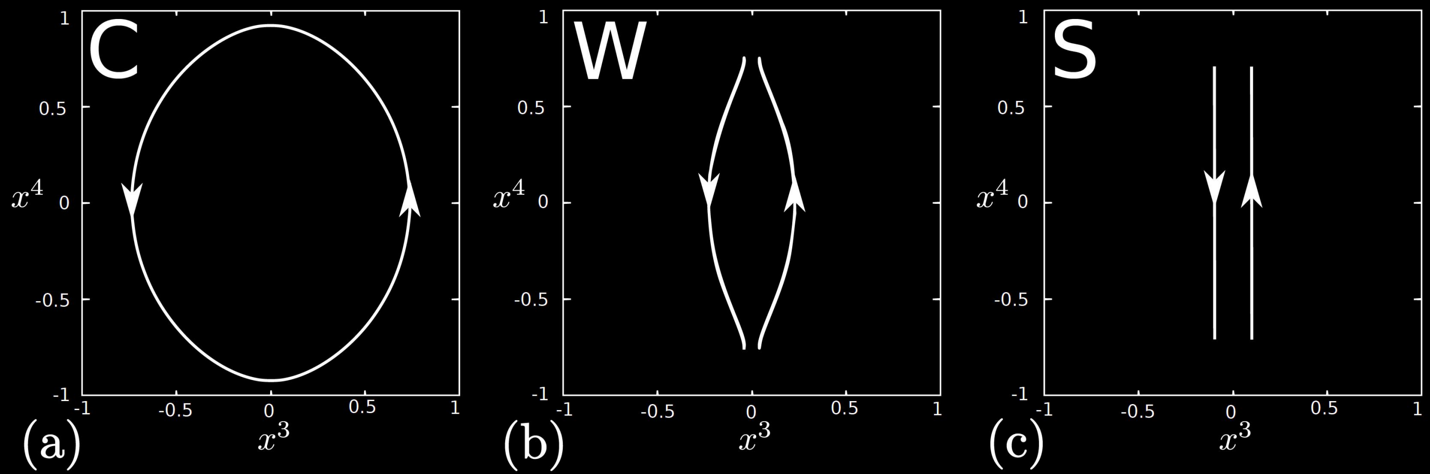
Thermal Schwinger Process



low T intermediate T high T

- SPS collisions: high T
- Instanton action $S_{\text{inst}} = \frac{2m}{T}\left(1 - \left(\frac{g^3 B}{4\pi m^2}\right)^{1/2}\right)$ (Gould&AR 2017)
- Gives mass bound $m \gtrsim \left(2.0 + 2.6\left(\frac{g}{g_D}\right)^{\frac{3}{2}}\right)$ GeV

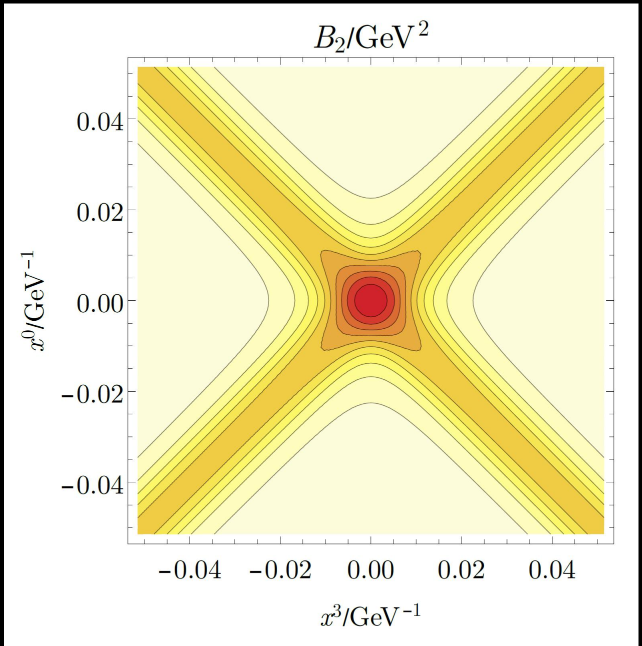
LHC Heavy Ion Collisions

- Pb-Pb collisions in Nov 2018
  - $\sqrt{s_{NN}} = 5.02\ \text{TeV}$,
- Time-dependent field

$$\vec{B} = \frac{B\hat{y}}{2}\left(\left(1+\omega^2\left(t-\frac{z}{v}\right)^2\right)^{-3/2} + \left(1+\omega^2\left(t+\frac{z}{v}\right)^2\right)^{-3/2}\right),$$

where $B \approx 8.6\ \text{GeV}^2$, $\omega \approx 62\ \text{GeV}$

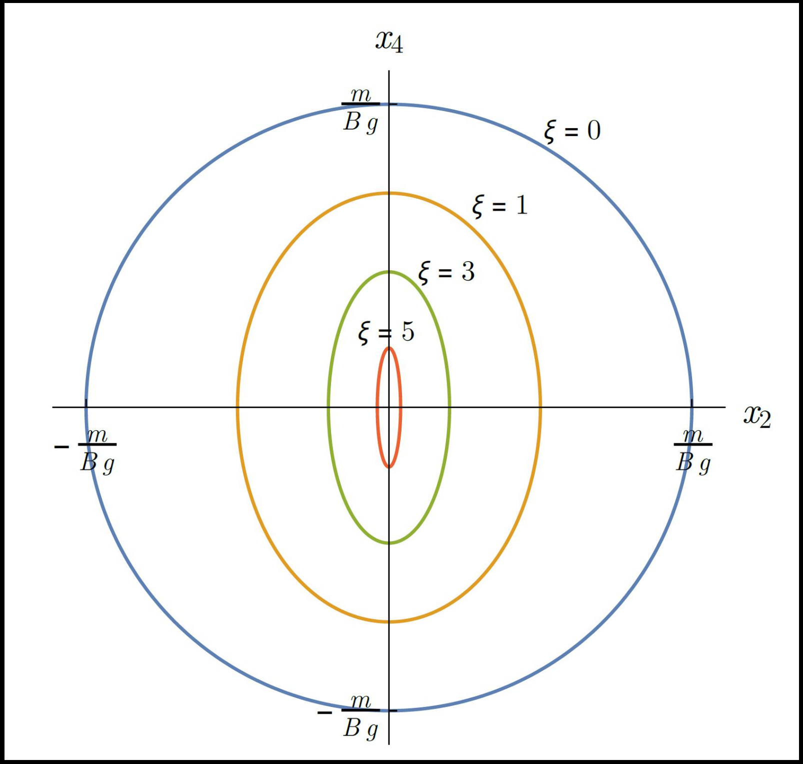
- Wick rotate to Euclidean space



$$\vec{B}_E = \frac{B\hat{y}}{2i}\left(\left(1+\omega^2\left(i\tau-\frac{z}{v}\right)^2\right)^{-3/2} + \left(1+\omega^2\left(i\tau+\frac{z}{v}\right)^2\right)^{-3/2}\right)$$

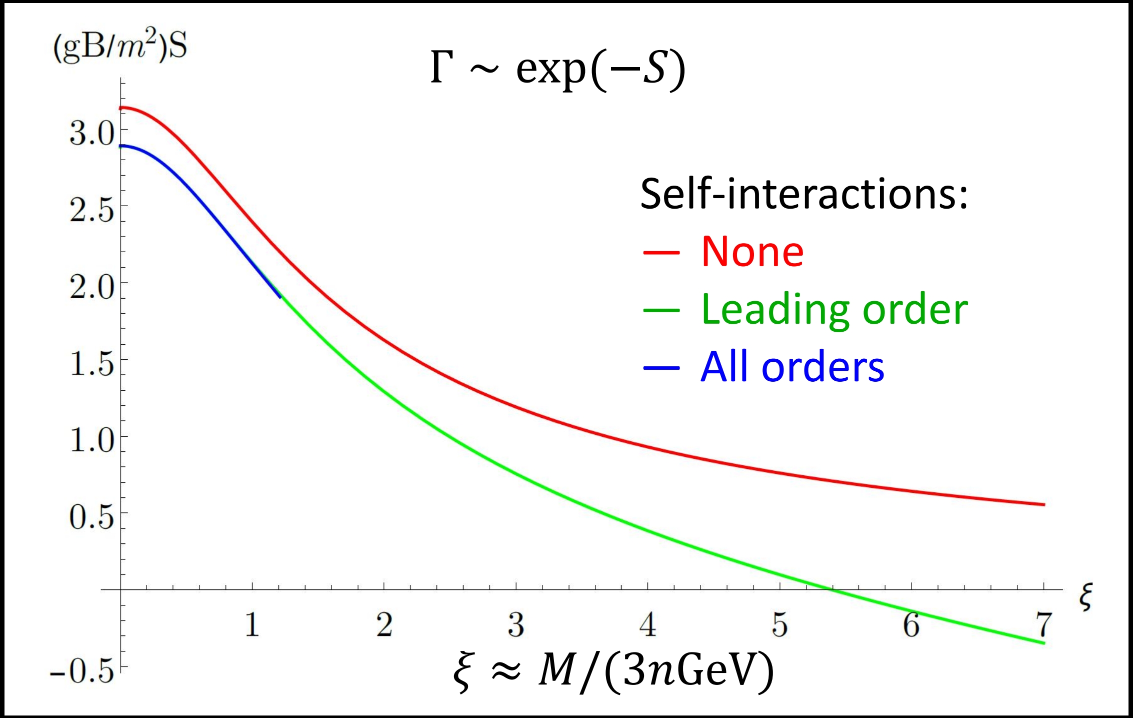
Spacetime Dependence

- In LHC heavy-ion collisions, fields are not even approximately constant
- Parameterised by
  $\xi = \frac{m\omega}{gB} \approx M/(3n\,\mathrm{GeV})$
  - Constant field: $\xi \to 0$
- Need to find worldline instanton in a time-dependent background (Gould, Ho & AR, arXiv:1902.04388)
- Can be done analytically to first order in self-interaction, numerically to all orders



Spacetime Dependence

- Space and time dependence enhances the production rate (Gould, Ho & AR, arXiv:1902.04388)
- This would strengthen mass bounds
- We cannot reach LHC parameters yet:
  - Self-interactions
  - Finite monopole size



- Ignoring self-interactions gives the (conservative) estimate $M \gtrsim 80$ GeV if no monopoles found

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Summary

- Colliders experiments constrain monopole production cross section:
  - To constrain monopole mass (or other parameters) one needs a theoretical calculation
  - Currently not possible for pp collisions because $g \gg 1$
- If monopoles exist, they are produced in strong magnetic fields by the Schwinger process:
  - Calculable with semiclassical instanton techniques even when $g \gg 1$
- Strongest magnetic fields in the Universe:
  - Magnetars $\Rightarrow M \gtrsim 1\ \mathrm{GeV}$
  - Heavy-ion collisions at SPS $\Rightarrow M \gtrsim 10\ \mathrm{GeV}$
  - Heavy-ion collisions at LHC $\Rightarrow M \gtrsim 100\ \mathrm{GeV}$?
- More calculations needed:
  - Strong time-dependence, finite monopole size