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ASTROPHYSICAL NEUTRINOS AND NEW PHYSICS

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OUTLINE

1. The detection of high energy astrophysical neutrinos by IceCube
2. What are neutrinos and their oscillations? What are the mechanisms for producing high energy astrophysical neutrinos?
3. What can we learn from the detection of these neutrinos? The flavour fraction of neutrinos at the source or the flavour transition mechanism occurring in the propagations of neutrinos?
4. Some recent results
5. Conclusion

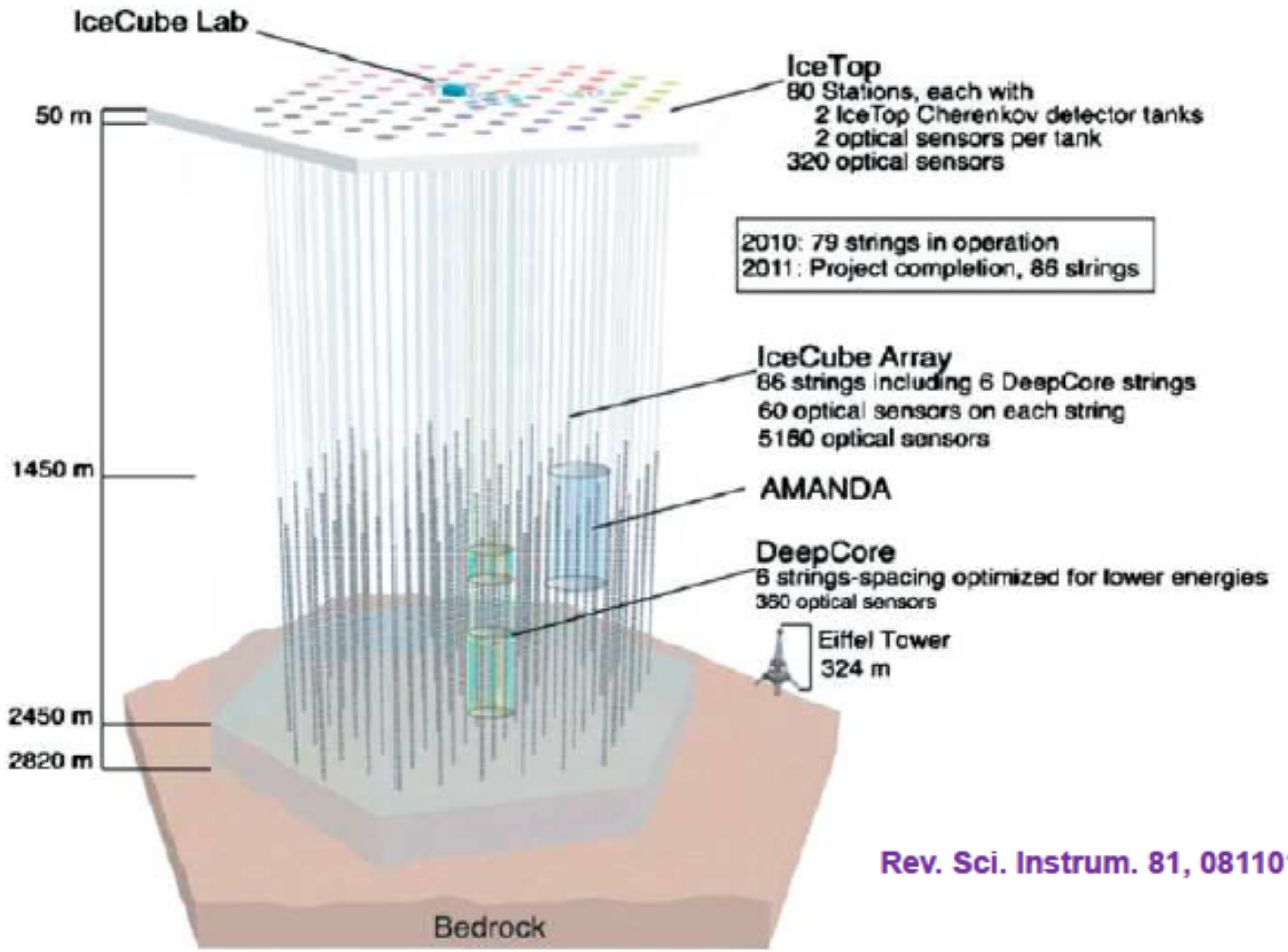
THE DETECTION OF HIGH ENERGY ASTROPHYSICAL NEUTRINOS BY ICECUBE

Science 22 Nov 2013:
Vol. 342, Issue 6161, 1242856

Quoted from the official website of IceCube Observatory

``Six years after its completion, IceCube has isolated more than 80 high-energy cosmic neutrinos, with energies between 100 TeV and 10 PeV, from more than a million atmospheric neutrinos and hundreds of billions of cosmic-ray muons."

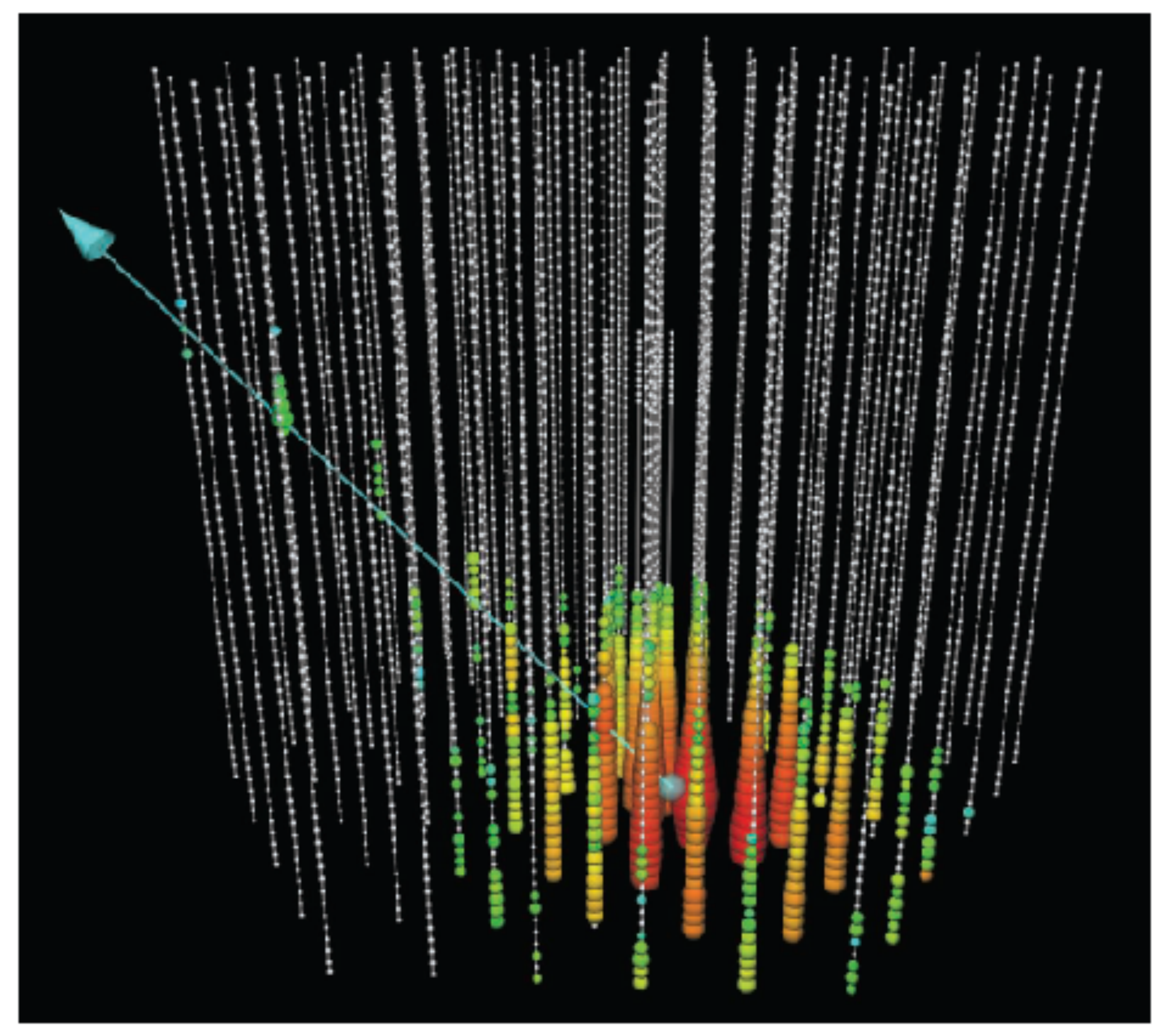
IceCube Neutrino Observatory

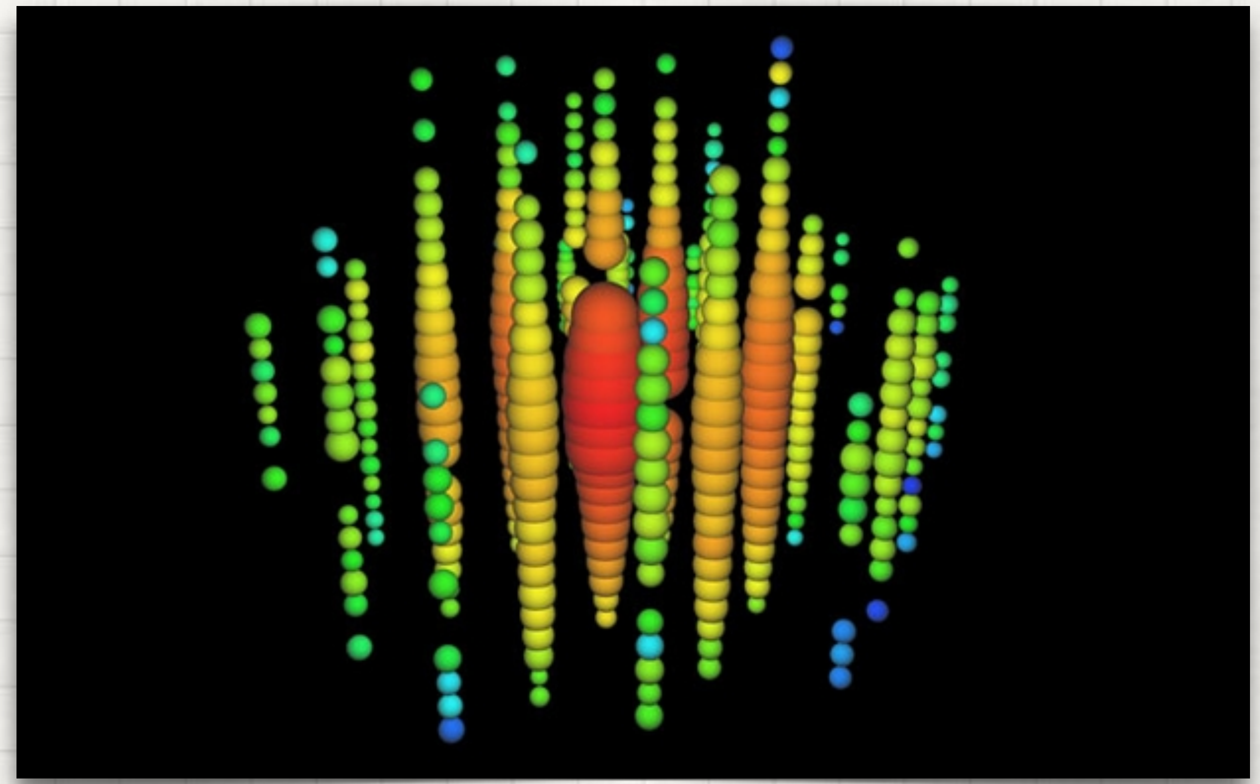


Rev. Sci. Instrum. 81, 081101 (2010)

Detection threshold energy of **Iceube** > **100GeV**

Detection threshold energy of **Iceube + DeepCore** ~ **10GeV**





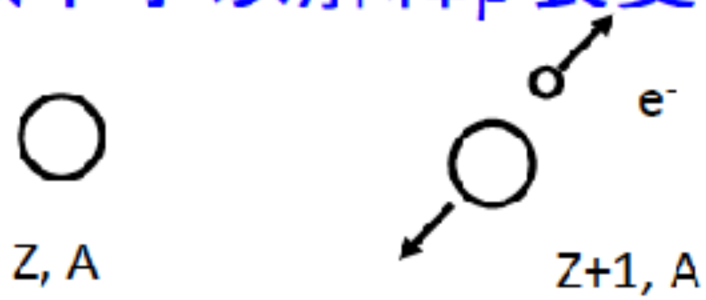
PeV cascade event

A 250 TeV neutrino interaction in IceCube
At the neutrino interaction point (bottom), a large particle shower is visible with a muon produced in the interaction leaving up and to the left. The direction of the muon indicates the direction of the original neutrino.

FACTS ABOUT NEUTRINOS

(1). 微中子發現經過:

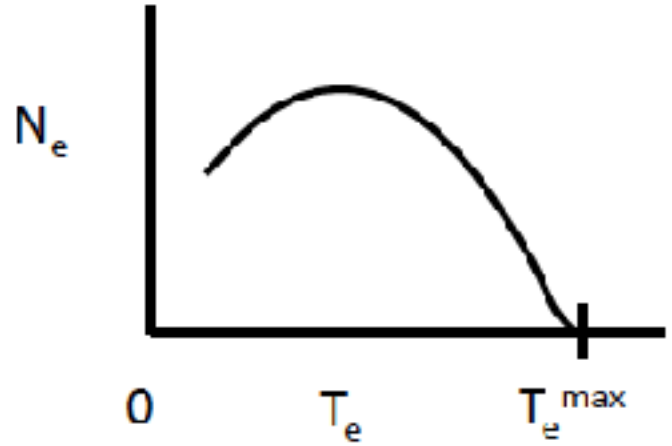
Pauli提出微中子以解釋β衰變之實驗結果



β decays

$$T_e^{\max} = \left(M_{Z+1,A} - M_{Z,A}\right)c^2$$

Electron kinetic energy spectrum



若末態只有兩顆粒子, 則根據能量動量守恆,
電子及末態核子必定瓜分初始態核子所有能量,
為何有電子能譜分佈呢？

1931年Pauli提出β衰變末態應有第三
個粒子--
微中子存在。Pauli假定微中子為
(1) 無質量
(2) 自旋 1/2
(3) 電中性
(4) 和其他粒子作用微弱

偵測微中子是實驗物理學家一大挑戰。
1956年Cowan和Reines偵測到微中子！
運用質子捕捉微中子而產生中子及正子:

$$\bar{\nu} + p \rightarrow n + e^{+}$$

微中子來自核子反應爐之β衰變, 上述反應截面積僅有10^{-44} cm^2！因此需要大量質子做標靶。

- We now know there are 3 types of neutrinos: ν_e, ν_μ, ν_τ
- We also know neutrinos oscillate from one kind to other kinds when they propagate.

- **PMNS (Pontecorvo-Maki-Nakagawa-Sakata) matrix**

Weak interaction states

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Mass eigenstates

$$U_{MNS} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix}}_{\text{Atmospheric}} \underbrace{\begin{pmatrix} \cos\theta_{13} & 0 & e^{-i\delta}\sin\theta_{13} \\ 0 & 1 & 0 \\ -e^{i\delta}\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix}}_{\text{Reactor}} \underbrace{\begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Solar}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}}_{\text{Majorana Phases}}$$

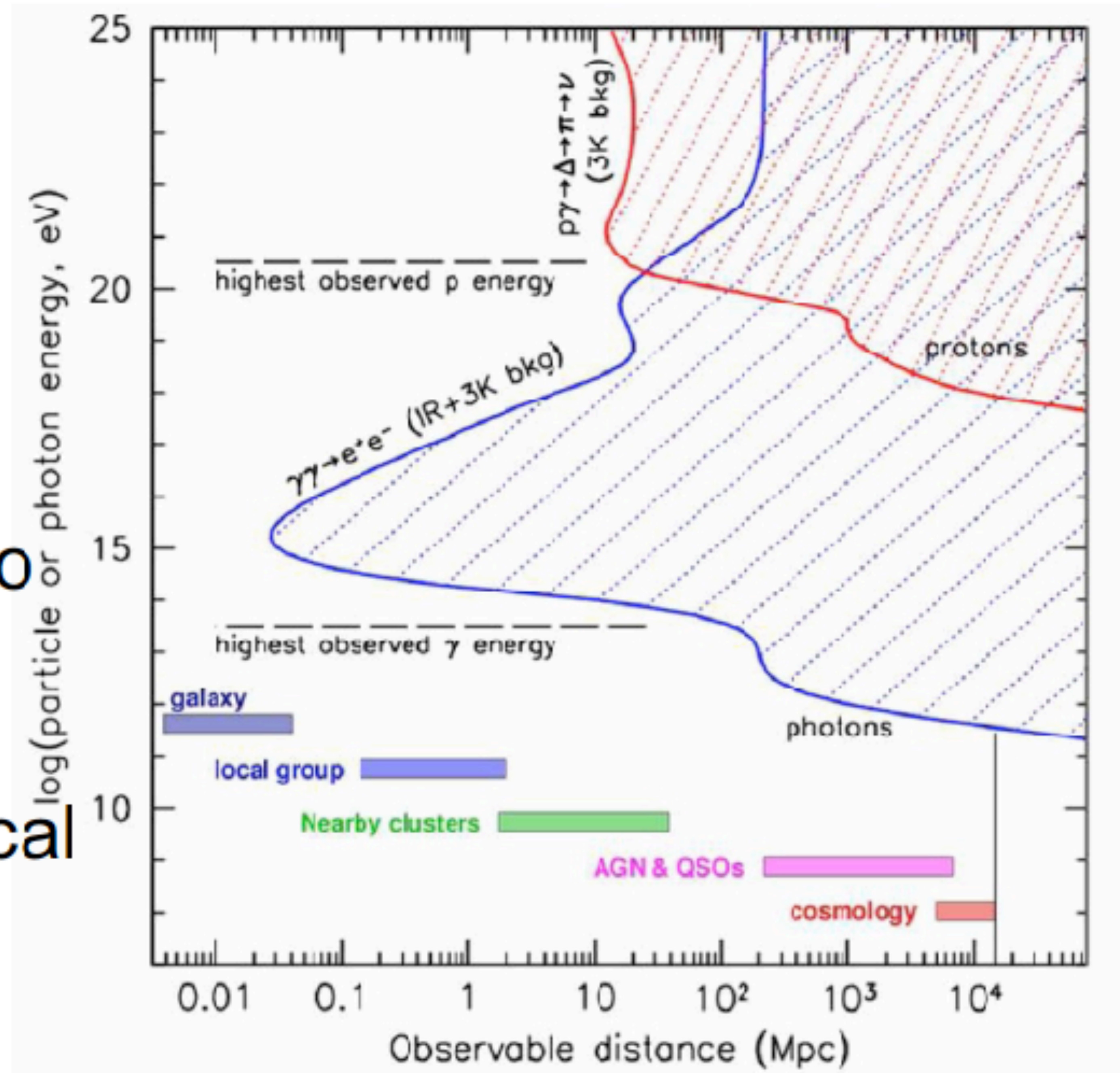
Daya Bay 2012

The motivation for detecting astrophysical neutrinos

- Both neutrinos and photons are produced by high energy hadronic collisions—likely to in AGN, GRB,…
- The universe becomes opaque for any photon with an energy >10^{14} eV
- On the other hand, a neutrino propagates freely due to its weak-interacting nature—

a complementary astrophysical probe

P. Allison et al., arXiv:0904.1309.



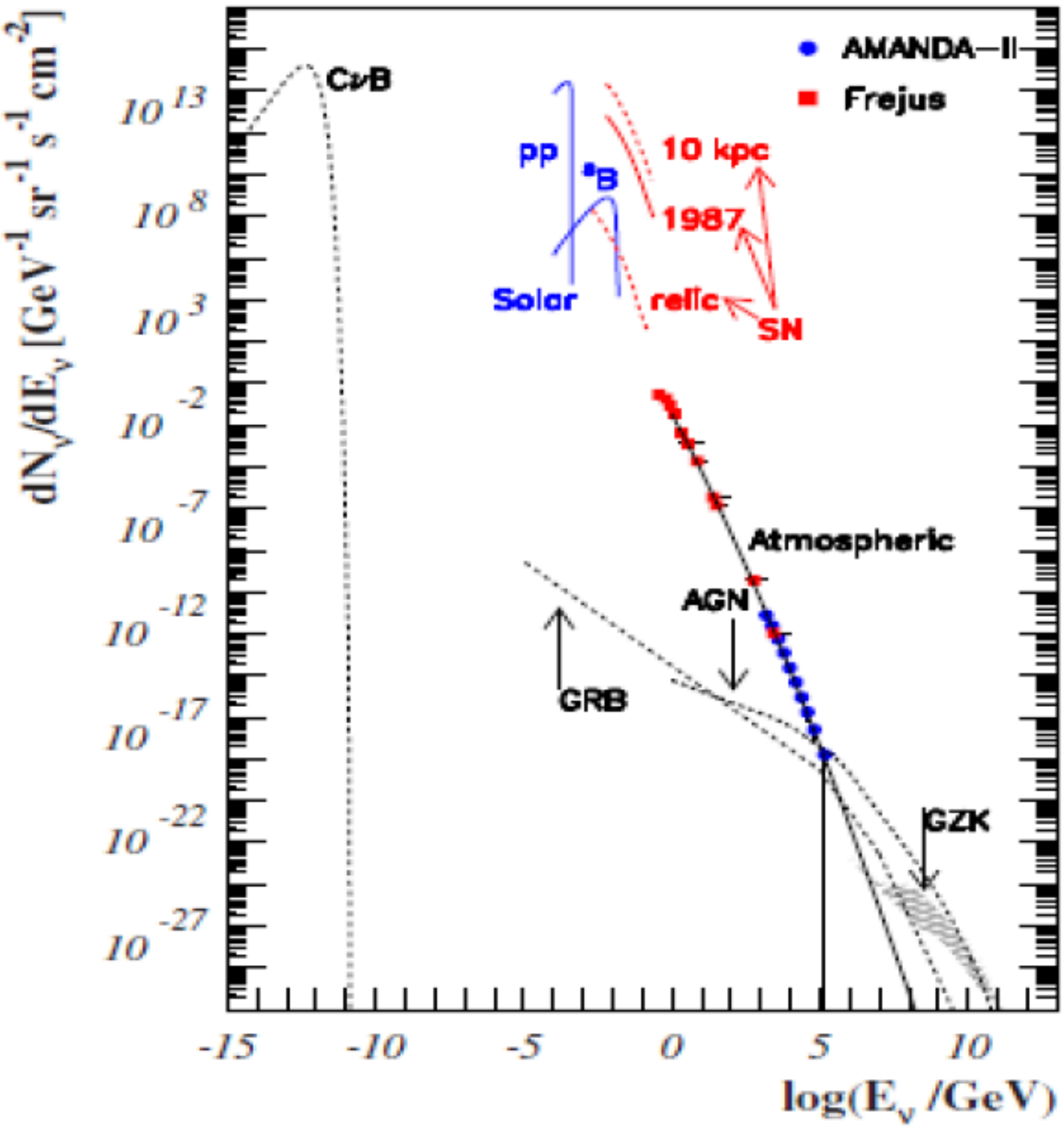
Fluxes of astrophysical neutrinos

Atm:
PA → π, *K*

GRB, AGN
*P*γ → Δ → *n*π

GZK
Pγ_{CMB} → Δ → *n*π

Neutrino arises from π, *K* decays



F. Halzen and S. R. Klein, 2010

Common astrophysical neutrino sources

(1) *pp* collisions: roughly the same number of **π^+ and π^-** are produced. Neutrinos and anti-neutrinos are produced equally

$$\pi^+(\pi^-) \to \mu^+(\mu^-)\nu_\mu(\bar{\nu}_\mu)$$

$$\mu^+(\mu^-) \to \nu_\mu(\bar{\nu}_\mu)e^+(e^-)\bar{\nu}_e(\nu_e)$$

(a) secondary muons decay immediately

Pion source

$$\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0 \qquad \bar{\nu}_e : \bar{\nu}_\mu : \bar{\nu}_\tau = 1 : 2 : 0$$

(b) secondary muons lose significant energies before decay

$$\nu_e : \nu_\mu : \nu_\tau = 0 : 1 : 0 \qquad \bar{\nu}_e : \bar{\nu}_\mu : \bar{\nu}_\tau = 0 : 1 : 0$$

Muon-damped source

(2) *pγ* collisions: leading contributions

$$p + \gamma \to \Delta^+ \to n + \pi^+$$

$$\pi^+ \to \mu^+ \nu_\mu$$

$$\mu^+ \to \bar{\nu}_\mu e^+ \nu_e$$

(a) secondary muons decay immediately   Pion source

$\nu_e : \nu_\mu : \nu_\tau = 1 : 1 : 0$ $\bar{\nu}_e : \bar{\nu}_\mu : \bar{\nu}_\tau = 0 : 1 : 0$

(b) secondary muons lose significant energies before decay

$\nu_e : \nu_\mu : \nu_\tau = 0 : 1 : 0$

Muon-damped source

(2) *pγ* collisions: sub-leading contributions

$$p\gamma \to p\pi^+\pi^-$$

Neutrinos and anti-neutrinos are produced equally

Non-negligible if gamma spectrum is hard enough

(10-15)%

(a) secondary muons decay immediately Pion source

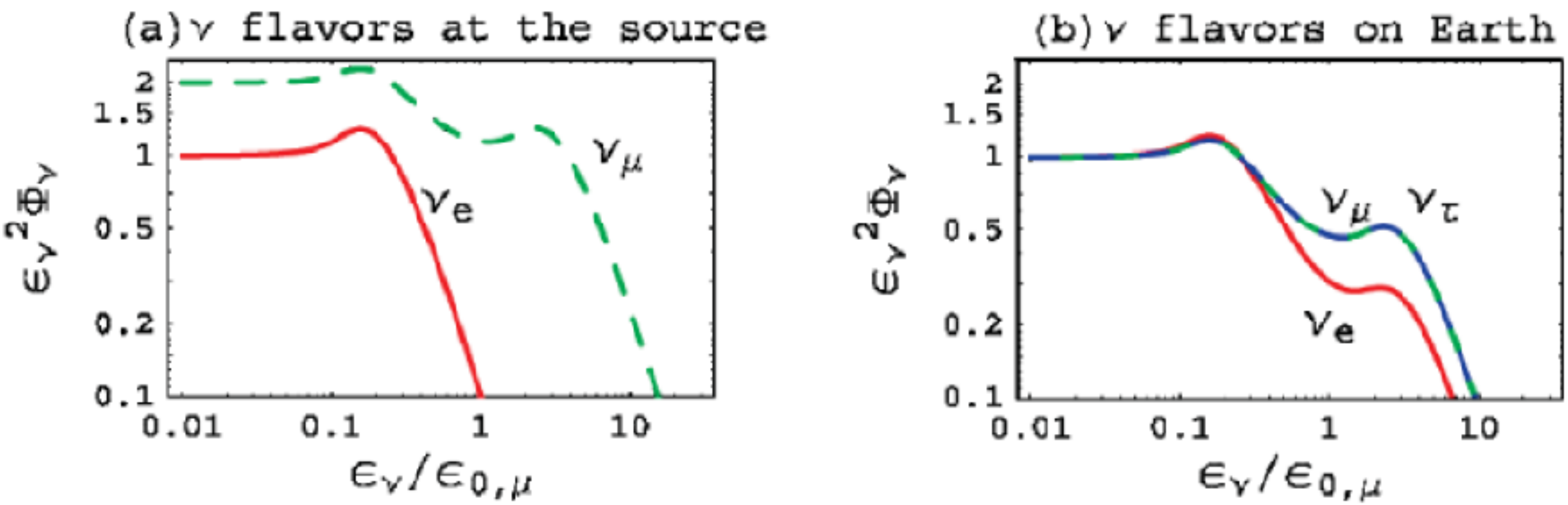
$$\nu_e : \nu_\mu : \nu_\tau = 1/3 : 2/3 : 0 \quad \bar{\nu}_e : \bar{\nu}_\mu : \bar{\nu}_\tau = 1/3 : 2/3 : 0$$

(b) secondary muons lose significant energies before decay

$$\nu_e : \nu_\mu : \nu_\tau = 0 : 1 : 0 \qquad \bar{\nu}_e : \bar{\nu}_\mu : \bar{\nu}_\tau = 0 : 1 : 0$$

Muon-damped source

In some cases, the neutrino flavour ratio at the source can be energy dependent

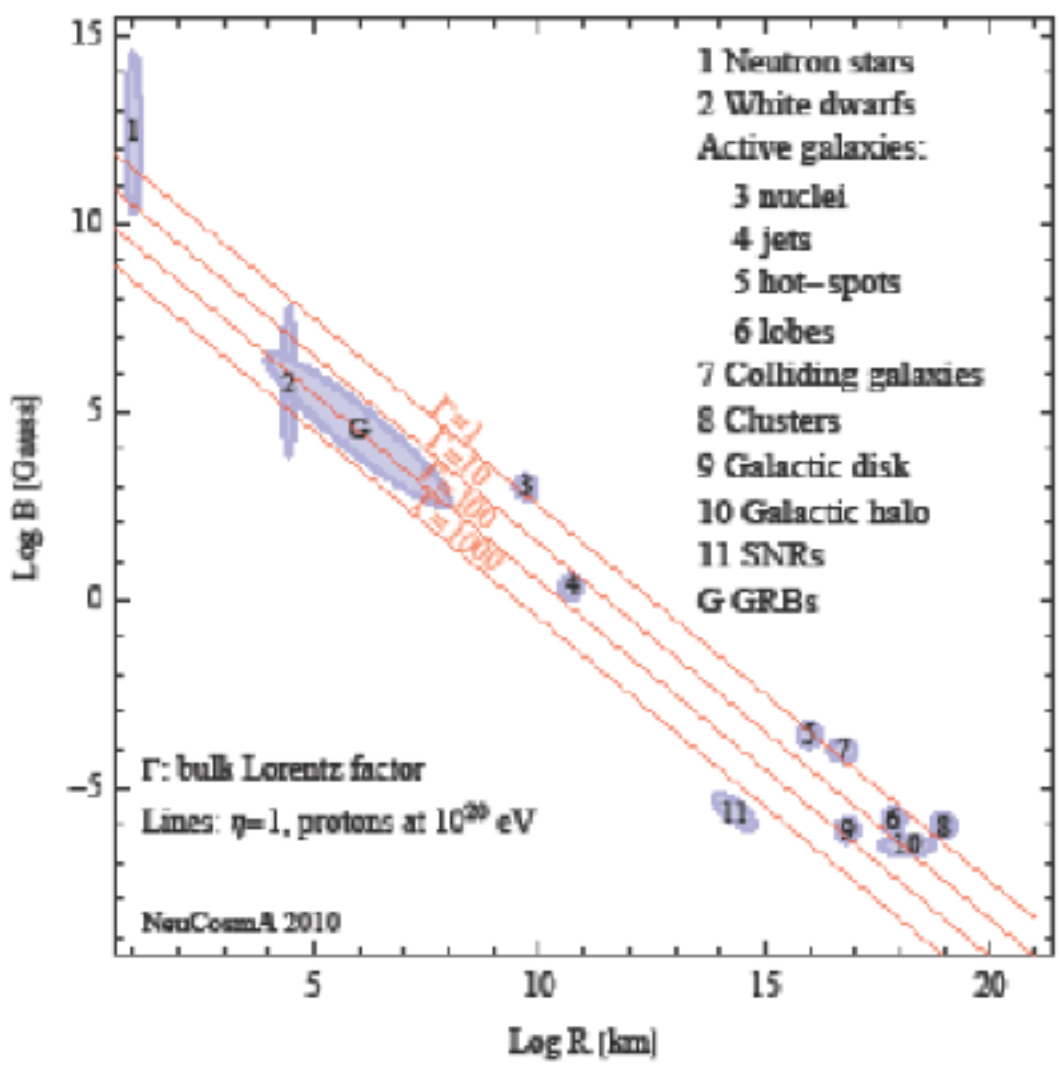


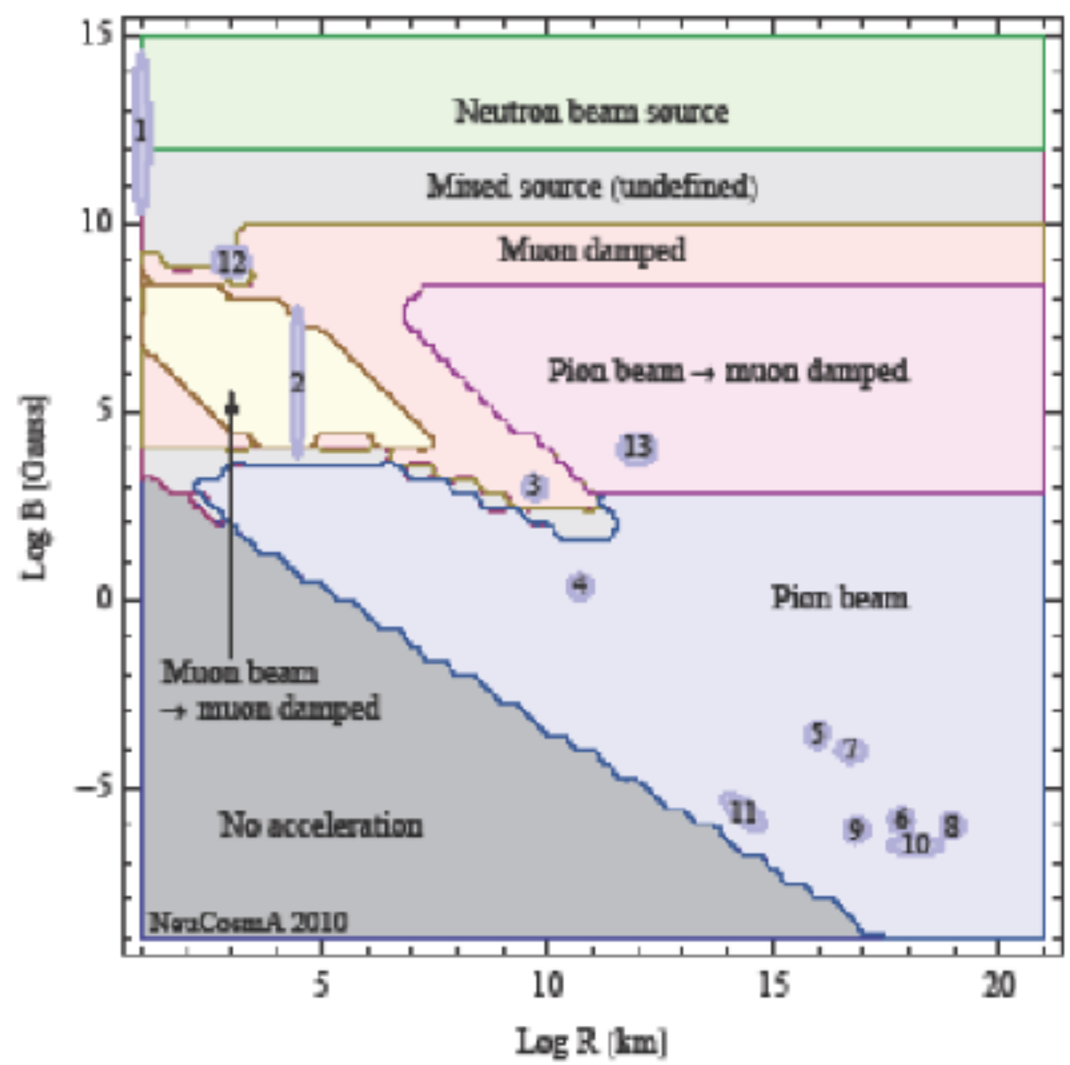
T. Kashti and E. Waxman Phy. Rev. Lett. 2005

The competition between decay and interaction time scales.

Systematically studying sources on Hillas plot

$$\phi(E_p) \propto E_p^{-2}$$





S. Hummer, M. Maltoni, W. Winter,and C. Yaguna, Astropart. Phys. 34, 205 (2010).

Neutrino oscillations

Take a simplified example

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

At t=0 when muon neutrinos are produced in the atmosphere

$$\left|\nu_\mu(0)\right\rangle \equiv \left|\nu_\mu\right\rangle = \cos\theta\left|\nu_1(0)\right\rangle + \sin\theta\left|\nu_2(0)\right\rangle$$

$$\left|\nu_\tau\right\rangle = -\sin\theta\left|\nu_1(0)\right\rangle + \cos\theta\left|\nu_2(0)\right\rangle$$

At time t

$$\left|\nu_\mu(t)\right\rangle = \exp\left(-iE_1 t/\hbar\right)\cos\theta\left|\nu_1(0)\right\rangle + \exp\left(-iE_2 t/\hbar\right)\sin\theta\left|\nu_2(0)\right\rangle,$$

with $E_i = \left(p^2c^2 + m_i^2c^4\right)^{1/2}$

$$P(\nu_\mu \to \nu_\tau) = \sin^2\theta \sin^2\left(\frac{\pi x}{l}\right),$$

Here *x* is the traveling distance of neutrino for time interval *t* while *l* is the oscillation length given by

$$l = \frac{2.5 P_\nu (\mathrm{MeV}\ / \mathrm{c})}{(m_2^2 - m_1^2)(\mathrm{eV/c}^2)^2}\ \mathrm{meters}$$

$$P_\nu = 1\,\mathrm{MeV/c},\ m_2^2 - m_1^2 = 1\,(\mathrm{eV/c}^2)^2, l = 2.5\ \mathrm{m};$$

$$P_\nu = 1\,\mathrm{GeV/c}\ ，m_2^2 - m_1^2 = 0.25\times10^{-3}\,(\mathrm{eV/c}^2)^2, l = 10^4\ \mathrm{km}.$$

$$P_\nu = 10\ \mathrm{GeV}/c, m_2^2 - m_1^2 = 2.5\times10^{-3}\left(\mathrm{eV}/c^2\right)^2, l = 10^4\ \mathrm{km}$$

The neutrino flavour transition probability

$$P_{\alpha\beta} \equiv P(\nu_\beta \to \nu_\alpha)$$

$$\beta \overset{i=1,2,3}{\text{———}} \alpha$$

3 channels interfere

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} \Re(U_{\beta j} U^*_{\beta i} U^*_{\alpha j} U_{\alpha i}) \sin^2(\Delta m^2_{ji} L/4E)$$

$$+2 \sum_{j>i} \Im(U_{\beta j} U^*_{\beta i} U^*_{\alpha j} U_{\alpha i}) \sin(\Delta m^2_{ji} L/2E),$$

For distant sources, the sinusoidal variations should be averaged so that

$$P_{\alpha\beta} = \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2$$

WHAT CAN WE LEARN FROM THE DETECTION OF THESE NEUTRINOS?

Reconstructing the neutrino flavor ratio at the source

$$\begin{pmatrix} \phi(\nu_e) \\ \phi(\nu_\mu) \\ \phi(\nu_\tau) \end{pmatrix} = \begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{\mu e} & P_{\mu\mu} & P_{\mu\tau} \\ P_{\tau e} & P_{\tau\mu} & P_{\tau\tau} \end{pmatrix} \begin{pmatrix} \phi_0(\nu_e) \\ \phi_0(\nu_\mu) \\ \phi_0(\nu_\tau) \end{pmatrix}$$

Standard neutrino oscillations

Measured flux Φ

Source flux Φ_0

$$P_{\alpha\beta} \equiv P(\nu_\beta \to \nu_\alpha) = \sum_{i=1}^{3} |U_{\beta i}|^2 |U_{\alpha i}|^2, \text{ where } \nu_\alpha = U^*_{\alpha i} \nu_i$$

Flavor Eigenstate

Mass Eigenstate

$\boldsymbol{U_{\alpha i}}$ contains 3 mixing angles--0_{12}, 0_{23}, and 0_{13}
one CP phase δ

Reconstructing the neutrino flavor ratio at the source--continued

- How well can we distinguish astrophysical sources with different neutrino flavor ratio, assuming three flavor neutrino oscillations?
- This depends on our understanding of neutrino mixing parameters and flavor discrimination capabilities in neutrino telescopes.
- $\sin^2\theta_{23}=0.386$, $\sin^2\theta_{12}=0.307$, $\sin^2\theta_{13}=0.0241$, $\delta_{cp}=1.08\pi$ Best fit for normal mass hierarchy

  G. L. Fogli *et al.* Phys. Rev. D 86, 013012 (2012).

$$P = \begin{pmatrix} 0.55 & 0.24 & 0.21 \\ 0.24 & 0.41 & 0.35 \\ 0.21 & 0.35 & 0.44 \end{pmatrix} \qquad P_{TBM} = \begin{pmatrix} 5/9 & 2/9 & 2/9 \\ 2/9 & 7/18 & 7/18 \\ 2/9 & 7/18 & 7/18 \end{pmatrix}$$

Second row (column) and third row (column) approximately the same $\Rightarrow \mu\tau$ symmetry

New basis

$$\Phi_a = R_{ab} \Phi_{0,b}$$

$$\Phi_1 = \frac{1}{3}\left(\phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau)\right)$$

$$\Phi_2 = \frac{1}{2}\left(\phi(\nu_\tau) - \phi(\nu_e)\right)$$

$$\Phi_3 = \frac{1}{3}\left(\phi(\nu_\mu) - (\phi(\nu_e) + \phi(\nu_\tau))/2\right)$$

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.285 & 0.165 \\ 0 & 0.055 & 0.115 \end{pmatrix} \qquad R_{TBM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/4 & 1/4 \\ 0 & 1/12 & 1/12 \end{pmatrix}$$

Φ_1 is just the normalization.
Measuring Φ_2 requires the tau neutrino identification ---rather challenging.
Measuring Φ_3 amounts to separating muon neutrino from the rest—this can be done by measuring track to shower event ratio for E_ν up to few tens of PeV.

J. F. Beacom *et al.* Phys. Rev. D 2003, arXiv: hep-ph/0307027v3
W. Winter, Phys. Rev. D 74, 033015 (2006).

Qualitatively, the reconstruction of initial flavor ratio is equivalent to inverting the matrix *R*.

$$R^{-1}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 4.85 & -6.96 \\ 0 & -2.32 & 12.0 \end{pmatrix}$$

K. C. Lai, G. L. Lin and T. C. Liu, Phys. Rev. D 80, 103005 (2009)

K. C. Lai, G. L. Lin and T. C. Liu, Phys. Rev. D 82, 103003 (2010)

large number!

$$\Phi_a=R_{ab}\Phi_{0,b} \qquad \Phi_{0,c}=R^{-1}{}_{cd}\Phi_d$$

$$\Phi_1=\frac{1}{3}\left(\phi(\nu_e)+\phi(\nu_\mu)+\phi(\nu_\tau)\right)$$

$$\Phi_2=\frac{1}{2}\left(\phi(\nu_\tau)-\phi(\nu_e)\right)$$

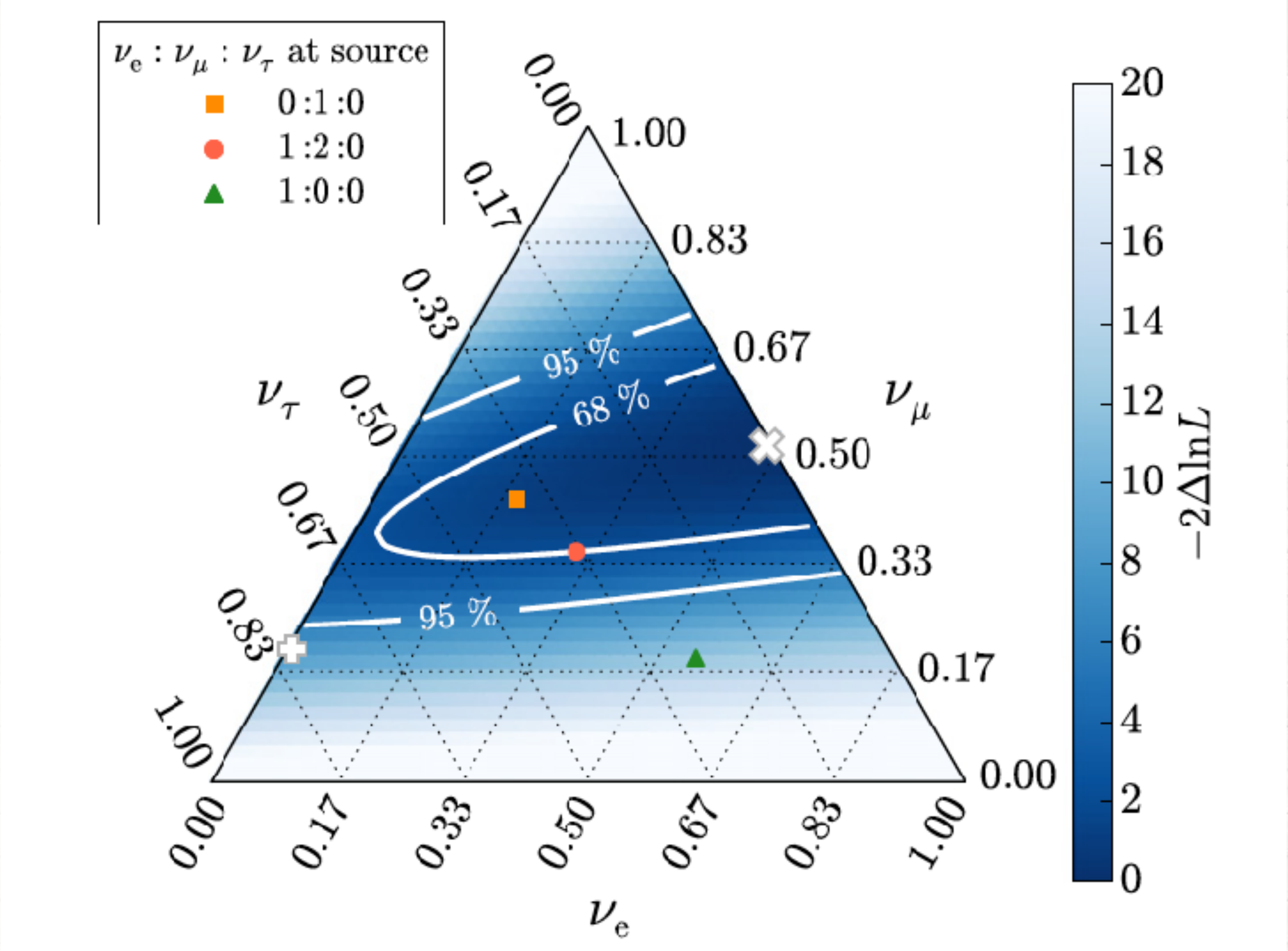
$$\Phi_3=\frac{1}{3}\left(\phi(\nu_\mu)-(\phi(\nu_e)+\phi(\nu_\tau))/2\right)$$

For 15% accuracy on measuring shower to track ratio (~100 events), the electron neutrino fraction of a pion source is reconstructed to be

$0 \leq \phi_{0,e} \leq 0.67$ -----compared to the true value 0.33

True fraction (0.33,0.67,0)

Disfavor (1,0,0)!



E_ν is between 25 TeV and 2.8 PeV (1:0:0) is disfavoured.

M. G. Aartsen et al. [IceCube Collaboration], Astrophys. J. 809, no. 1, 98 (2015).

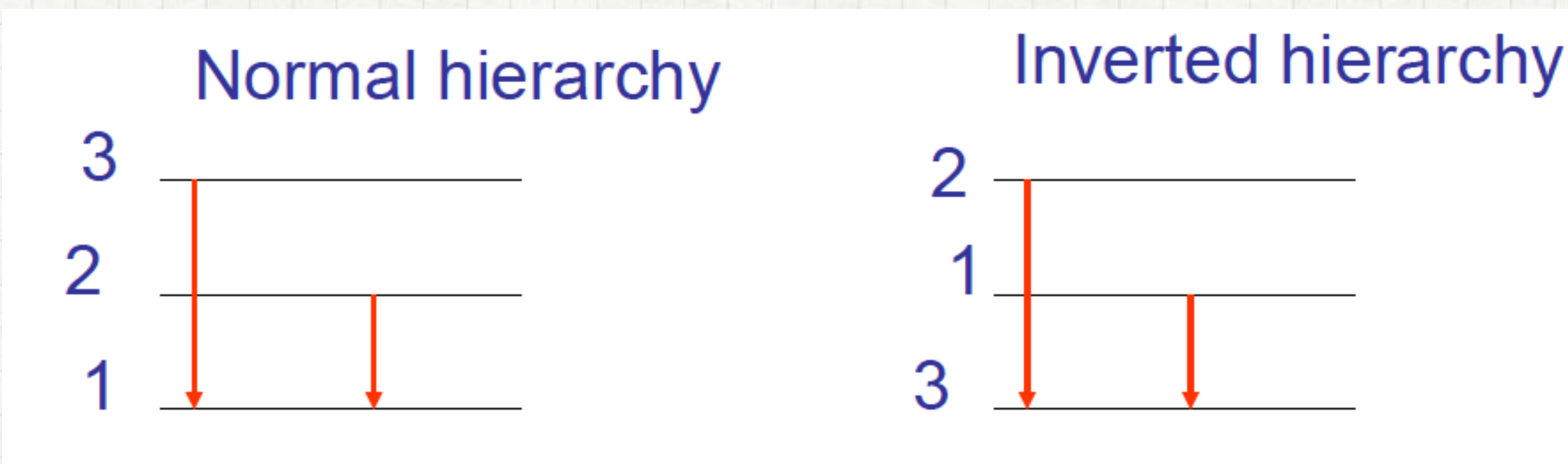
Understanding the flavour transition mechanisms during neutrino propagations from the source to terrestrial detector

$$P_{\alpha\beta} = \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2$$

Standard 3-flavour transition

β $\overline{\quad i = 1, 2, 3 \quad}$ α

If neutrino mass eigenstates are not stable…



Normal hierarchy

$$P_{\alpha\beta} = |U_{\alpha 1}|^2 \left(\sum_{i=1}^{3} |U_{\beta i}|^2 \right) = |U_{\alpha 1}|^2$$

The flavor fraction on the Earth Independent of flavor fraction at the source

$$(|U_{e1}|^2, |U_{\mu 1}|^2, |U_{\tau 1}|^2) \approx (0.67, 0.17, 0.17)$$

Disfavored by more than 2σ

Inverted hierarchy

$$P_{\alpha\beta} = |U_{\alpha 3}|^2 \left(\sum_{i=1}^{3} |U_{\beta i}|^2 \right) = |U_{\alpha 3}|^2$$

If neutrino Hamiltonian contains additional beyond standard model terms, then we have the following generalization:

- **PMNS (Pontecorvo-Maki-Nakagawa-Sakata) matrix**

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U_{\alpha i} \to V_{\alpha i}$$

Hence $$P_{\alpha\beta} = \sum_{i=1}^{3} |V_{\alpha i}|^2 |V_{\beta i}|^2$$

Let us consider Lorentz violation as the example of new physics that modifies the PMNS matrix

CONSTRAINING THE MASS SCALE OF A LORENTZ VIOLATION HAMILTONIAN WITH THE MEASUREMENT OF ASTROPHYSICAL NEUTRINO FLAVOR FRACTIONS

K.-C. Lai, W.-H. Lai and G.-L. Lin, Phys. Rev. D 96 (2017) no. 11, 115026

- Violations of Lorentz symmetry could arise in Planck scale physics

V. A. Kostelecky and S. Samuel, Phys. Rev. D 39, 683 (1989)

V. A. Kostelecky and R. Potting, Nucl. Phys. B 359, 545 (1991)

- The effects of Lorentz violations (LV) to neutrino oscillations have been studied before

V. A. Kostelecky and M. Mewes, Phys. Rev. D 69, 016005 (2004)

V. A. Kostelecky and M. Mewes, Phys. Rev. D 70, 031902 (2004)

V. A. Kostelecky and M. Mewes, Phys. Rev. D 70, 076002 (2004)

- The standard model neutrino Hamiltonian in vacuum

$$H_{\mathrm{SM}} = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} U^\dagger / 2E$$

U : PMNS matrix

H_{SM} behaves as $1/E$

- With Lorentz violation

$$H = H_{\mathrm{SM}} + H_{\mathrm{LV}}$$

H_{LV} contains E^0 and E^1 terms

- We shall study the LV effects with high energy astrophysical neutrino source. The neutrino flavor transition probability this case is

$$P(\nu_\alpha \to \nu_\beta) = |V_{\alpha i}|^2 |V_{\beta i}|^2,$$

where *V* is the matrix that diagonalizes the full Hamiltonian

$$H = H_{\rm SM} + H_{\rm LV}$$

Diagonalized by *V* (→ H)

Diagonalized by *U* (→ $H_{\rm SM}$)

$H_{\rm SM}$: $1/E$ $H_{\rm LV}$: E^0 and E^1

- *V* approaches to PMNS matrix *U* for $H_{\rm LV}$=0.
- When the neutrino energy is sufficiently high, the structure of *V* is dictated by $H_{\rm LV}$.

We shall focus on pion source from *pp* collisions, i.e.,

$$\nu_e : \nu_\mu : \nu_\tau = 1:2:0 \quad \bar{\nu}_e : \bar{\nu}_\mu : \bar{\nu}_\tau = 1:2:0$$

Defining neutrino flavor fraction:

$$f^0_\alpha \equiv \Phi^0(\nu_\alpha)/(\Phi^0(\nu_e) + \Phi^0(\nu_\mu) + \Phi^0(\nu_\tau))$$

total flux of neutrinos and anti-neutrinos of flavor α at the source

Hence

$$(f^0_e, f^0_\mu, f^0_\tau) = (1/3,\ 2/3,\ 0)$$

$$f_\alpha \equiv \Phi(\nu_\alpha)/(\Phi(\nu_e) + \Phi(\nu_\mu) + \Phi(\nu_\tau))$$

total flux of neutrinos and anti-neutrinos
of flavor at the terrestrial detector

$$f_\alpha = P_{\alpha\beta} f^0_\beta \qquad P_{\alpha\beta} \equiv P(\nu_\beta \to \nu_\alpha)$$

With $(f^0_e, f^0_\mu, f^0_\tau) = (1/3,\ 2/3,\ 0)$

$$f_e = 1/3 + (P_{e\mu} - P_{e\tau})/3$$

A test of μτ symmetry breaking

$$f_\mu = 1/3 + (P_{\mu\mu} - P_{\mu\tau})/3$$

$$f_\tau = 1/3 + (P_{\mu\tau} - P_{\tau\tau})/3$$

μτ symmetry breaking effects are small in the standard model neutrino Hamiltonian

$$(P_{e\mu} - P_{e\tau}) = 2\epsilon \quad (P_{\mu\mu} - P_{\mu\tau}) = (P_{\mu\tau} - P_{\tau\tau}) = -\epsilon$$

Here $\epsilon = 2\cos 2\theta_{23}/9 + \sqrt{2}\sin\theta_{13}\cos\delta/9$

(1) Taking $\sin^2\theta_{12} = 1/3$
and keeping only the leading-order symmetry breaking terms

(2) Lorentz violating Hamiltonian may contain large μτ symmetry breaking effects

LV EFFECTS TO NEUTRINO FLAVOR TRANSITIONS

For neutrinos, the general form of LV Hamiltonian

$$H^{\nu}_{\rm LV} = \frac{p_\lambda}{E}\begin{pmatrix} a^\lambda_{ee} & a^\lambda_{e\mu} & a^\lambda_{e\tau} \\ a^{\lambda *}_{e\mu} & a^\lambda_{\mu\mu} & a^\lambda_{\mu\tau} \\ a^{\lambda *}_{e\tau} & a^{\lambda *}_{\mu\tau} & a^\lambda_{\tau\tau} \end{pmatrix} - \frac{p^\rho p^\lambda}{E}\begin{pmatrix} c^{\rho\lambda}_{ee} & c^{\rho\lambda}_{e\mu} & c^{\rho\lambda}_{e\tau} \\ c^{\rho\lambda *}_{e\mu} & c^{\rho\lambda}_{\mu\mu} & c^{\rho\lambda}_{\mu\tau} \\ c^{\rho\lambda *}_{e\tau} & c^{\rho\lambda *}_{\mu\tau} & c^{\rho\lambda}_{\tau\tau} \end{pmatrix}$$

For rotationally invariant LV effects

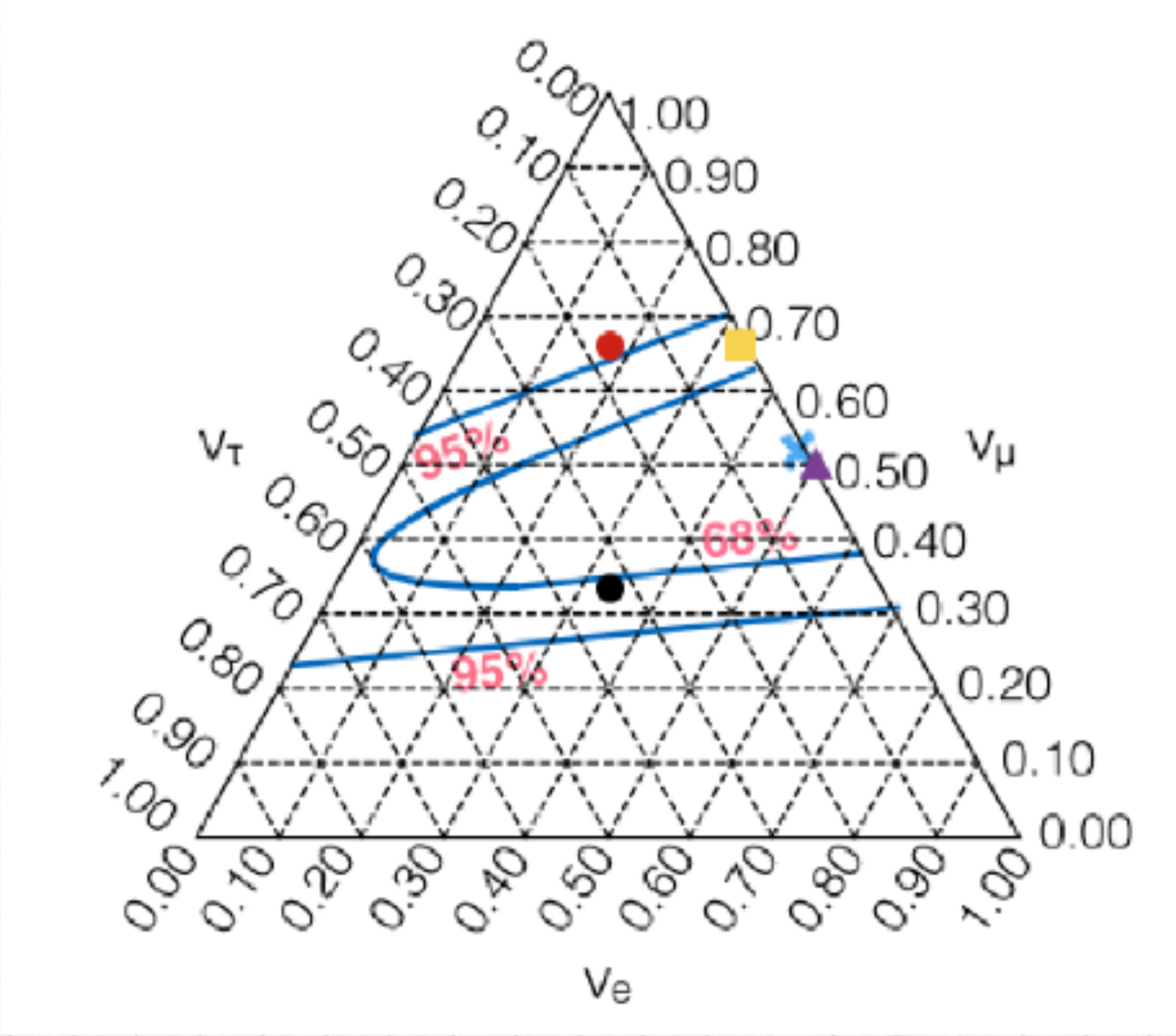
$$H^{\nu}_{\rm LV} = \begin{pmatrix} a^T_{ee} & a^T_{e\mu} & a^T_{e\tau} \\ a^{T*}_{e\mu} & a^T_{\mu\mu} & a^T_{\mu\tau} \\ a^{T*}_{e\tau} & a^{T*}_{\mu\tau} & a^T_{\tau\tau} \end{pmatrix} - \frac{4E}{3}\begin{pmatrix} c^{TT}_{ee} & c^{TT}_{e\mu} & c^{TT}_{e\tau} \\ c^{TT*}_{e\mu} & c^{TT}_{\mu\mu} & c^{TT}_{\mu\tau} \\ c^{TT*}_{e\tau} & c^{TT*}_{\mu\tau} & c^{TT}_{\tau\tau} \end{pmatrix}$$

$$H^{\bar\nu}_{\rm LV} = -\begin{pmatrix} a^T_{ee} & a^T_{e\mu} & a^T_{e\tau} \\ a^{T*}_{e\mu} & a^T_{\mu\mu} & a^T_{\mu\tau} \\ a^{T*}_{e\tau} & a^{T*}_{\mu\tau} & a^T_{\tau\tau} \end{pmatrix}^* - \frac{4E}{3}\begin{pmatrix} c^{TT}_{ee} & c^{TT}_{e\mu} & c^{TT}_{e\tau} \\ c^{TT*}_{e\mu} & c^{TT}_{\mu\mu} & c^{TT}_{\mu\tau} \\ c^{TT*}_{e\tau} & c^{TT*}_{\mu\tau} & c^{TT}_{\tau\tau} \end{pmatrix}^*$$

Sun-centered celestial equatorial frame

$(T,\ X,\ Y,\ Z)$ Let us first focus on $a^T_{\alpha\beta}$

LORENTZ VIOLATIONS AND CURRENT ICECUBE RESULTS ON ASTROPHYSICAL NEUTRINO FLAVOR COMPOSITIONS



E_ν is between 25 TeV and 2.8 PeV

$H_{SM} \approx \Delta m^2_{31}/2E_\nu$

Hence H_{SM} is between

$$5 \times 10^{-26}\ \text{GeV}$$

and

$$4.5 \times 10^{-28}\ \text{GeV}$$

M. G. Aartsen et al. [IceCube Collaboration], Astrophys. J. 809, no. 1, 98 (2015)

Can Lorentz violation play role in this data?

CURRENT BOUNDS ON LORENTZ VIOLATION PARAMETERS SUPER-KAMIOKANDE MEASUREMENTS

LV parameter		Limit at 95% C.L.	Best fit	No LV $\Delta\chi^2$	Previous limit
$e\mu$	$\mathrm{Re}(a^T)$	1.8×10^{-23} GeV	1.0×10^{-23} GeV	1.4	4.2×10^{-20} GeV [61]
	$\mathrm{Im}(a^T)$	1.8×10^{-23} GeV	4.6×10^{-24} GeV		
	$\mathrm{Re}(c^{TT})$	8.0×10^{-27}	1.0×10^{-28}	0.0	9.6×10^{-20} [61]
	$\mathrm{Im}(c^{TT})$	8.0×10^{-27}	1.0×10^{-28}		
$e\tau$	$\mathrm{Re}(a^T)$	4.1×10^{-23} GeV	2.2×10^{-24} GeV	0.0	7.8×10^{-20} GeV [62]
	$\mathrm{Im}(a^T)$	2.8×10^{-23} GeV	1.0×10^{-28} GeV		
	$\mathrm{Re}(c^{TT})$	9.3×10^{-25}	1.0×10^{-28}	0.3	1.3×10^{-17} [62]
	$\mathrm{Im}(c^{TT})$	1.0×10^{-24}	3.5×10^{-25}		
$\mu\tau$	$\mathrm{Re}(a^T)$	6.5×10^{-24} GeV	3.2×10^{-24} GeV	0.9	...
	$\mathrm{Im}(a^T)$	5.1×10^{-24} GeV	1.0×10^{-28} GeV		
	$\mathrm{Re}(c^{TT})$	4.4×10^{-27}	1.0×10^{-28}	0.1	...
	$\mathrm{Im}(c^{TT})$	4.2×10^{-27}	7.5×10^{-28}		

$$H_{\mathrm{SM}} < \; 5 \times 10^{-26} \; \mathrm{GeV}$$

K. Abe *et al.* [Super-Kamiokande Collaboration], Phys. Rev. D 91, no. 5, 052003 (2015).

Significant room for H_{LV} to play an important role

SPECIAL STRUCTURES OF $H_{\rm LV}$ AND THE RESULTING ASTROPHYSICAL NEUTRINO FLAVOR COMPOSITION ON EARTH

(A) only $a^T_{e\mu}(a^{T*}_{e\mu})$ are non-vanishing

$$H^\nu_{\rm LV} = \begin{pmatrix} 0 & a^T_{e\mu} & 0 \\ a^{T*}_{e\mu} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad P = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

For anti-neutrinos,

$$a^T_{e\mu} \to -a^{T*}_{e\mu}$$

Large breaking of $\mu\tau$ symmetry

$$(P_{e\mu} - P_{e\tau}) = (P_{\mu\mu} - P_{\mu\tau}) = 1/2$$

$$(P_{\mu\tau} - P_{\tau\tau}) = -1$$

$$(f_e, f_\mu, f_\tau) = (1/2, 1/2, 0)$$

SPECIAL STRUCTURES OF H_{LV} AND THE RESULTING ASTROPHYSICAL NEUTRINO FLAVOR COMPOSITION ON EARTH

(B) only $a^T_{e\tau}(a^{T*}_{e\tau})$ are non-vanishing

$$H^\nu_{\rm LV} = \begin{pmatrix} 0 & 0 & a^T_{e\tau} \\ 0 & 0 & 0 \\ a^{T*}_{e\tau} & 0 & 0 \end{pmatrix} \quad P = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{pmatrix}$$

For anti-neutrinos,

$$a^T_{e\tau} \to -a^{T*}_{e\tau}$$

$$(P_{e\mu} - P_{e\tau}) = (P_{\mu\tau} - P_{\tau\tau}) = -1/2$$

$$(P_{\mu\mu} - P_{\mu\tau}) = 1$$

$$(f_e,\ f_\mu,\ f_\tau) = (1/6,\ 2/3,\ 1/6)$$

Large breaking of $\mu\tau$ symmetry

(C) only $a^T_{\mu\tau}(a^{T*}_{\mu\tau})$ are non-vanishing

$$H^{\nu}_{\rm LV} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & a^T_{\mu\tau} \\ 0 & a^{T*}_{\mu\tau} & 0 \end{pmatrix} \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{pmatrix}$$

For anti-neutrinos,

$$a^T_{\mu\tau} \to -a^{T*}_{\mu\tau}$$

$$(P_{e\mu} - P_{e\tau}) = (P_{\mu\mu} - P_{\mu\tau}) = (P_{\mu\tau} - P_{\tau\tau}) = 0$$

μτ symmetric case

$$(f_e,\ f_\mu,\ f_\tau) = (1/3,\ 1/3,\ 1/3)$$

SPECIAL STRUCTURES OF $H_{\rm LV}$ AND THE RESULTING ASTROPHYSICAL NEUTRINO FLAVOR COMPOSITION ON EARTH

(D) only $a^T_{\mu\mu}, a^T_{\tau\tau}$ are non-vanishing, $a^T_{\mu\mu} \neq a^T_{\tau\tau}$

$$H^\nu_{\rm LV} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & a^T_{\mu\mu} & 0 \\ 0 & 0 & a^T_{\tau\tau} \end{pmatrix} \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

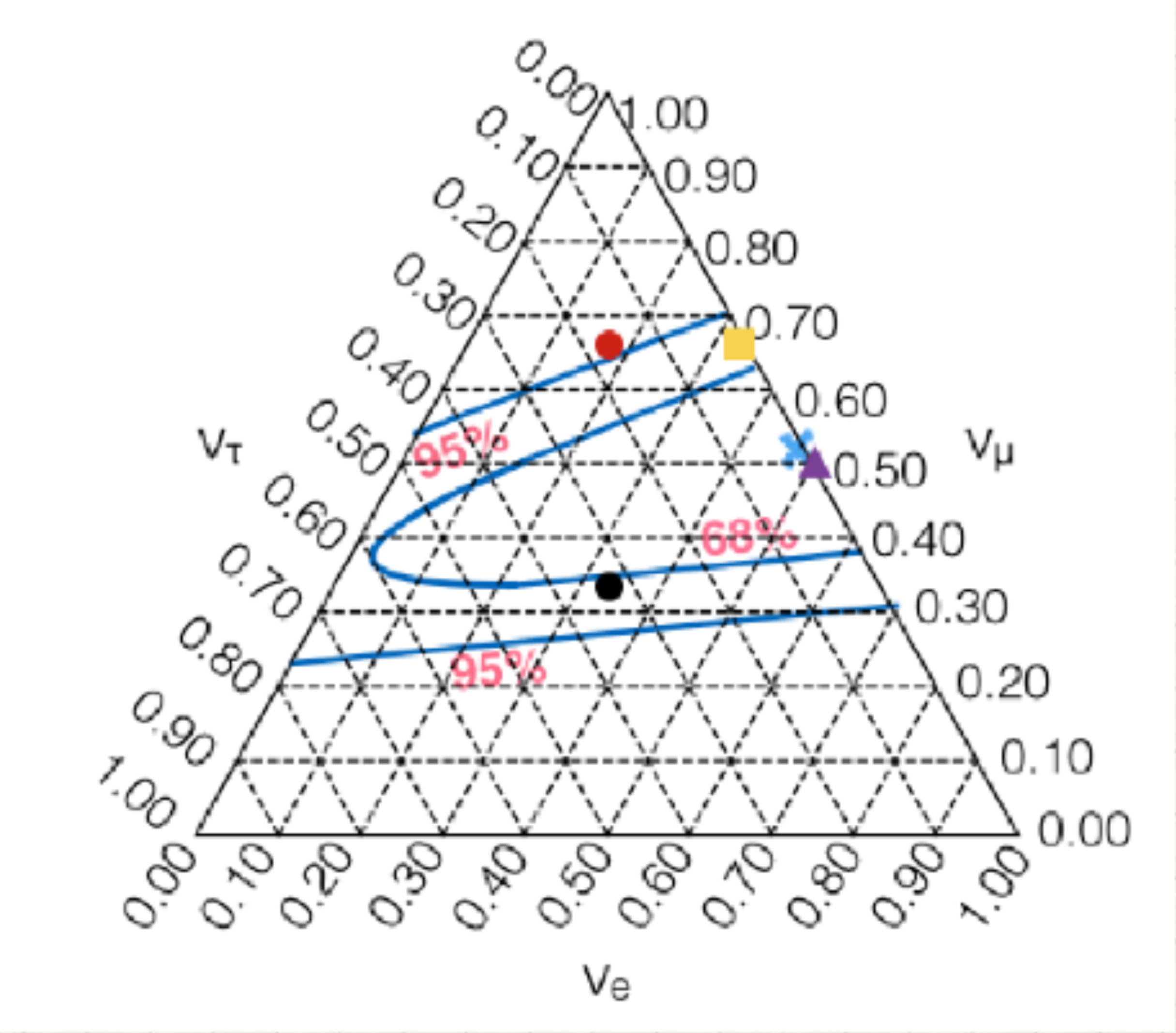
$$(P_{e\mu} - P_{e\tau}) = 0$$

Large breaking of μτ symmetry

$$(P_{\mu\mu} - P_{\mu\tau}) = 1 \quad (P_{\mu\tau} - P_{\tau\tau}) = -1$$

$$(f_e,\ f_\mu,\ f_\tau) = (1/3,\ 2/3,\ 0)$$

COMPARISONS OF SPECIAL CASES WITH RECENT ICECUBE MEASUREMENT OF ASTROPHYSICAL NEUTRINO FLAVOR COMPOSITION



Red: $a^T_{e\tau}, a^{T*}_{e\tau} \neq 0$

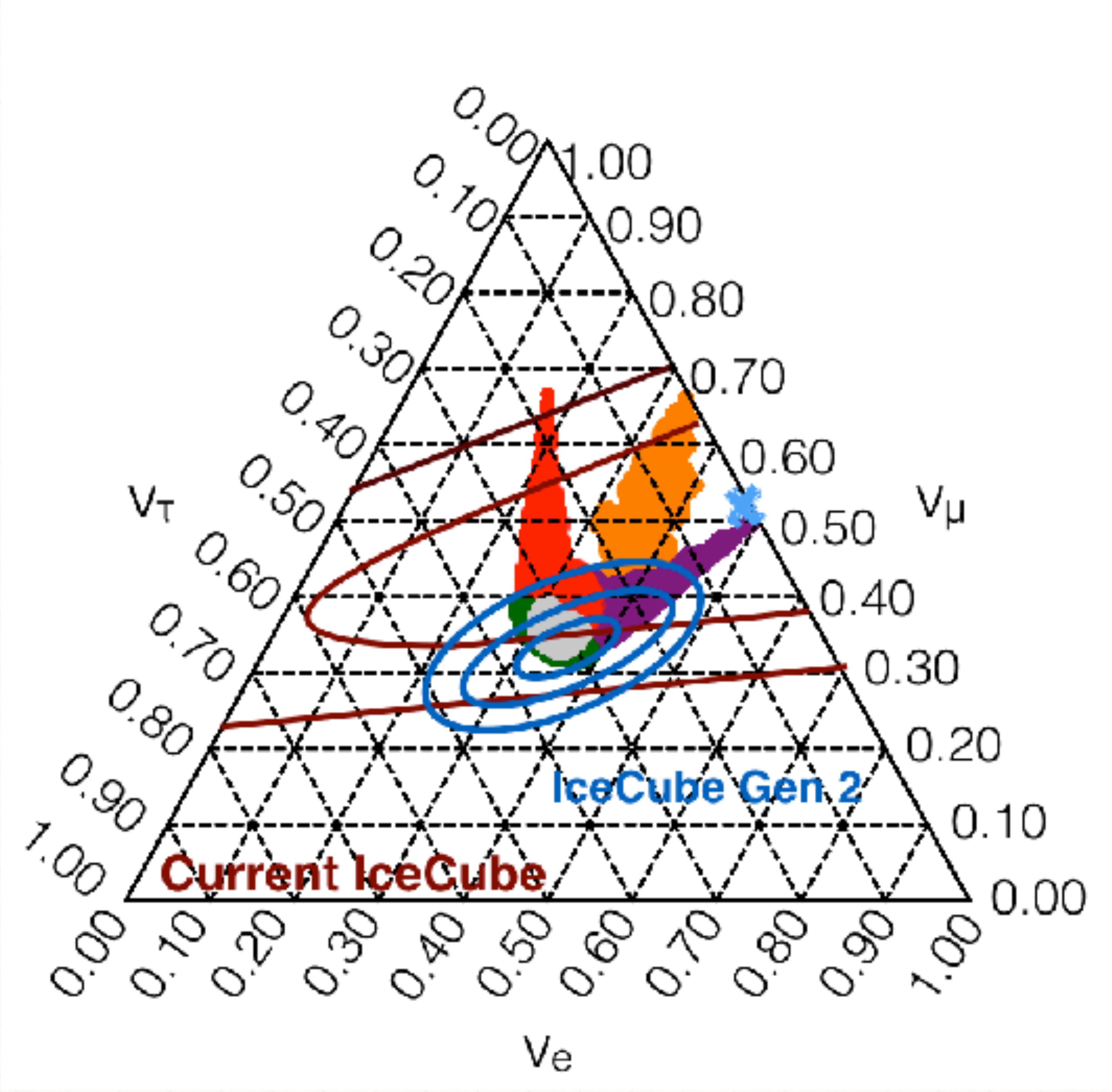
Yellow: $a^T_{\mu\mu}, a^T_{\tau\tau} \neq 0$

Purple: $a^T_{e\mu}, a^{T*}_{e\mu} \neq 0$

Black: $a^T_{\mu\tau}, a^{T*}_{\mu\tau} \neq 0$

All cases fall into 2**σ** region as other elements grow from zero

What happens if we consider the full Hamiltonian $H_{\rm SM}^{\nu,\bar{\nu}} + H_{\rm LV}^{\nu,\bar{\nu}}$



Red: $a^{T}_{e\tau}, a^{T*}_{e\tau} \neq 0$

Orange: $a^{T}_{\mu\mu}, a^{T}_{\tau\tau} \neq 0$

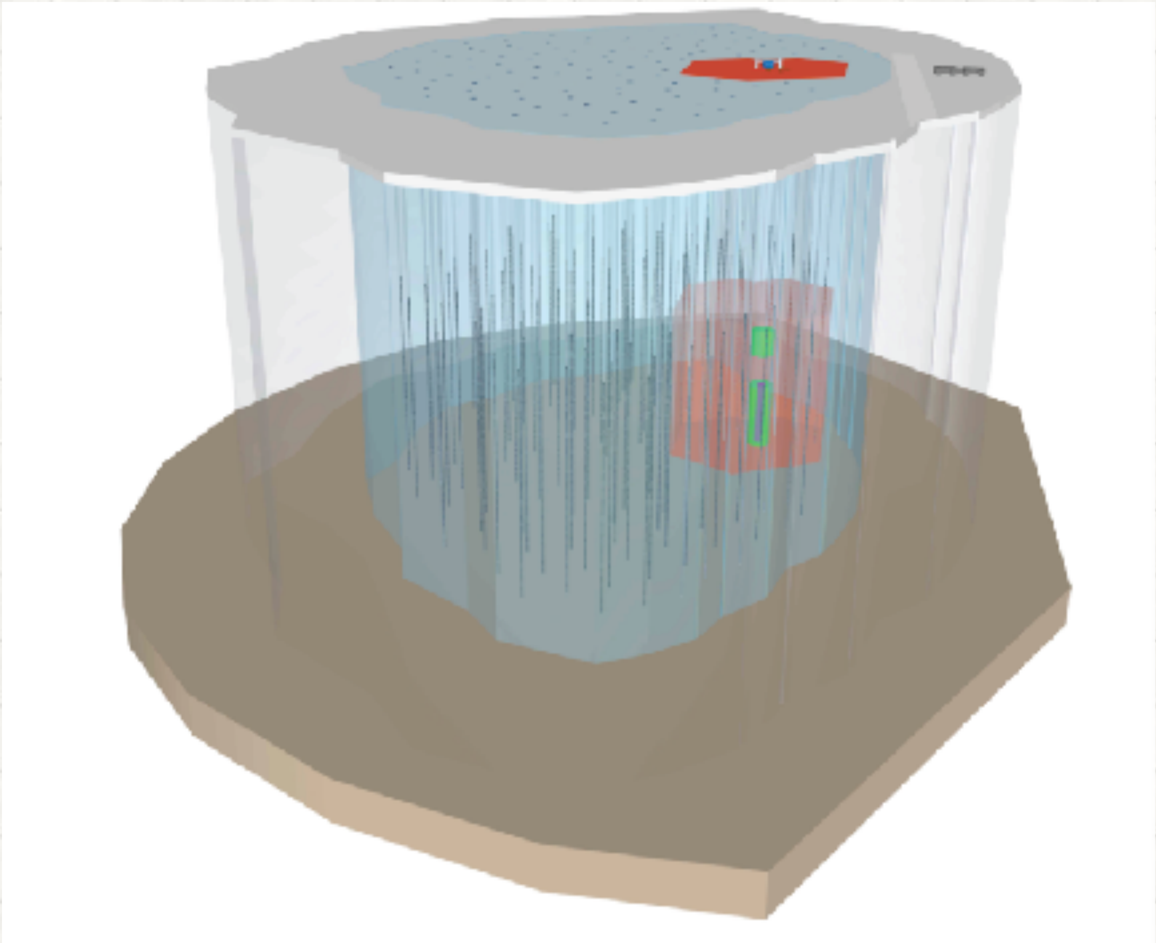
Purple: $a^{T}_{e\mu}, a^{T*}_{e\mu} \neq 0$

Gray: $a^{T}_{\mu\tau}, a^{T*}_{\mu\tau} \neq 0$

$|a^{T}_{\alpha\beta}|$ in each scenario is varied from 0 To SK 95% C.L. Limit

Clearly IceCube Gen2 is needed to constrain LV Hamiltonian

4. ICECUBE GEN2 AND ITS POTENTIAL OF CONSTRAINING LORENTZ VIOLATION HAMILTONIAN



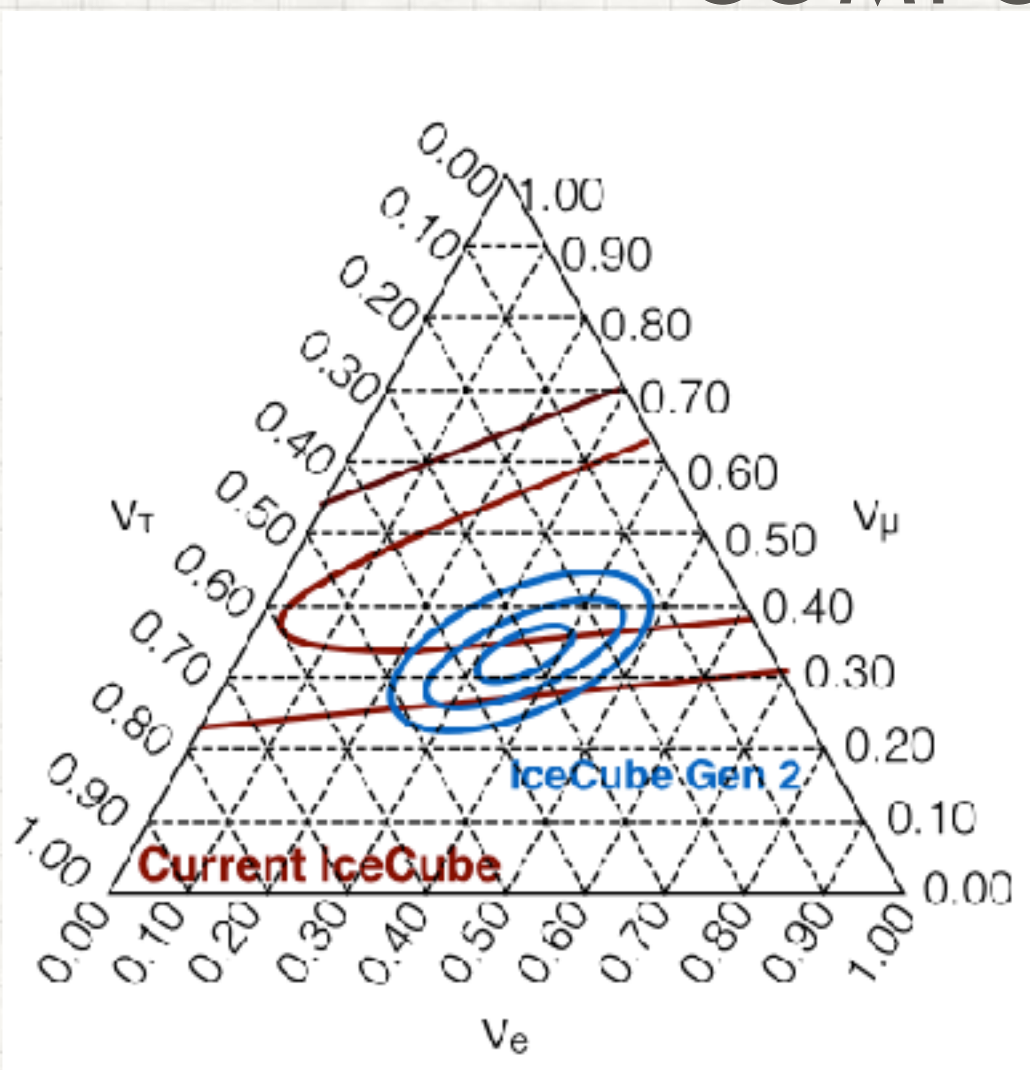
IceCube Collaboration (M.G. Aartsen (Adelaide U.) et al.), arXiv:1412.5106

~10 km^3 instrumented volume

~250 m spacing of photo sensors

(1) A possible IceCube-Gen2 configuration.
(2) IceCube, in red, and the infill sub-detector DeepCore, in green.
(3) blue volume shows the full instrumented next-generation detector, with PINGU displayed in grey as a denser infill extension within DeepCore.

SENSITIVITIES OF ICECUBE-GEN2 ON ASTROPHYSICAL NEUTRINO FLAVOR COMPOSITIONS



$$\Phi_\nu(E) = \Phi_0 \left(\frac{100\ \mathrm{TeV}}{E}\right)^\gamma$$

$$\gamma = 2.2 \pm 0.2$$

$$\Phi_0 = (5.1 \pm 1.8) \times 10^{-18} \mathrm{GeV}^{-1} \mathrm{cm}^{-2} \mathrm{s}^{-1} \mathrm{sr}^{-1}$$

Pion source from *pp* collision is assumed

$$E_{\mathrm{th}} = 100\ \mathrm{TeV}$$

10 years of exposure

$$1\sigma,\ 2\sigma,\ \text{and } 3\sigma \text{ regions}$$

I. M. Shoemaker and K. Murase, Phys. Rev. D 93 085004 (2016) IceCube Gen2 regions

Consider general LV Hamiltonian

$$H^{\nu}_{\mathrm{LV}} = H^{\nu}_1 + H^{\nu}_2$$

$$H^{\nu}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & a^T_{\mu\mu} & a^T_{\mu\tau} \\ 0 & a^{T*}_{\mu\tau} & a^T_{\tau\tau} \end{pmatrix}, \qquad H^{\nu}_2 = \begin{pmatrix} 0 & a^T_{e\mu} & a^T_{e\tau} \\ a^{T*}_{e\mu} & 0 & 0 \\ a^{T*}_{e\tau} & 0 & 0 \end{pmatrix}.$$

The structure of H_{SM} in the limit: $\theta_{23} = 45^\circ,\ \theta_{13} = 0$

$$H_{\mathrm{SM}} = \begin{pmatrix} s^2_{12}\frac{\Delta m^2_{21}}{2E} & \frac{\sqrt{2}}{2}s_{12}c_{12}\frac{\Delta m^2_{21}}{2E} & -\frac{\sqrt{2}}{2}s_{12}c_{12}\frac{\Delta m^2_{21}}{2E} \\ \frac{\sqrt{2}}{2}s_{12}c_{12}\frac{\Delta m^2_{21}}{2E} & \frac{1}{2}(c^2_{12}\frac{\Delta m^2_{21}}{2E} + \frac{\Delta m^2_{31}}{2E}) & \frac{1}{2}(-c^2_{12}\frac{\Delta m^2_{21}}{2E} + \frac{\Delta m^2_{31}}{2E}) \\ -\frac{\sqrt{2}}{2}s_{12}c_{12}\frac{\Delta m^2_{21}}{2E} & \frac{1}{2}(-c^2_{12}\frac{\Delta m^2_{21}}{2E} + \frac{\Delta m^2_{31}}{2E}) & \frac{1}{2}(c^2_{12}\frac{\Delta m^2_{21}}{2E} + \frac{\Delta m^2_{31}}{2E}) \end{pmatrix}$$

It can be written as the sum of two distinctive structures.

Taking the example

$$H^{\nu}_{\rm LV} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & a^T_{\mu\mu} & a^T_{\mu\tau} \\ 0 & a^{T*}_{\mu\tau} & a^T_{\tau\tau} \end{pmatrix} = H^{\nu}_1$$

$$= \frac{a^T_{\mu\mu} + a^T_{\tau\tau}}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} a^T_{\mu\mu} + a^T_{\tau\tau} & 0 & 0 \\ 0 & a^T_{\tau\tau} - a^T_{\mu\mu} & -2a^T_{\mu\tau} \\ 0 & -2a^{T*}_{\mu\tau} & a^T_{\mu\mu} - a^T_{\tau\tau}) \end{pmatrix}$$

↑ Relevant

The relevant part of the Hamiltonian can be written as

$$H = H_{\rm SM} + H^{\nu}_{\rm LV}$$

Set E in H_{SM} to be 100 TeV

$$H^{\nu}_{\rm LV} = -M \begin{pmatrix} \gamma & 0 & 0 \\ 0 & \cos 2\alpha & e^{i\beta}\sin 2\alpha \\ 0 & e^{-i\beta}\sin 2\alpha & -\cos 2\alpha \end{pmatrix}$$

Here

$$M = \frac{1}{2}\sqrt{(a^T_{\tau\tau} - a^T_{\mu\mu})^2 + 4a^T_{\mu\tau}a^{T*}_{\mu\tau}}$$

$$\gamma = \frac{a^T_{\mu\mu} + a^T_{\tau\tau}}{\sqrt{(a^T_{\tau\tau} - a^T_{\mu\mu})^2 + 4a^T_{\mu\tau}a^{T*}_{\mu\tau}}}$$

$$\cos 2\alpha = \frac{a^T_{\tau\tau} - a^T_{\mu\mu}}{\sqrt{(a^T_{\tau\tau} - a^T_{\mu\mu})^2 + 4a^T_{\mu\tau}a^{T*}_{\mu\tau}}}$$

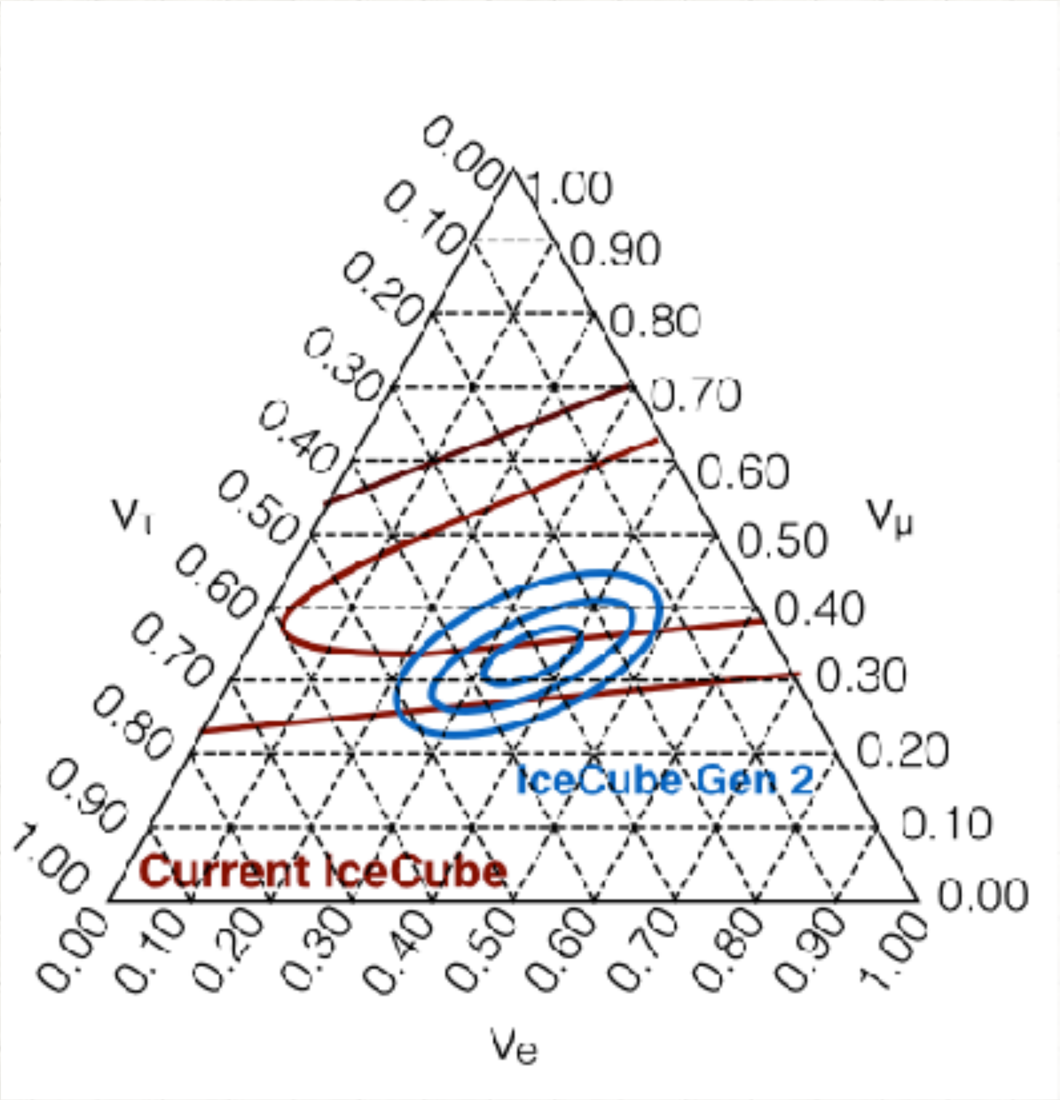
$$\sin 2\alpha = \frac{2|a^T_{\mu\tau}|}{\sqrt{(a^T_{\tau\tau} - a^T_{\mu\mu})^2 + 4a^T_{\mu\tau}a^{T*}_{\mu\tau}}}$$

$\mu\tau$ symmetry limit
$\alpha = \pi/4$

The constraint on M depends on α

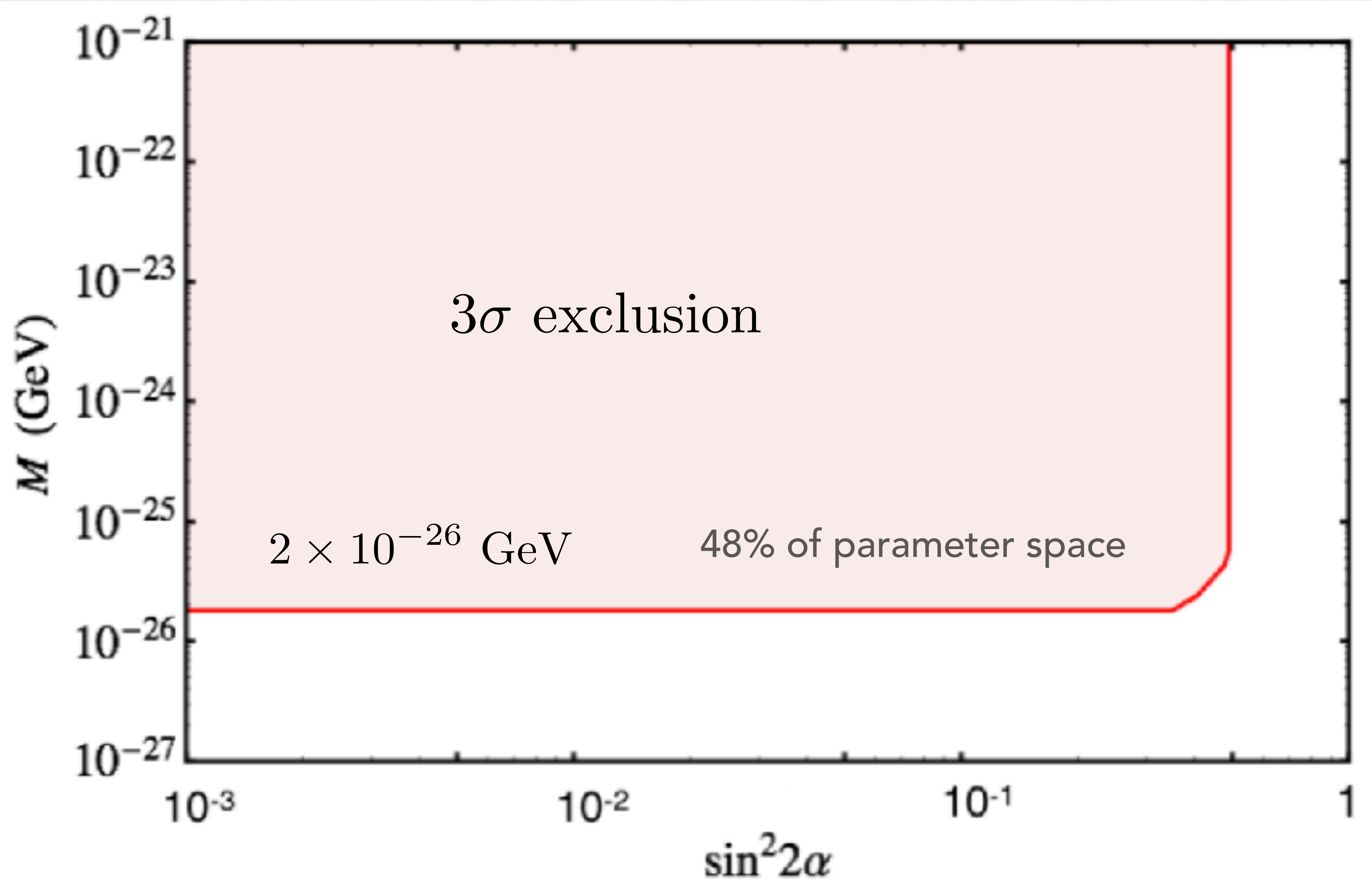
$$H = H_{\rm SM} + H^{\nu}_{\rm LV}$$

$$H^{\nu}_{\rm LV} = -M \begin{pmatrix} \gamma & 0 & 0 \\ 0 & \cos 2\alpha & e^{i\beta}\sin 2\alpha \\ 0 & e^{-i\beta}\sin 2\alpha & -\cos 2\alpha \end{pmatrix}$$



$$M \to -M, \beta \to -\beta \text{ for } \bar{\nu}$$

Increasing *M* until the predicted flavor fraction is out of the IceCube Gen2 3σ region.



Constraint valid up to $\sin^2 2\alpha = 0.46.$

K. Abe *et al.* [Super-Kamiokande Collaboration], Phys. Rev. D 91, no. 5, 052003 (2015).

LV parameter		Limit at 95% C.L.	Best fit	No LV $\Delta\chi^2$	Previous limit
$e\mu$	$\mathrm{Re}(a^T)$	1.8×10^{-23} GeV	1.0×10^{-23} GeV	1.4	4.2×10^{-20} GeV [61]
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	$\mathrm{Re}(c^{TT})$	8.0×10^{-27}	1.0×10^{-28}	0.0	9.6×10^{-20} [61]
	$\mathrm{Im}(c^{TT})$	8.0×10^{-27}	1.0×10^{-28}		
$e\tau$	$\mathrm{Re}(a^T)$	4.1×10^{-23} GeV	2.2×10^{-24} GeV	0.0	7.8×10^{-20} GeV [62]
	$\mathrm{Im}(a^T)$	2.8×10^{-23} GeV	1.0×10^{-28} GeV		
	$\mathrm{Re}(c^{TT})$	9.3×10^{-25}	1.0×10^{-28}	0.3	1.3×10^{-17} [62]
	$\mathrm{Im}(c^{TT})$	1.0×10^{-24}	3.5×10^{-25}		
$\mu\tau$	$\mathrm{Re}(a^T)$	6.5×10^{-24} GeV	3.2×10^{-24} GeV	0.9	...
	$\mathrm{Im}(a^T)$	5.1×10^{-24} GeV	1.0×10^{-28} GeV		
	$\mathrm{Re}(c^{TT})$	4.4×10^{-27}	1.0×10^{-28}	0.1	...
	$\mathrm{Im}(c^{TT})$	4.2×10^{-27}	7.5×10^{-28}		

M. G. Aartsen *et al.* [IceCube Collaboration], arXiv:1709.03434 [hep-ex].

99% C.L. bounds

$$\mathrm{Re}(a^T_{\mu\tau}) < 2.9 \times 10^{-24}\ \mathrm{GeV} \quad \mathrm{Im}(a^T_{\mu\tau}) < 2.9 \times 10^{-24}\ \mathrm{GeV}$$

$$\mathrm{Re}(c^{TT}_{\mu\tau}) < 3.9 \times 10^{-28} \quad \mathrm{Im}(c^{TT}_{\mu\tau}) < 3.9 \times 10^{-28}$$

$$H^{\nu}_{\rm LV} = H^{\nu}_{2} = \begin{pmatrix} 0 & a^{T}_{e\mu} & a^{T}_{e\tau} \\ a^{T*}_{e\mu} & 0 & 0 \\ a^{T*}_{e\tau} & 0 & 0 \end{pmatrix}$$

$\mu - \tau$ symmetry limit

$\rho = \pi/4$

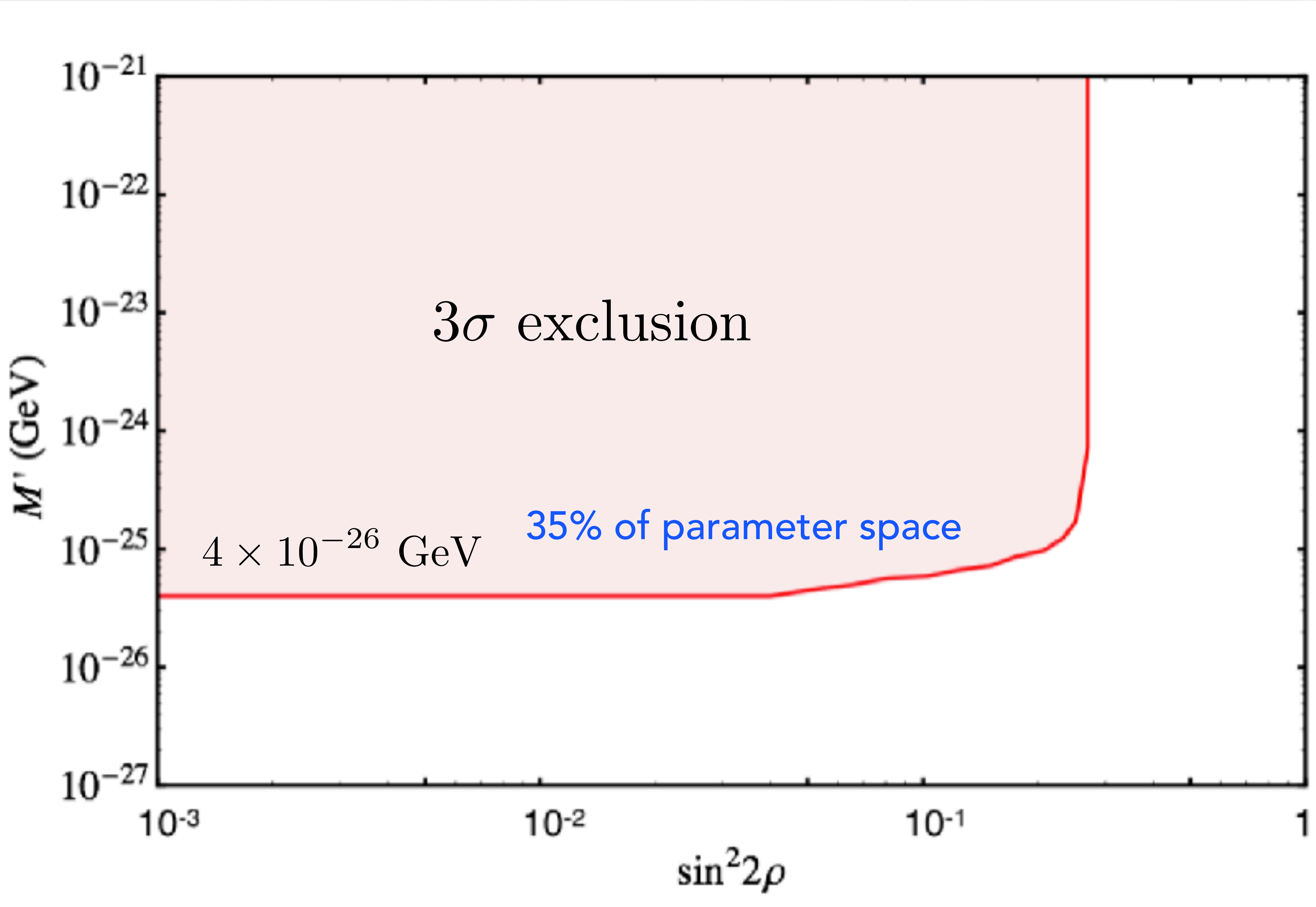
Maximal Symmetry Breaking

$\rho = 0$ or $\pi/2$

$$H^{\nu}_{2} = M' \begin{pmatrix} 0 & e^{i\sigma}\cos\rho & e^{i\lambda}\sin\rho \\ e^{-i\sigma}\cos\rho & 0 & 0 \\ e^{-i\lambda}\sin\rho & 0 & 0 \end{pmatrix}$$

$$M' = \sqrt{a^{T}_{e\mu}a^{T*}_{e\mu} + a^{T}_{e\tau}a^{T*}_{e\tau}},\ \cos\rho = |a^{T}_{e\mu}|/M',\ \sin\rho = |a^{T}_{e\tau}|/M'$$

For anti-neutrino Hamiltonian $M' \to -M'$, $\sigma \to -\sigma$, and $\lambda \to -\lambda$.

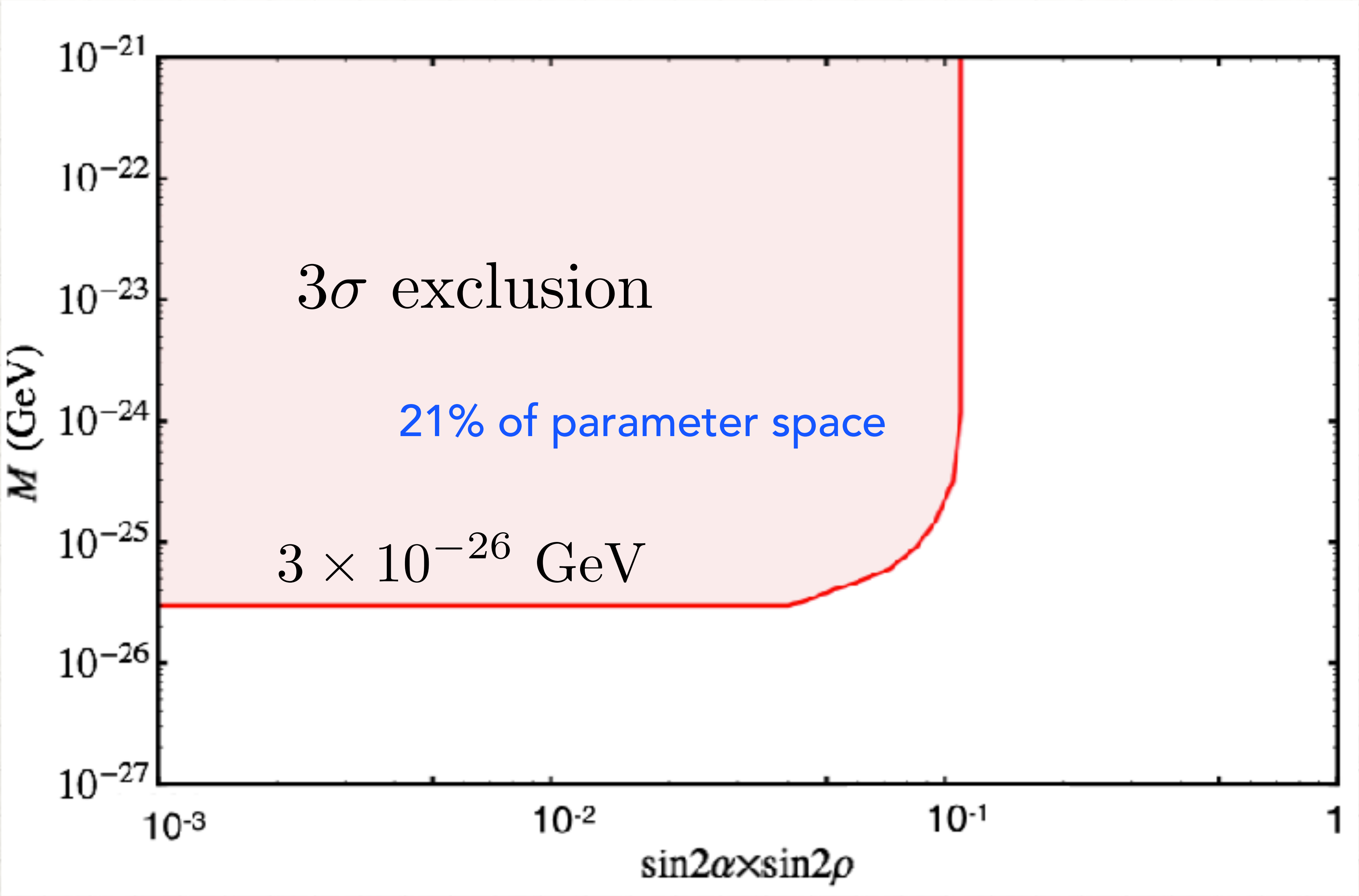


Constraint valid up to $\sin^2 2\rho = 0.27$

$$H^{\nu}_{\rm LV} = H^{\nu}_{1} + H^{\nu}_{2}$$

M *M′*

Assume *M*=*M′*



Constraint valid up to $\sin 2\alpha \times \sin 2\rho = 0.11$

$$H = H_{\rm SM} + H^{\nu}_{\rm LV}$$

Constraints CPT-even $c^{TT}_{\alpha\beta}$

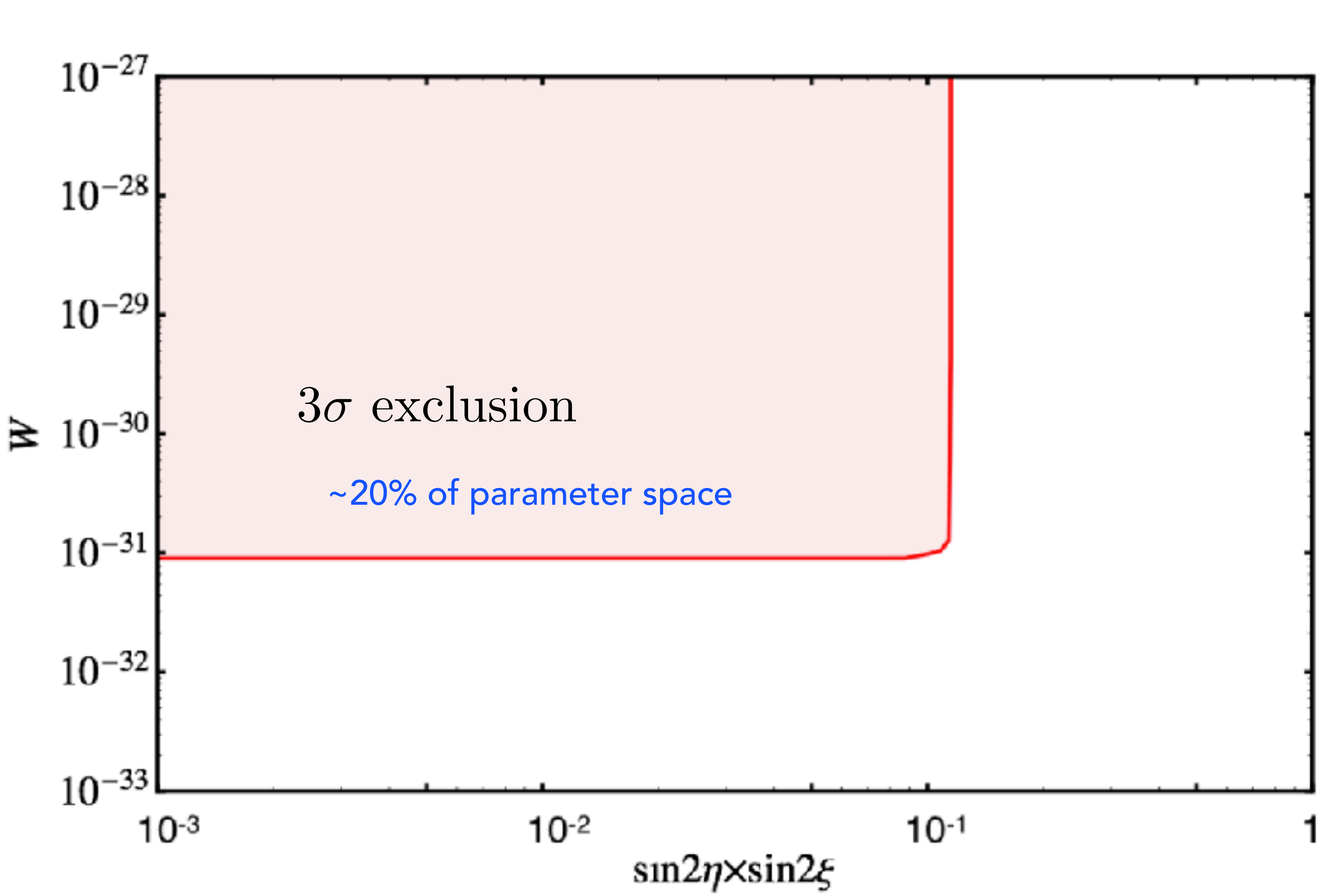
Recall

$$H^{\nu}_{\rm LV} = \begin{pmatrix} a^T_{ee} & a^T_{e\mu} & a^T_{e\tau} \\ a^{T*}_{e\mu} & a^T_{\mu\mu} & a^T_{\mu\tau} \\ a^{T*}_{e\tau} & a^{T*}_{\mu\tau} & a^T_{\tau\tau} \end{pmatrix} - \frac{4E}{3} \begin{pmatrix} c^{TT}_{ee} & c^{TT}_{e\mu} & c^{TT}_{e\tau} \\ c^{TT*}_{e\mu} & c^{TT}_{\mu\mu} & c^{TT}_{\mu\tau} \\ c^{TT*}_{e\tau} & c^{TT*}_{\mu\tau} & c^{TT}_{\tau\tau} \end{pmatrix}$$

$$H^{\bar{\nu}}_{\rm LV} = - \begin{pmatrix} a^T_{ee} & a^T_{e\mu} & a^T_{e\tau} \\ a^{T*}_{e\mu} & a^T_{\mu\mu} & a^T_{\mu\tau} \\ a^{T*}_{e\tau} & a^{T*}_{\mu\tau} & a^T_{\tau\tau} \end{pmatrix}^* - \frac{4E}{3} \begin{pmatrix} c^{TT}_{ee} & c^{TT}_{e\mu} & c^{TT}_{e\tau} \\ c^{TT*}_{e\mu} & c^{TT}_{\mu\mu} & c^{TT}_{\mu\tau} \\ c^{TT*}_{e\tau} & c^{TT*}_{\mu\tau} & c^{TT}_{\tau\tau} \end{pmatrix}^*$$

$$W \equiv \sqrt{(c^{TT}_{\tau\tau} - c^{TT}_{\mu\mu})^2 + 4c^{TT}_{\mu\tau}c^{TT*}_{\mu\tau}}/2 \qquad \sin 2\eta = 2|c^{TT}_{\mu\tau}|/\sqrt{(c^{TT}_{\tau\tau} - c^{TT}_{\mu\mu})^2 + 4c^{TT}_{\mu\tau}c^{TT*}_{\mu\tau}}$$

$$W' \equiv \sqrt{c^{TT}_{e\mu}c^{TT*}_{e\mu} + c^{TT}_{e\tau}c^{TT*}_{e\tau}} \qquad \sin\xi = |c^{TT}_{e\tau}|/W'$$


10^{-27}
10^{-28}
10^{-29}
10^{-30}
10^{-31}
10^{-32}
10^{-33}
W
3σ exclusion
~20% of parameter space
10^{-3}
10^{-2}
10^{-1}
1
$\sin 2\eta \times \sin 2\xi$

CONCLUSION

- We have discussed the mechanisms for generating the high energy astrophysical neutrinos.
- The detections of these neutrinos with the measurement of neutrino flavor fractions on Earth provide us some understanding about astrophysical source.
- We can also take advantage of the above detection to study non-standard physics that affects the flavor transition of astrophysical neutrinos during their propagations from the source to the terrestrial detector.
- We have shown that some scenarios of neutrino decays can already be constrained by the current IceCube data.
- We have seen that the current IceCube measurement on the astrophysical neutrino flavor fraction still not cannot constrain LV Hamiltonian better than the current SK measurement
- On the other hand, IceCube-Gen2 is expected to probe deeper than SK into LV Hamiltonian.