Exploring high energy hadronic scatterings by quantum computing

邢宏喜

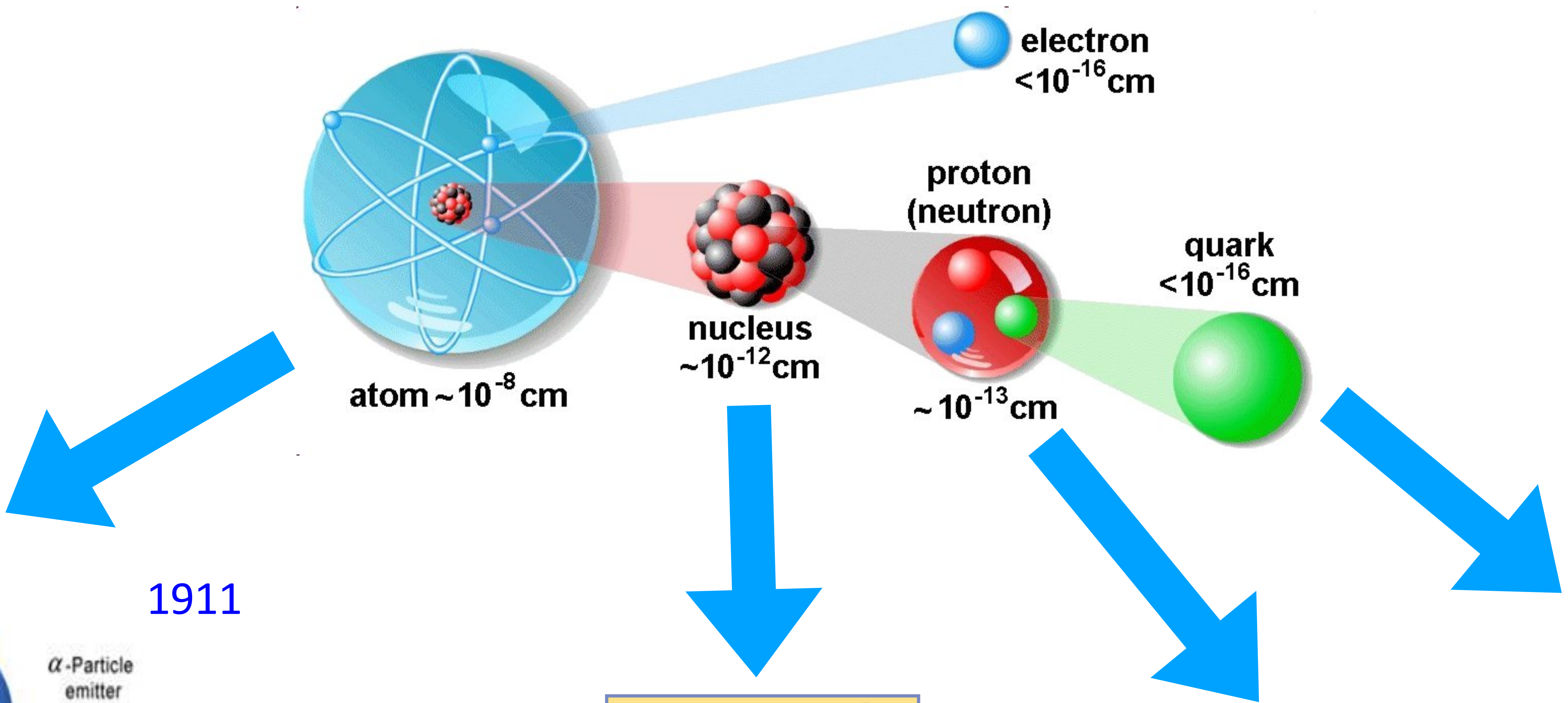
arXiv: 2301.04179, 2207.13258, 2106.03865

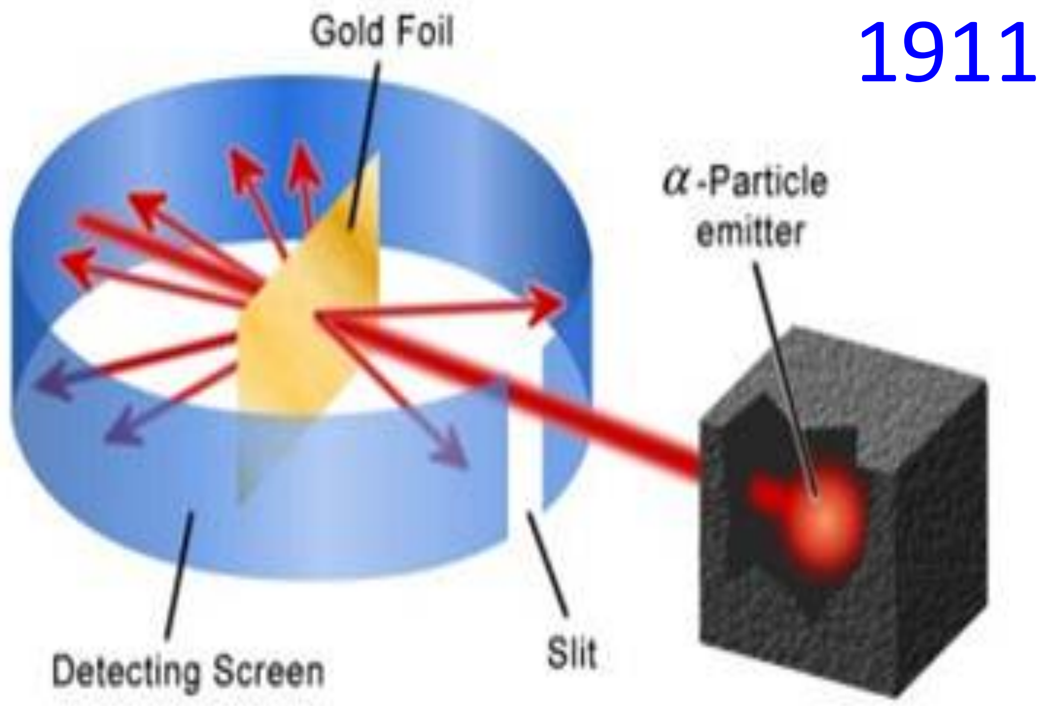
Peng Huanwu Center for Fundamental Theory，2023.11.30

Outline

- ✦ Introduction
- ✦ Simulate hadronic scatterings from quantum computing
  - ➡ parton distribution in hadron
  - ➡ partonic scatterings
  - ➡ hadronization
- ✦ Summary and outlook

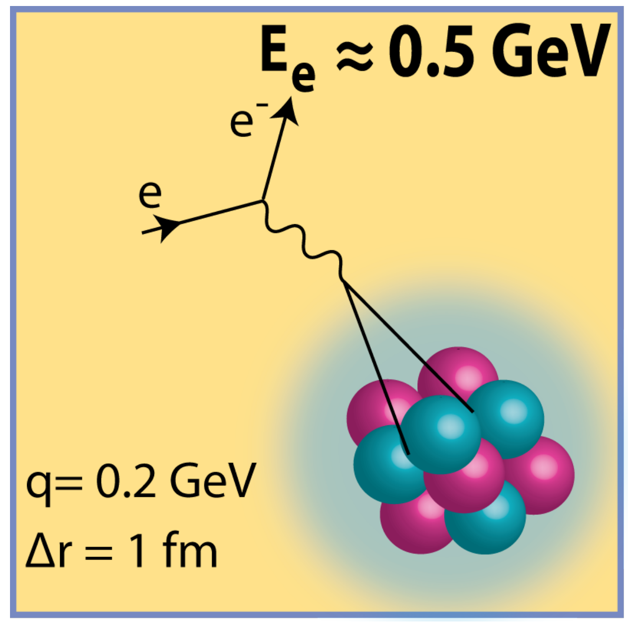
Probing nuclear structure at different energy scales

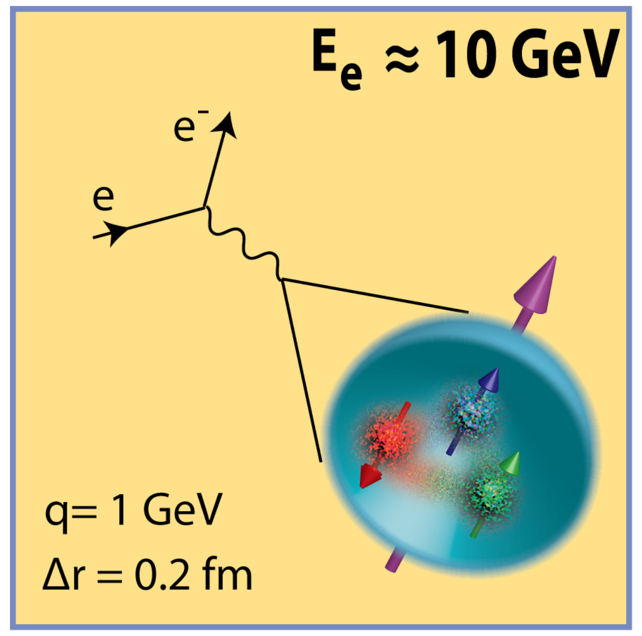




1911

Rutherford scattering

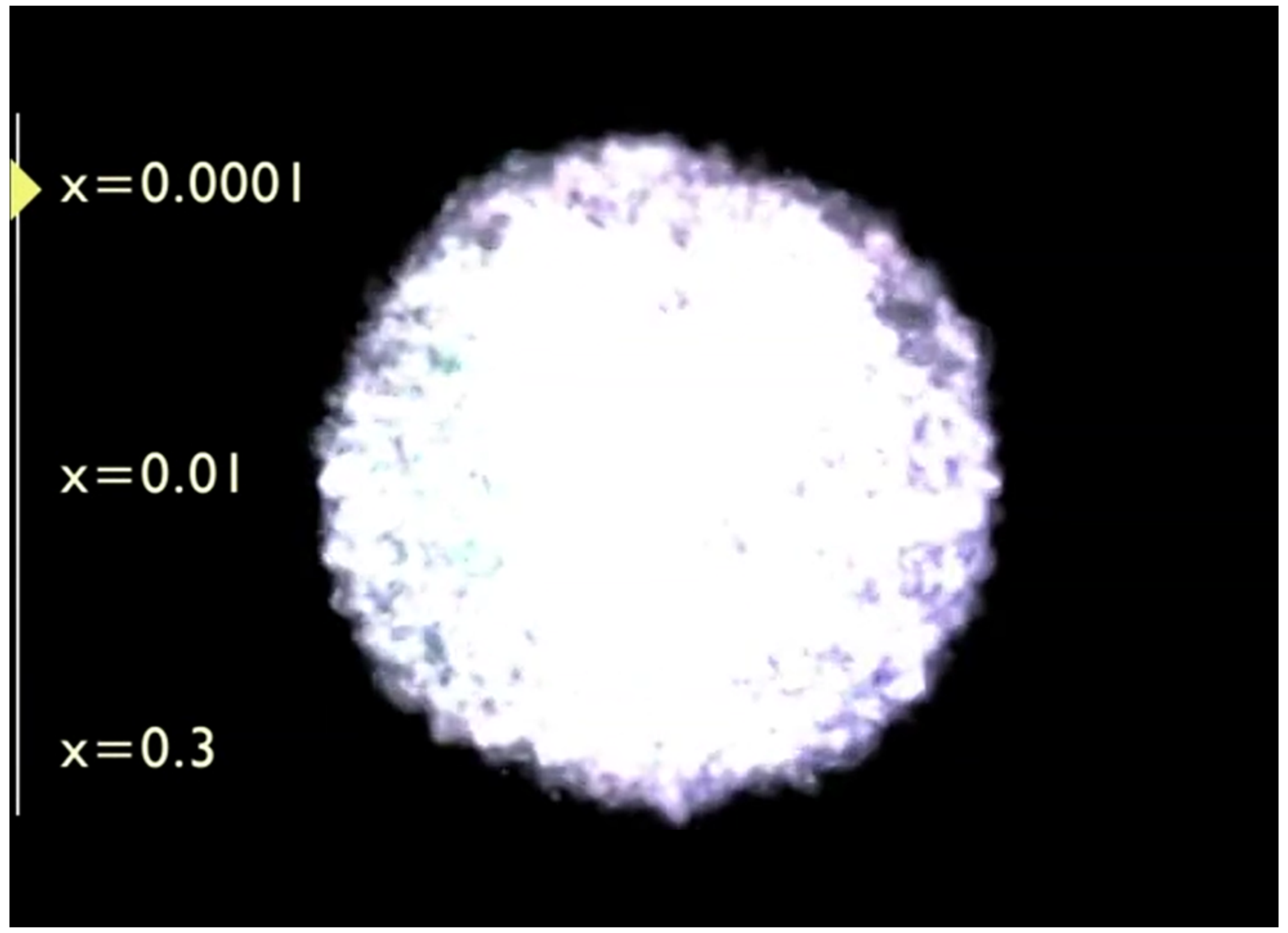




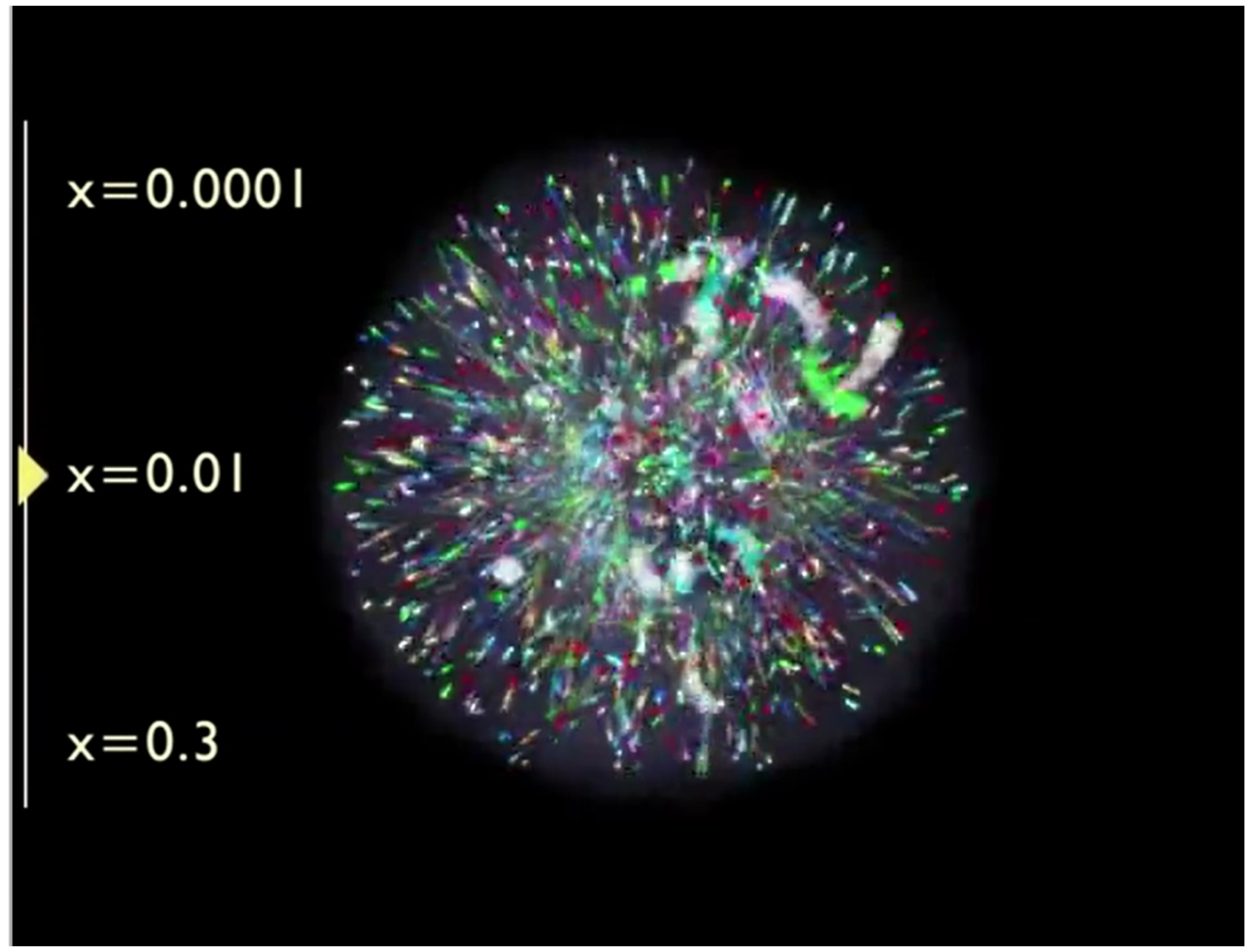
scattering: a fundamental tool to explore the nuclear structure!

The benefits from hadronic scattering

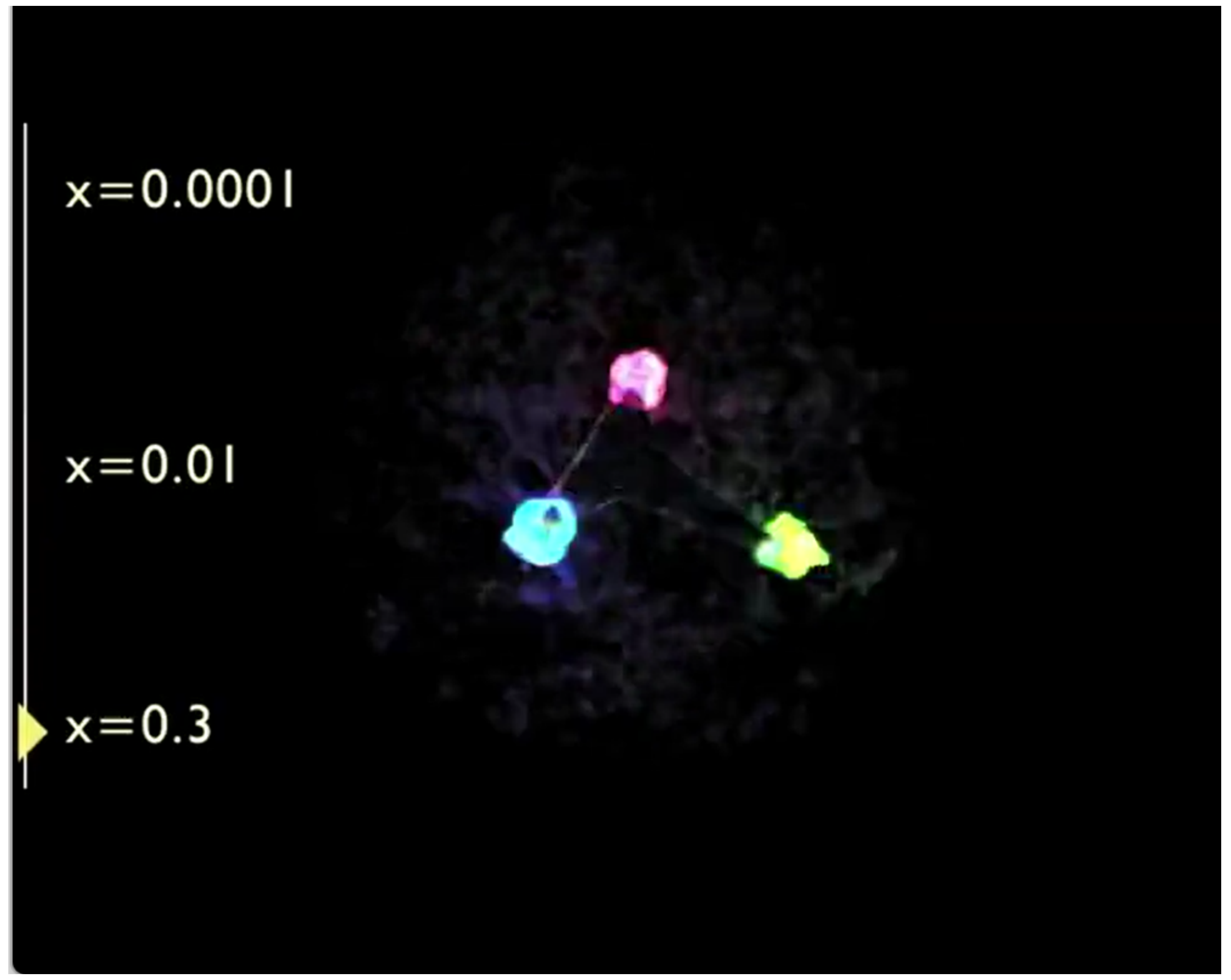
- ✦ Extract proton PDFs from world data



gluon



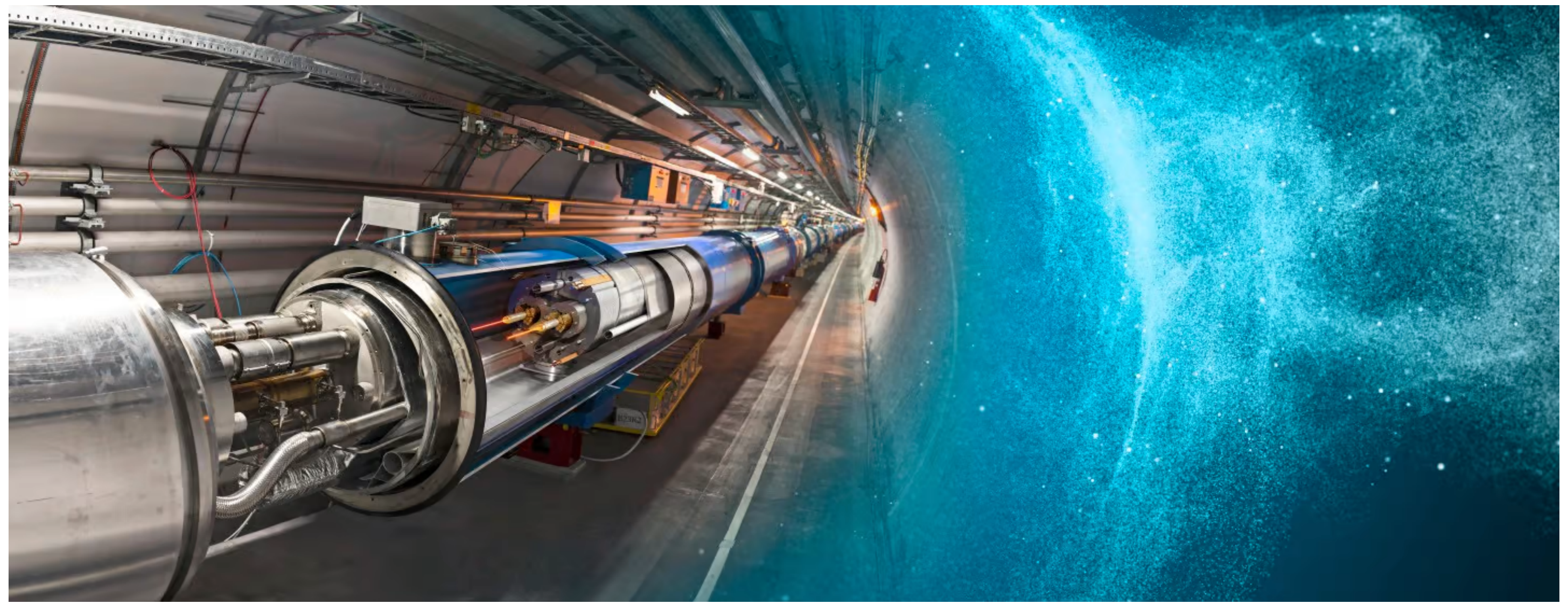
sea quark



valence quark

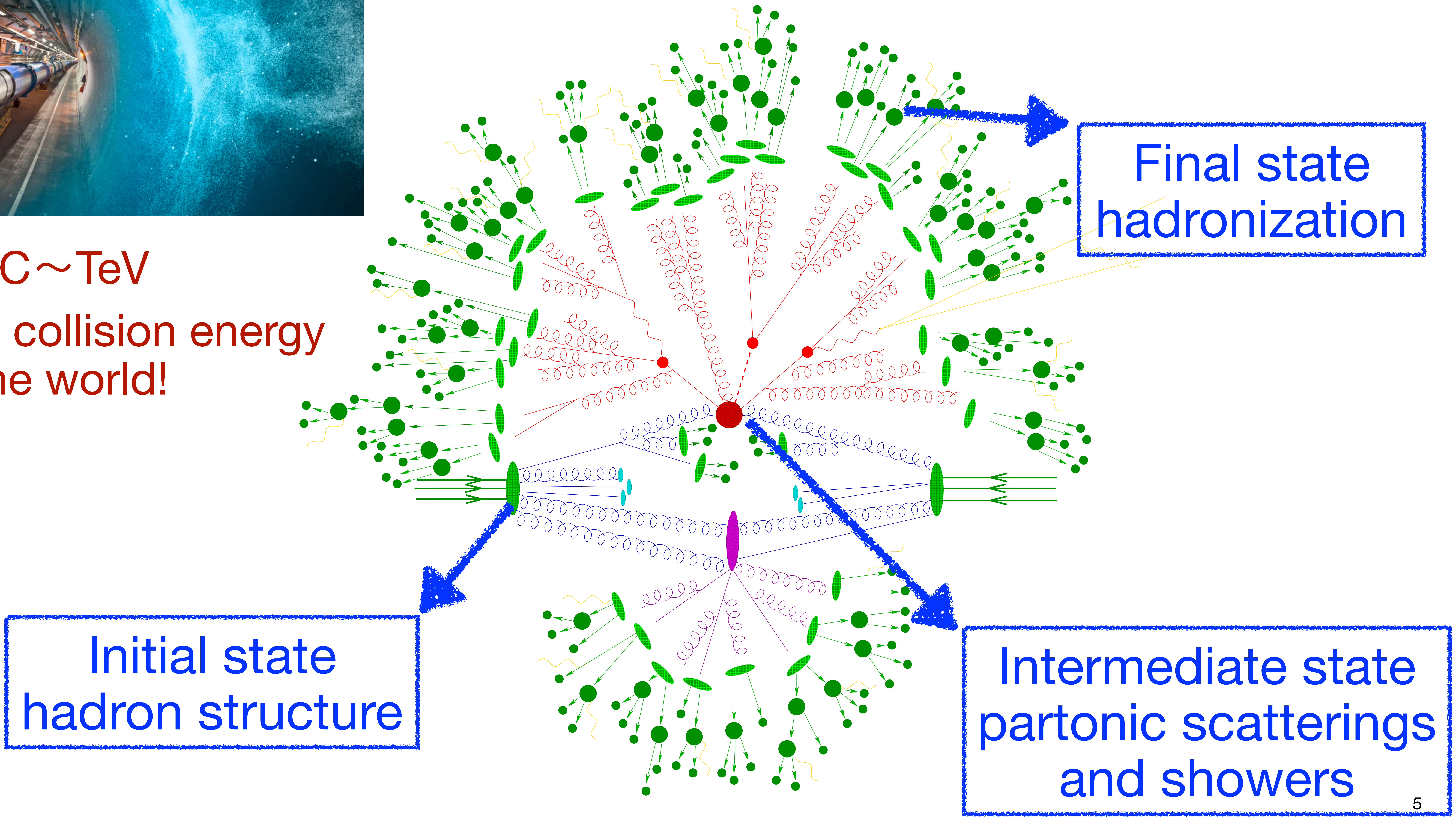
- ✦ High precision test of standard model

High energy hadronic scatterings



LHC～TeV

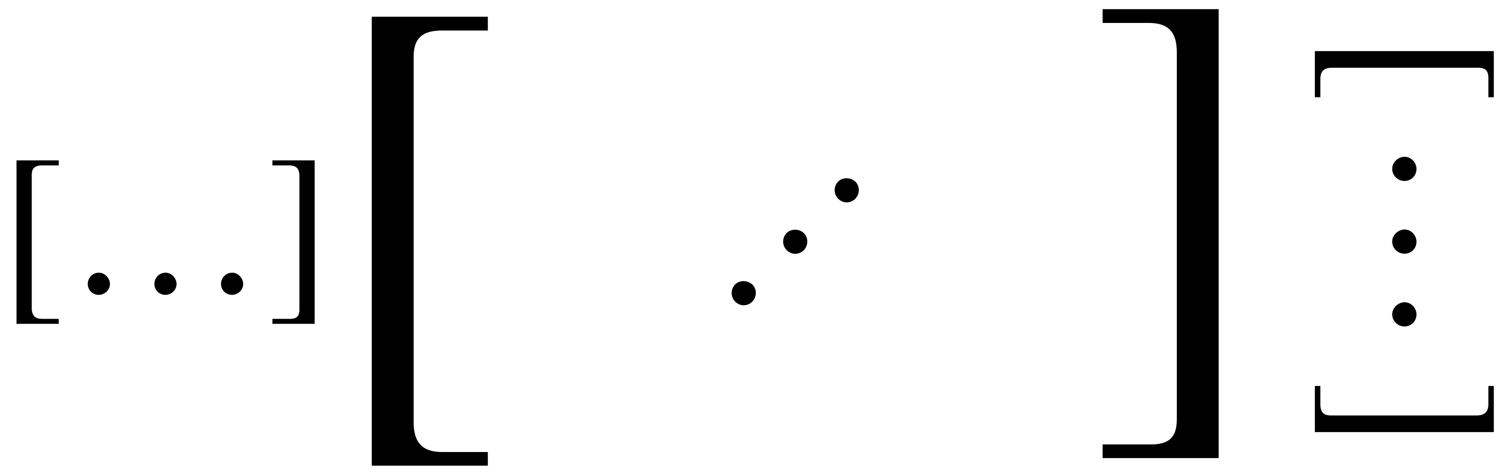
the highest collision energy in the world!

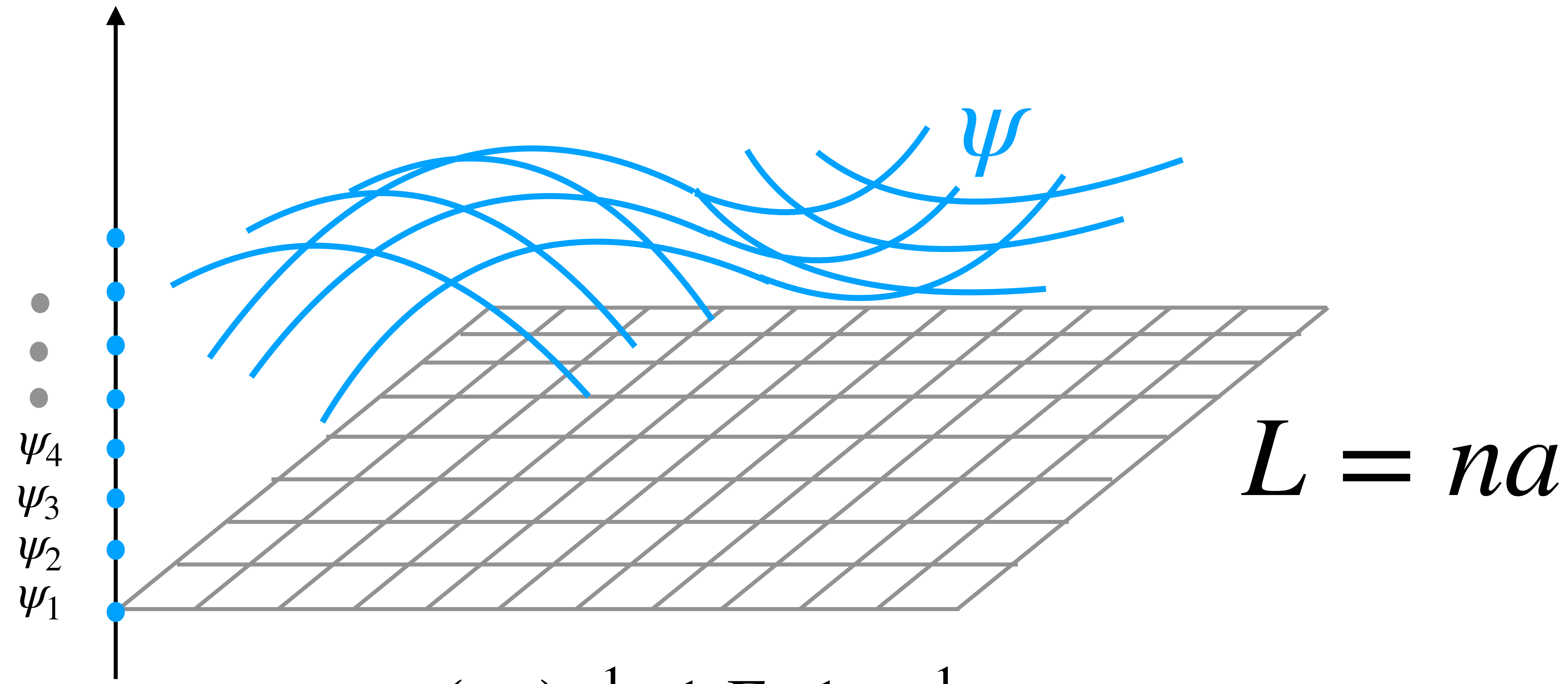


Simulate hadronic scatterings

✦ S-matrix in high energy scatterings

$$\langle \text{out} | e^{-iH[\psi]t} | \text{in} \rangle$$





$$(na)^{-1} \lesssim E \lesssim a^{-1}$$

- Hilbert space dimension: $n_H = (n_\psi)^{n^{D_{sp}}}$
- Classically: gigantic size, diagonalize H with infinite dim, impossible/hard !
- Quantum computing: requires qubits $n_q = (\mathrm{Log}_2 n_H) = (n^{D_{sp}} \mathrm{Log}_2 n_\psi)$, $n_{q,\mathrm{LHC}} \sim 5 \times 10^{12}$, $n_{q,\mathrm{hadron}} \sim 5000$, reasonable size

For the LHC $100\mathrm{MeV} \lesssim E \lesssim 13\mathrm{TeV}$

$$n^{D_{sp}} \sim 10^{12}$$

For the hadron: $100\mathrm{MeV} \lesssim E \lesssim 1\mathrm{GeV}$

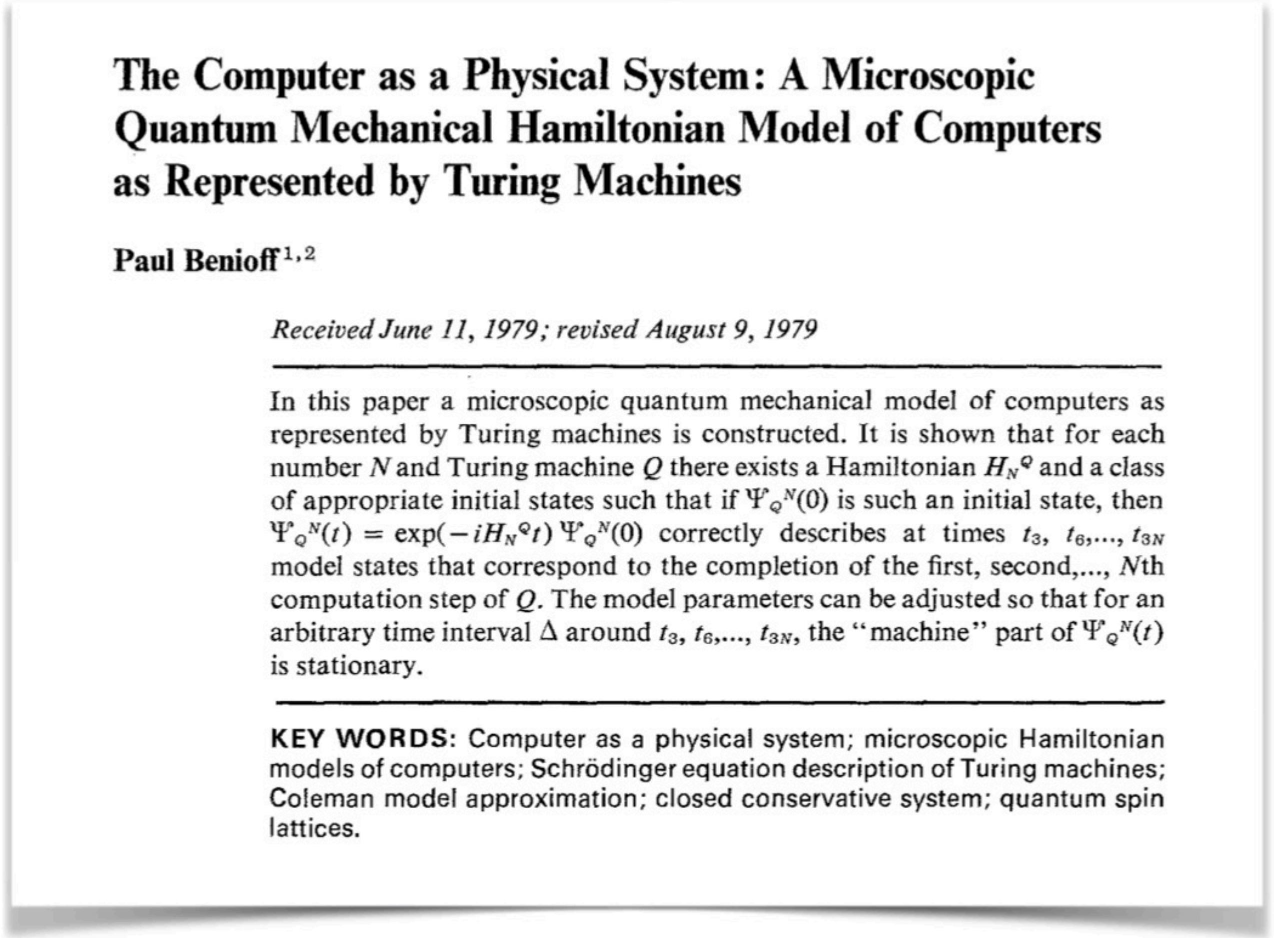
$$n^{D_{sp}} \sim 10^{3}$$

Suppose $n_\psi = 2^5 = 32$

dim $\to \infty$

Quantum computing

✦ A bit history



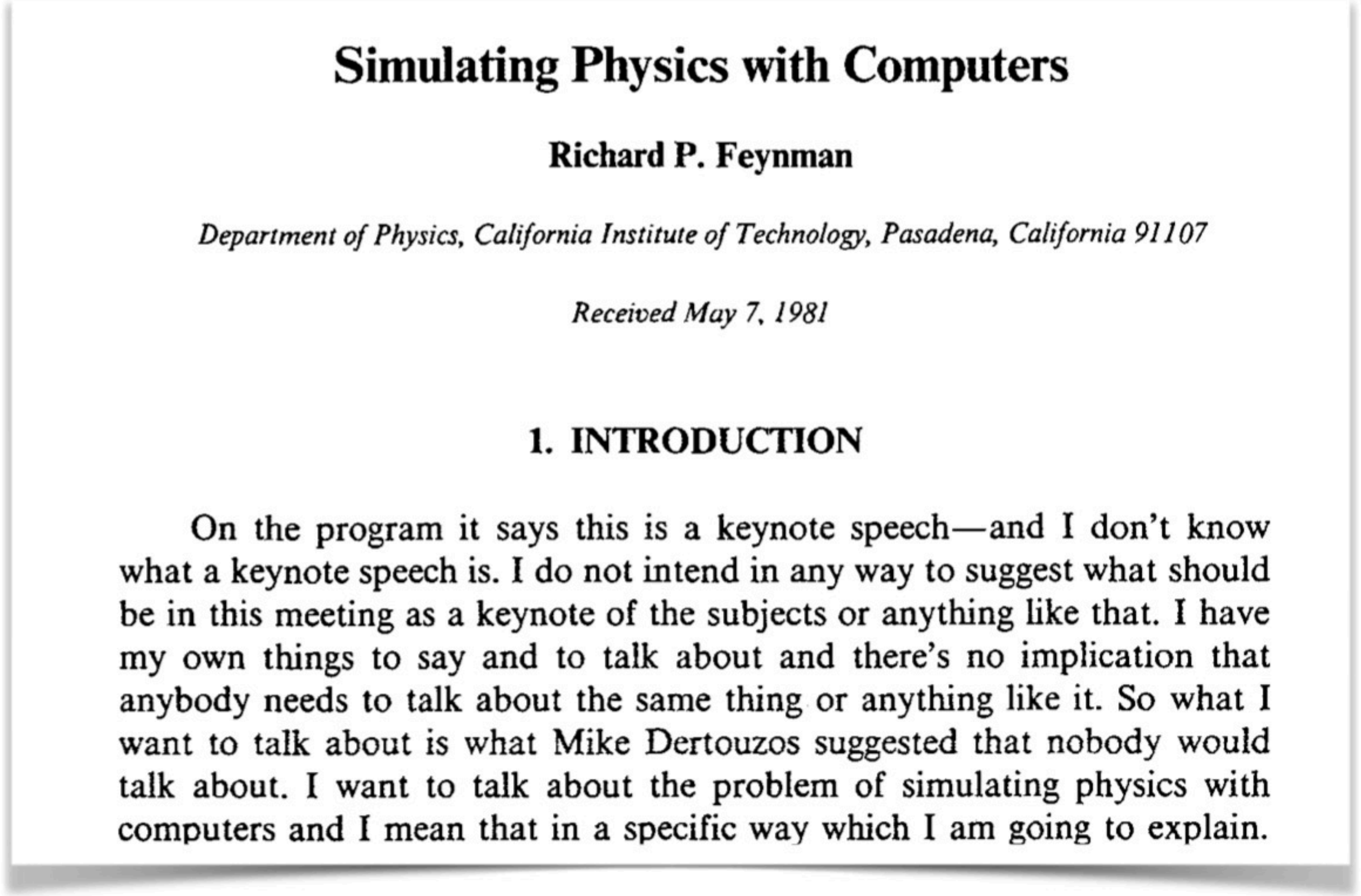
The Computer as a Physical System: A Microscopic Quantum Mechanical Hamiltonian Model of Computers as Represented by Turing Machines

Paul Benioff[1,2]

Received June 11, 1979; revised August 9, 1979

In this paper a microscopic quantum mechanical model of computers as represented by Turing machines is constructed. It is shown that for each number N and Turing machine Q there exists a Hamiltonian $H_N{}^Q$ and a class of appropriate initial states such that if $\Psi_Q{}^N(0)$ is such an initial state, then $\Psi_Q{}^N(t) = \exp(-iH_N{}^Q t)\Psi_Q{}^N(0)$ correctly describes at times $t_3, t_6, \ldots, t_{3N}$ model states that correspond to the completion of the first, second,..., Nth computation step of Q. The model parameters can be adjusted so that for an arbitrary time interval Δ around $t_3, t_6, \ldots, t_{3N}$, the "machine" part of $\Psi_Q{}^N(t)$ is stationary.

KEY WORDS: Computer as a physical system; microscopic Hamiltonian models of computers; Schrödinger equation description of Turing machines; Coleman model approximation; closed conservative system; quantum spin lattices.



Simulating Physics with Computers

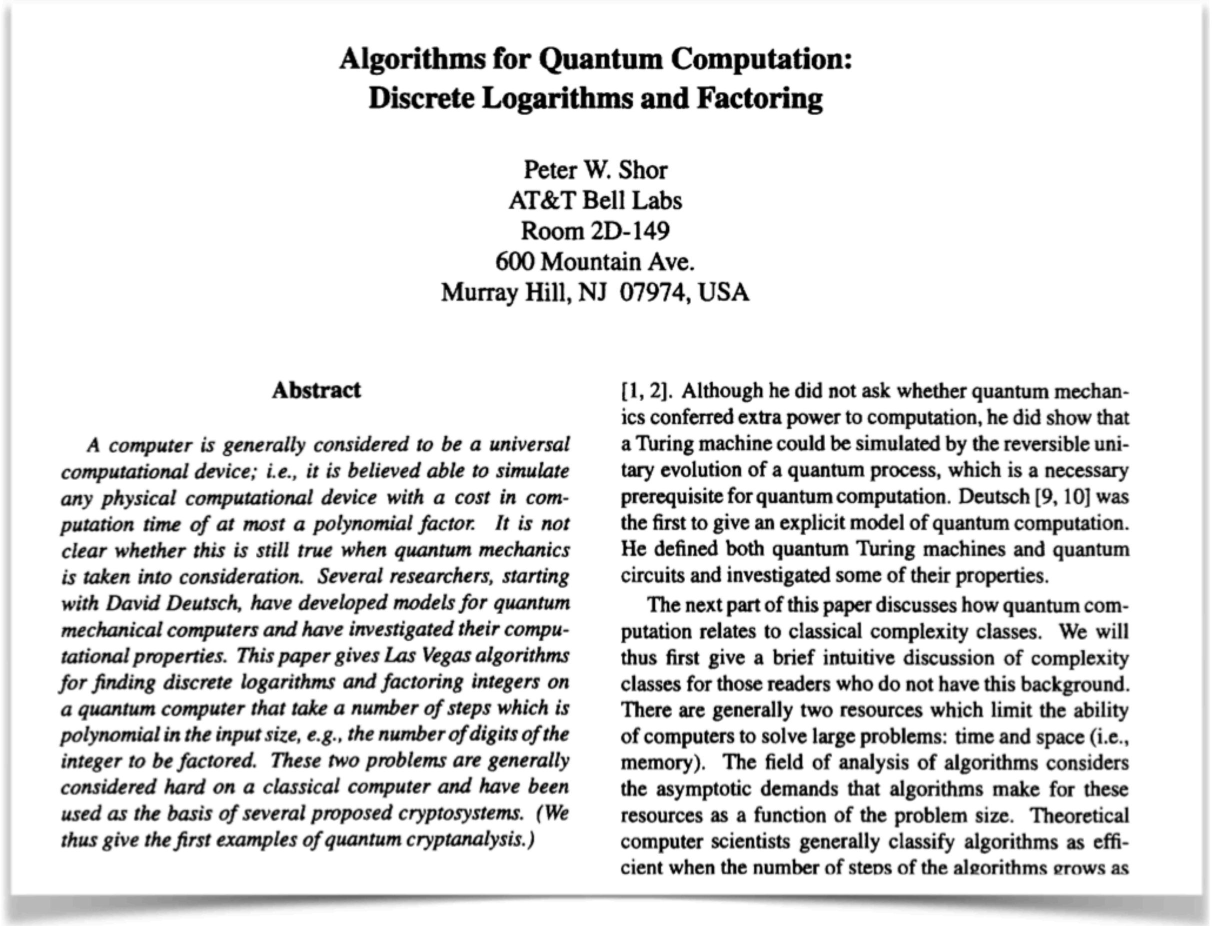
Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

1. INTRODUCTION

On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should be in this meeting as a keynote of the subjects or anything like that. I have my own things to say and to talk about and there's no implication that anybody needs to talk about the same thing or anything like it. So what I want to talk about is what Mike Dertouzos suggested that nobody would talk about. I want to talk about the problem of simulating physics with computers and I mean that in a specific way which I am going to explain.



Algorithms for Quantum Computation: Discrete Logarithms and Factoring

Peter W. Shor
AT&T Bell Labs
Room 2D-149
600 Mountain Ave.
Murray Hill, NJ 07974, USA

Abstract

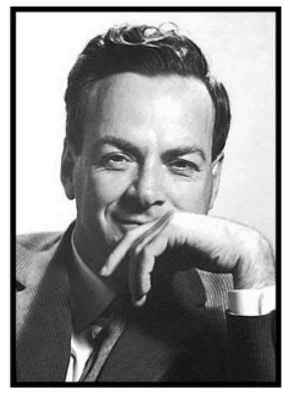
A computer is generally considered to be a universal computational device; i.e., it is believed able to simulate any physical computational device with a cost in computation time of at most a polynomial factor. It is not clear whether this is still true when quantum mechanics is taken into consideration. Several researchers, starting with David Deutsch, have developed models for quantum mechanical computers and have investigated their computational properties. This paper gives Las Vegas algorithms for finding discrete logarithms and factoring integers on a quantum computer that take a number of steps which is polynomial in the input size, e.g., the number of digits of the integer to be factored. These two problems are generally considered hard on a classical computer and have been used as the basis of several proposed cryptosystems. (We thus give the first examples of quantum cryptanalysis.)

[1, 2]. Although he did not ask whether quantum mechanics conferred extra power to computation, he did show that a Turing machine could be simulated by the reversible unitary evolution of a quantum process, which is a necessary prerequisite for quantum computation. Deutsch [9, 10] was the first to give an explicit model of quantum computation. He defined both quantum Turing machines and quantum circuits and investigated some of their properties.

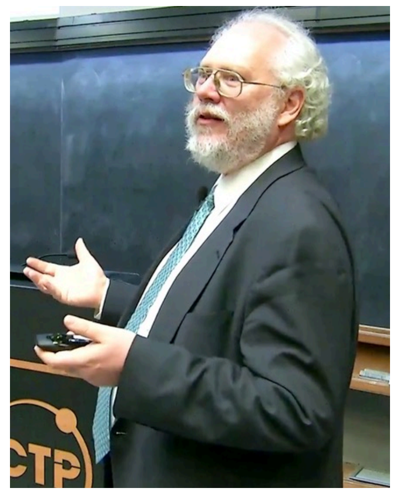
The next part of this paper discusses how quantum computation relates to classical complexity classes. We will thus first give a brief intuitive discussion of complexity classes for those readers who do not have this background. There are generally two resources which limit the ability of computers to solve large problems: time and space (i.e., memory). The field of analysis of algorithms considers the asymptotic demands that algorithms make for these resources as a function of the problem size. Theoretical computer scientists generally classify algorithms as efficient when the number of steps of the algorithms grows as

P. Benioff, 1979

R. Feynman, 1981



P. Shor, 1994



IBM Q System One (2019), the first circuit-based commercial quantum computer

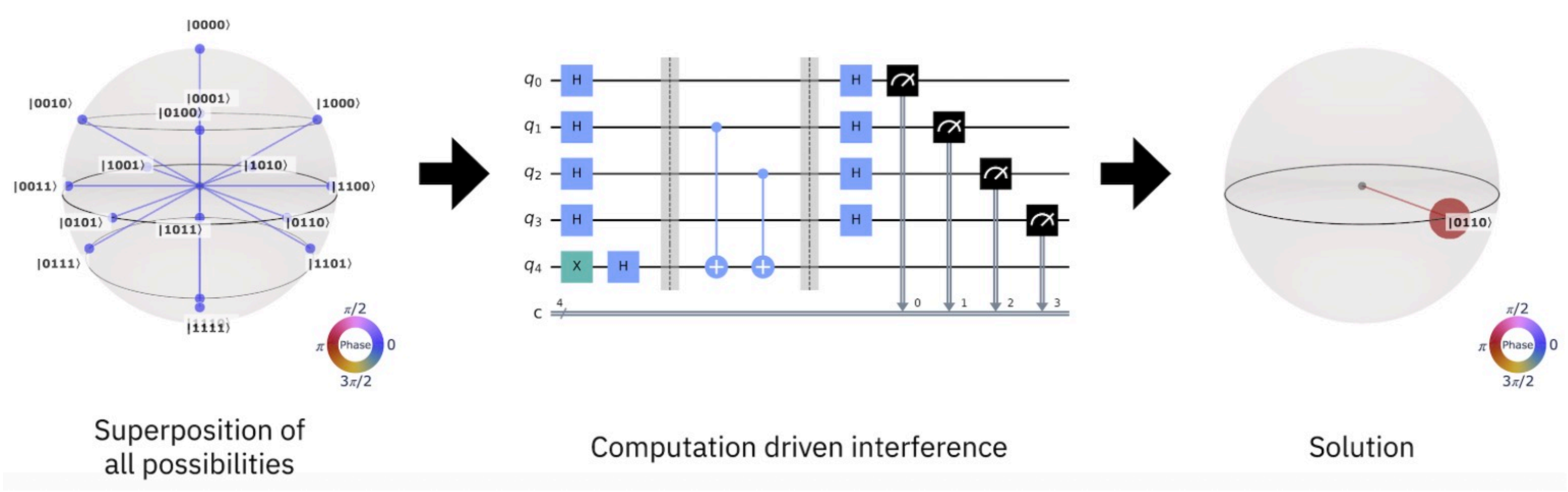
"... and if you want to make a simulation of nature, you'd better make it quantum mechanical, ..."

—Feynman

Quantum computing

Quantum circuit

https://qiskit.org/



✦ Building blocks of quantum computing

- Qubit: takes infinitely many different values $|\psi\rangle := \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$
- Quantum gate: unitary operators (X、Y、Z、CNOT)

$\alpha|0\rangle + \beta|1\rangle$ —[X]— $\beta|0\rangle + \alpha|1\rangle$

$|0\rangle$ —[H]— $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$

$|x\rangle$ —●— $|x\rangle$
$|y\rangle$ —⊕— $|y \oplus x\rangle$

- Measurements: Hermitian

Increasing interest in HEP and NP using quantum computing

Solving a Higgs optimization problem with quantum annealing for machine learning

Alex Mott, Joshua Job, Jean-Roch Vlimant, Daniel Lidar & Maria Spiropulu

Nature **550**, 375–379 (2017) | Cite this article

Abstract

The discovery of Higgs-boson decays in a background of standard-model processes was assisted by machine learning methods[1,2]. The classifiers used to separate signals such as these from background are trained using highly unerring but not completely perfect simulations of the physical processes involved, often resulting in incorrect labelling of background processes or signals (label noise) and systematic errors. Here we use quantum[3,4,5,6] and classical[7,8] annealing (probabilistic techniques for approximating the global maximum or minimum of a given function) to solve a Higgs-signal-versus-background machine learning optimization problem, mapped to a problem of finding the ground state of a corresponding Ising spin model. We build a set of weak classifiers based on the kinematic observables of the Higgs decay photons, which we then use to construct a

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Cloud Quantum Computing of an Atomic Nucleus

E. F. Dumitrescu, A. J. McCaskey, G. Hagen, G. R. Jansen, T. D. Morris, T. Papenbrock, R. C. Pooser, D. J. Dean, and P. Lougovski

Phys. Rev. Lett. **120**, 210501 – Published 23 May 2018

Physics See Viewpoint: Cloud Quantum Computing Tackles Simple Nucleus

Article | References | Citing Articles (127) | PDF | HTML | Export Citation

ABSTRACT

We report a quantum simulation of the deuteron binding energy on quantum processors accessed via cloud servers. We use a Hamiltonian from pionless effective field theory at leading order. We design a low-depth version of the unitary coupled-cluster ansatz, use the variational quantum eigensolver algorithm, and compute the binding energy to within a few percent. Our work is the first step towards scalable nuclear structure computations on a quantum processor via the cloud, and it sheds light on how to map scientific computing applications onto nascent quantum devices.

Quantum Algorithm for High Energy Physics Simulations

Benjamin Nachman, Davide Provasoli, Wibe A. de Jong, and Christian W. Bauer

Phys. Rev. Lett. **126**, 062001 – Published 10 February 2021

Article | References | Citing Articles (6) | Supplemental Material | PDF | HTML | Export Ci

ABSTRACT

Simulating quantum field theories is a flagship application of quantum computing. However, calculating experimentally relevant high energy scattering amplitudes entirely on a quantum computer is prohibitively difficult. It is well known that such high energy scattering processes can be factored into pieces that can be computed using well established perturbative techniques, and pieces which currently have to be simulated using classical Markov chain algorithms. These classical Markov chain simulation approaches work well to capture many of the salient features, but cannot capture all quantum effects. To exploit quantum resources in the most efficient way, we introduce a new paradigm for quantum algorithms in field theories. This approach uses quantum computers only for those parts of the problem which are not computable using existing techniques. In particular, we develop a polynomial time quantum final state shower that accurately models the effects of intermediate spin states similar to those present in high energy electroweak showers with a global evolution variable. The algorithm is explicitly demonstrated for a simplified quantum field theory on a quantum computer.

Letter | Open Access | Access by South

Quantum simulation of open quantum systems in heavy-ion collisions

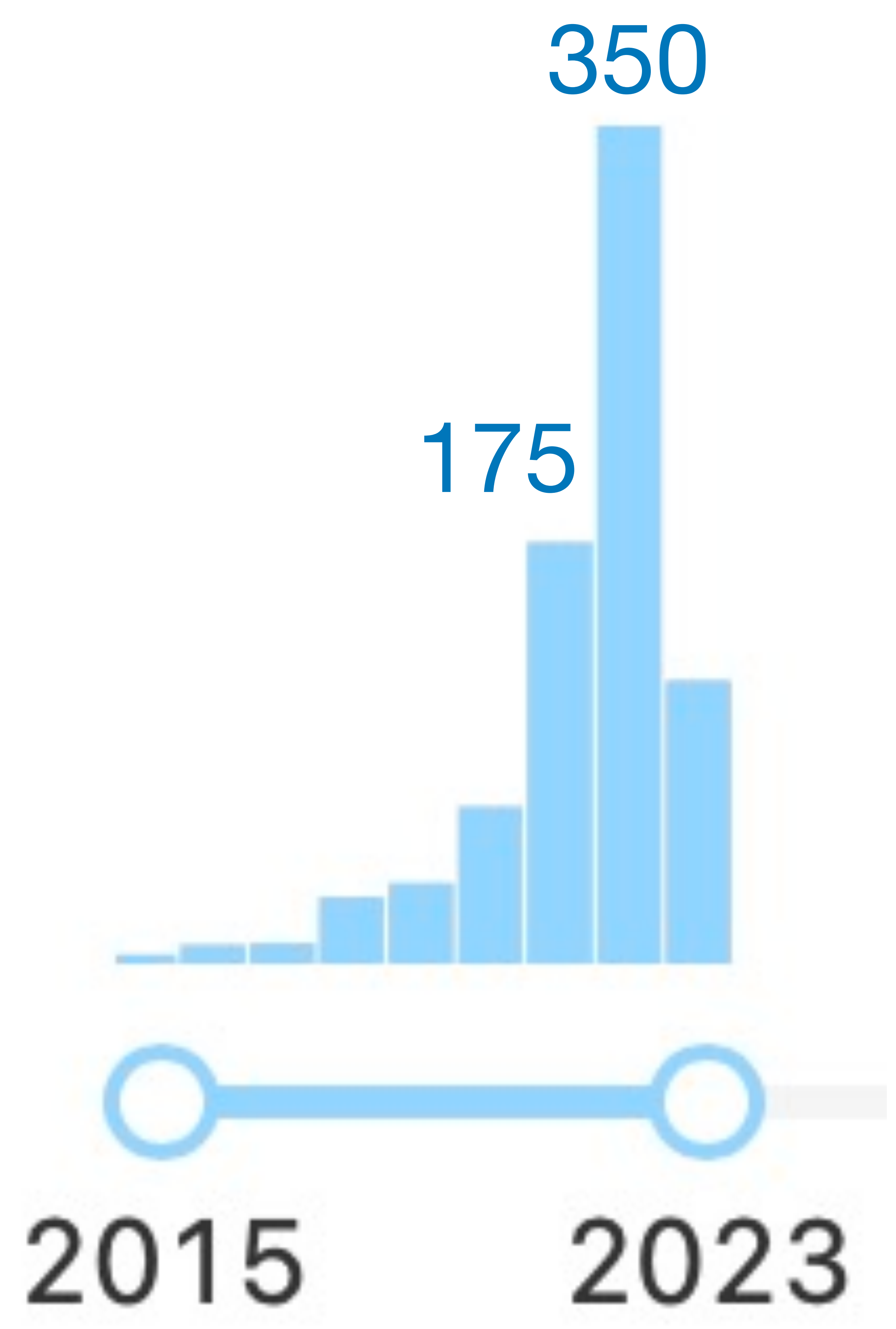
Wibe A. de Jong, Mekena Metcalf, James Mulligan, Mateusz Płoskoń, Felix Ringer, and Xiaojun Yao

Phys. Rev. D **104**, L051501 – Published 7 September 2021

Article | References | No Citing Articles | Supplemental Material | PDF | HTML | Export Citati

ABSTRACT

We present a framework to simulate the dynamics of hard probes such as heavy quarks or jets in a hot, strongly coupled quark-gluon plasma (QGP) on a quantum computer. Hard probes in the QGP can be treated as open quantum systems governed in the Markovian limit by the Lindblad equation. However, due to large computational costs, most current phenomenological calculations of hard probes evolving in the QGP use semiclassical approximations of the quantum evolution. Quantum computation can mitigate these costs and offers the potential for a fully quantum treatment with exponential speed-up over classical techniques. We report a simplified demonstration of our framework on IBM Q quantum devices and apply the random identity insertion method to account for CNOT depolarization noise, in addition to measurement error mitigation. Our work demonstrates the feasibility of simulating open quantum systems on current and near-term quantum devices, which is of broad relevance to applications in nuclear physics, quantum information, and other fields.



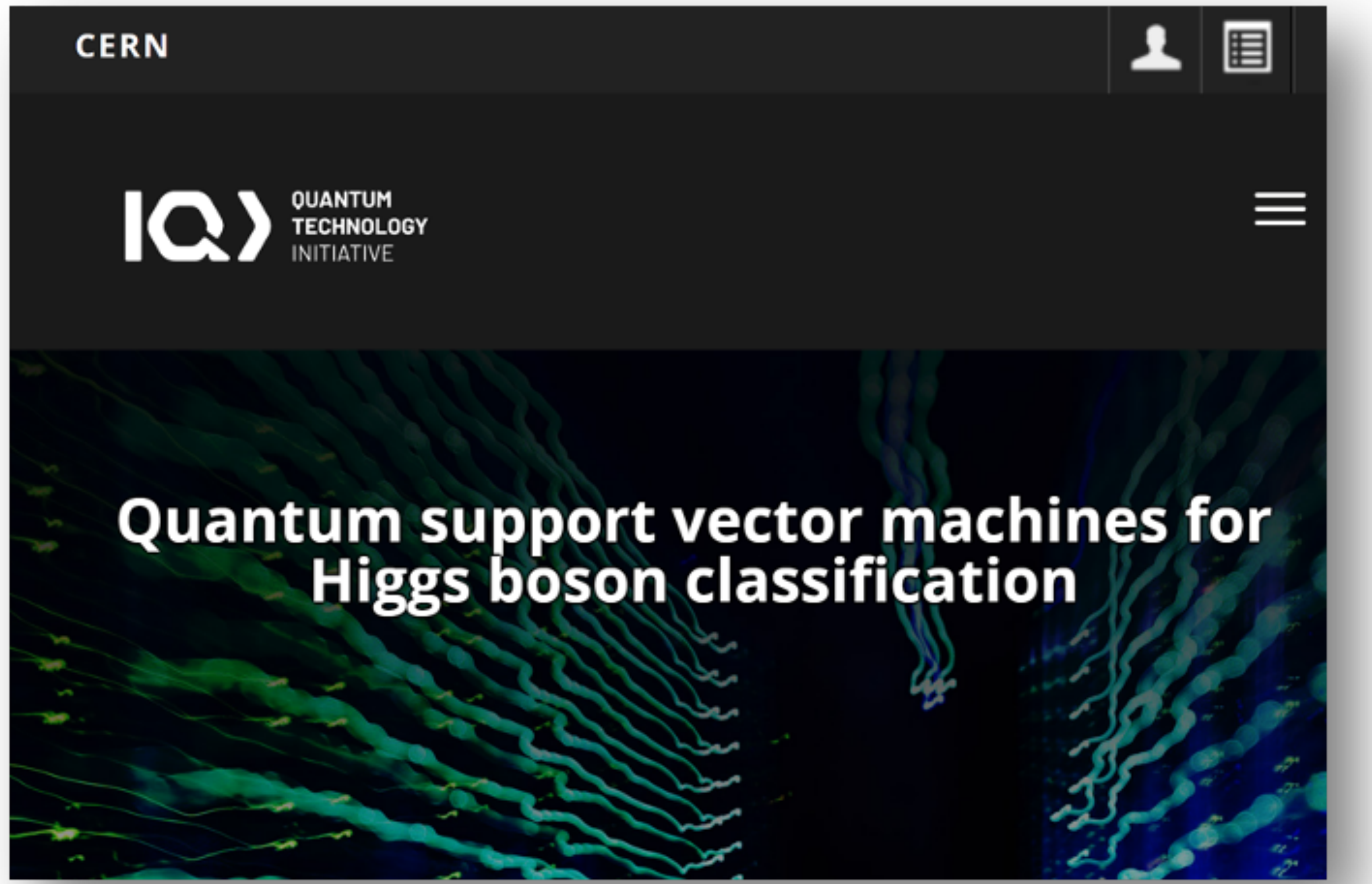
Inspire:

find t quantum computing and date>2015

Community-wide efforts

QUANTUM COMPUTING FOR THEORETICAL NUCLEAR PHYSICS

A White Paper prepared for the U.S. Department of Energy, Office of Science, Office of Nuclear Physics



arXiv > quant-ph > arXiv:2209.14839

Quantum Physics

[Submitted on 29 Sep 2022]

Report of the Snowmass 2021 Theory Frontier Topical Group on Quantum Information Science

Simon Catterall, Roni Harnik, Veronika E. Hubeny, Christian W. Bauer, Asher Berlin, Zohreh Davoudi, Thomas Faulkner, Thomas Hartman, Matthew Headrick, Yonatan F. Kahn, Henry Lamm, Yannick Meurice, Surjeet Rajendran, Mukund Rangamani, Brian Swingle

arXiv > quant-ph > arXiv:2307.03236

Quantum Physics

[Submitted on 6 Jul 2023]

Quantum Computing for High-Energy Physics: State of the Art and Challenges. Summary of the QC4HEP Working Group

Alberto Di Meglio, Karl Jansen, Ivano Tavernelli, Constantia Alexandrou, Srinivasan Arunachalam, Christian W. Bauer, Kerstin Borras, Stefano Carrazza, Arianna Crippa, Vincent Croft, Roland de Putter, Andrea Delgado, Vedran Dunjko, Daniel J. Egger, Elias Fernandez-Combarro, Elina Fuchs, Lena Funcke, Daniel Gonzalez-Cuadra, Michele Grossi, Jad C. Halimeh, Zoe Holmes, Stefan Kuhn,

arXiv > nucl-ex > arXiv:2303.00113

Nuclear Experiment

[Submitted on 28 Feb 2023]

Quantum Information Science and Technology for Nuclear Physics. Input into U.S. Long-Range Planning, 2023

Douglas Beck, Joseph Carlson, Zohreh Davoudi, Joseph Formaggio, Sofia Quaglioni, Martin Savage, Joao Barata, Tanmoy Bhattacharya, Michael Bishof, Ian Cloet, Andrea Delgado, Michael DeMarco, Caleb Fink, Adrien Florio, Marianne Francois, Dorota Grabowska, Shannon Hoogerheide, Mengyao Huang, Kazuki Ikeda, Marc Illa, Kyungseon Joo, Dmitri Kharzeev, Karol Kowalski, Wai Kin Lai, Kyle Leach, Ben Loer, Ian Low, Joshua Martin, David Moore, Thomas

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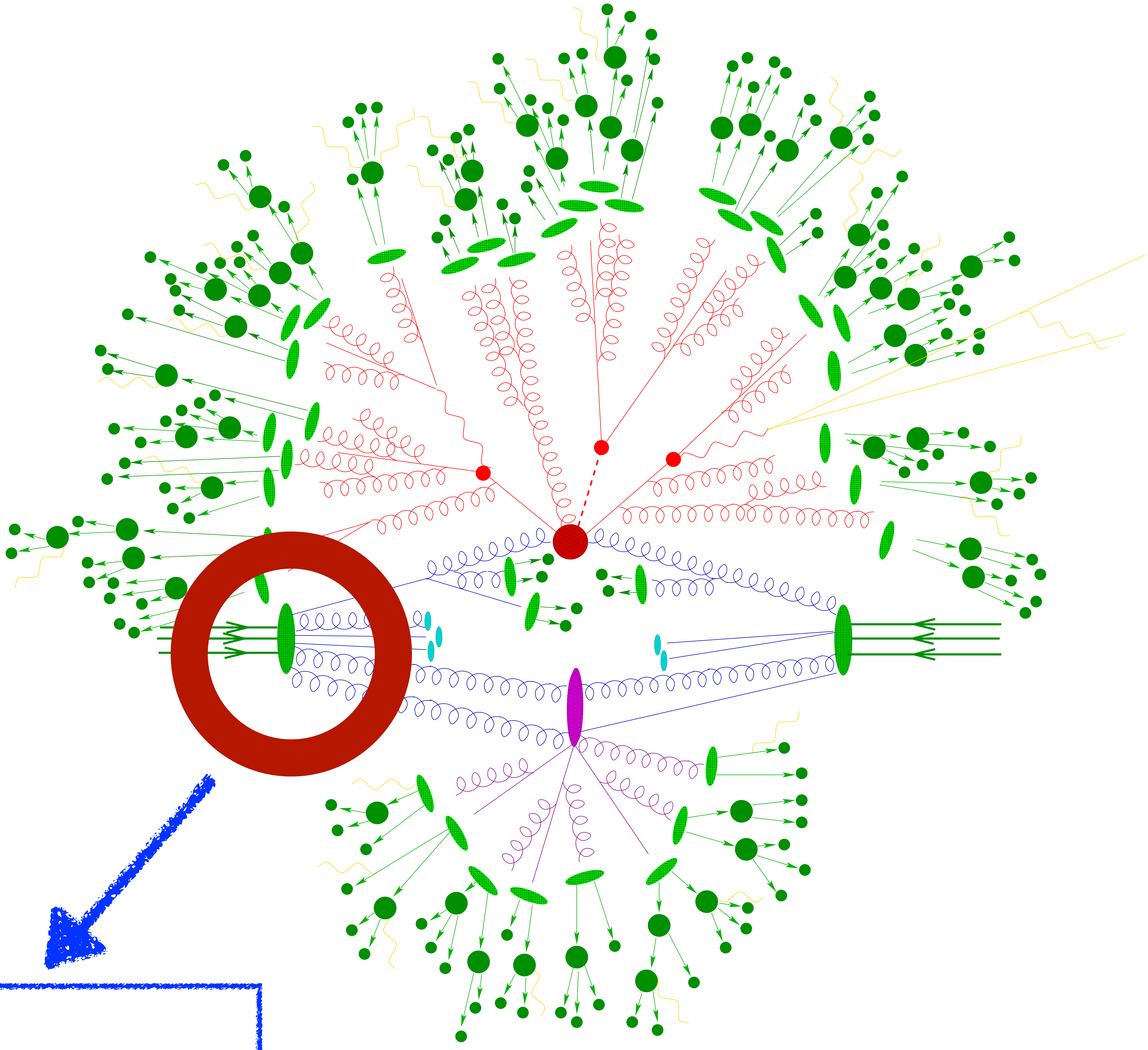


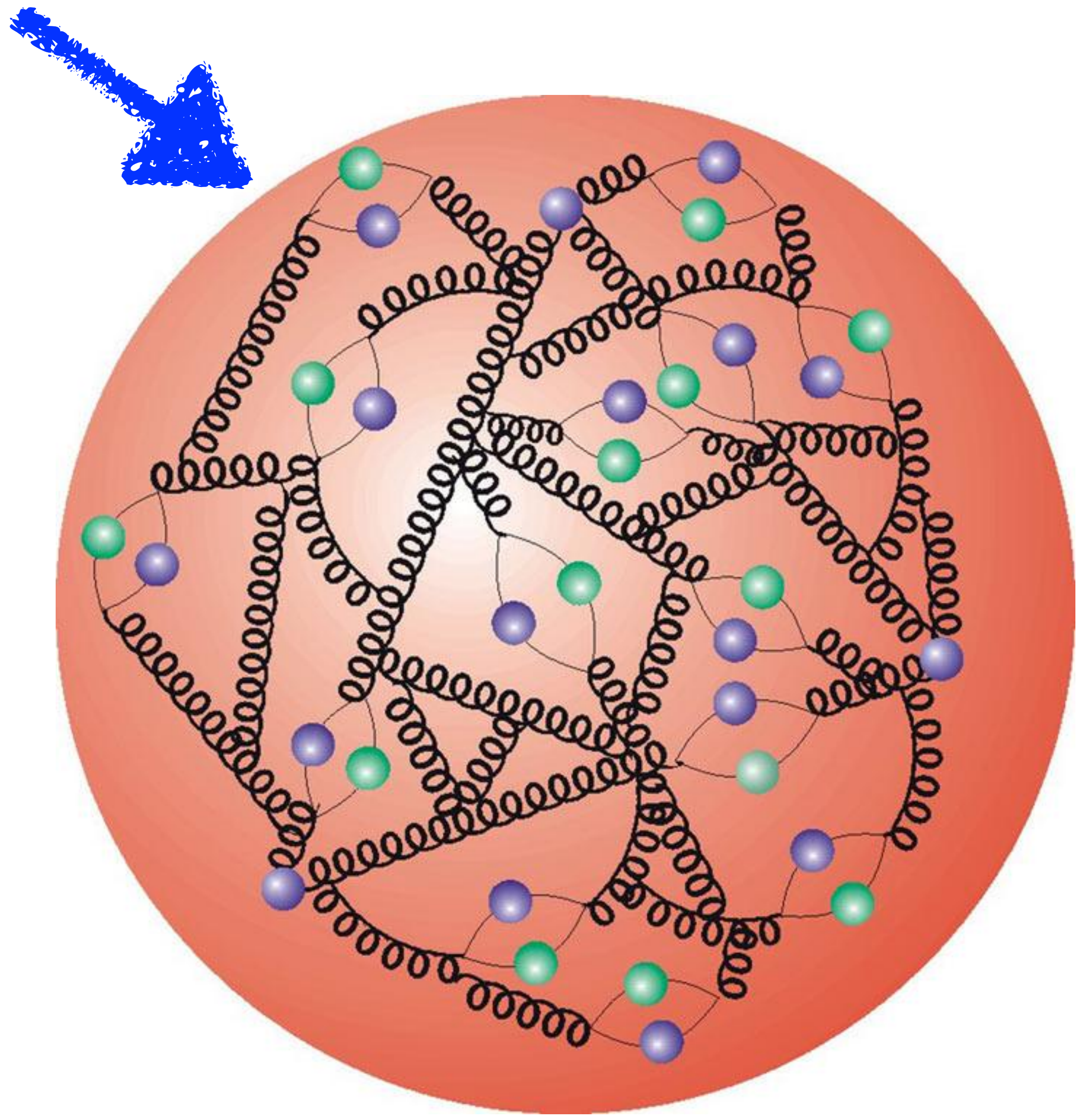
Initial state

parton distribution function f

Simulate hadron partonic structure on quantum computer

✦ Operator definition of quark PDF

$$\sigma \sim f_A \otimes H \otimes D$$



$$f_{q/p}(x) = \int_{-\infty}^{\infty} \frac{dy^-}{2\pi} e^{ixp^+y^-} \langle p | \bar{\psi}(0) \frac{\gamma^+}{2} \mathscr{W}(0,y^-) \psi(y^-) | p \rangle$$

- Light cone momentum fraction:
  $x = k^+/p^+,\ k^+ = (k^0 + k^z)/\sqrt{2}$
- Wilson line to ensure gauge invariance
  $\mathscr{W}(0,y^-) = \mathscr{P} e^{-ig\int_0^{y^-} d\eta^- A^+(\eta^-)}$

✦ The probability density of finding a parton inside a proton

✦ PDFs are extremely challenge to simulate in classical/Euclidean lattice calculation, due to multidimensional oscillating integral.

✦ QC can naturally simulate real-time dynamics.

Simulate hadron partonic structure on quantum computer

✦ A toy model - 1+1D NJL (Gross, Neveu, 1974), no gauge

$$\mathcal{L} = \bar{\psi}_\alpha (i\gamma^\mu \partial_\mu - m_\alpha)\psi_\alpha + g(\bar{\psi}_\alpha \psi_\alpha)^2$$

$$f(x) = \int dz^- e^{-ixM_h z^-} \langle h | \bar{\psi}(z^-)\gamma^+ \psi(0) | h \rangle = \int dz^- e^{-ixM_h z^-} \langle h | e^{iHz} \bar{\psi}(0, -z) e^{-iHz} \gamma^+ \psi(0) | h \rangle$$

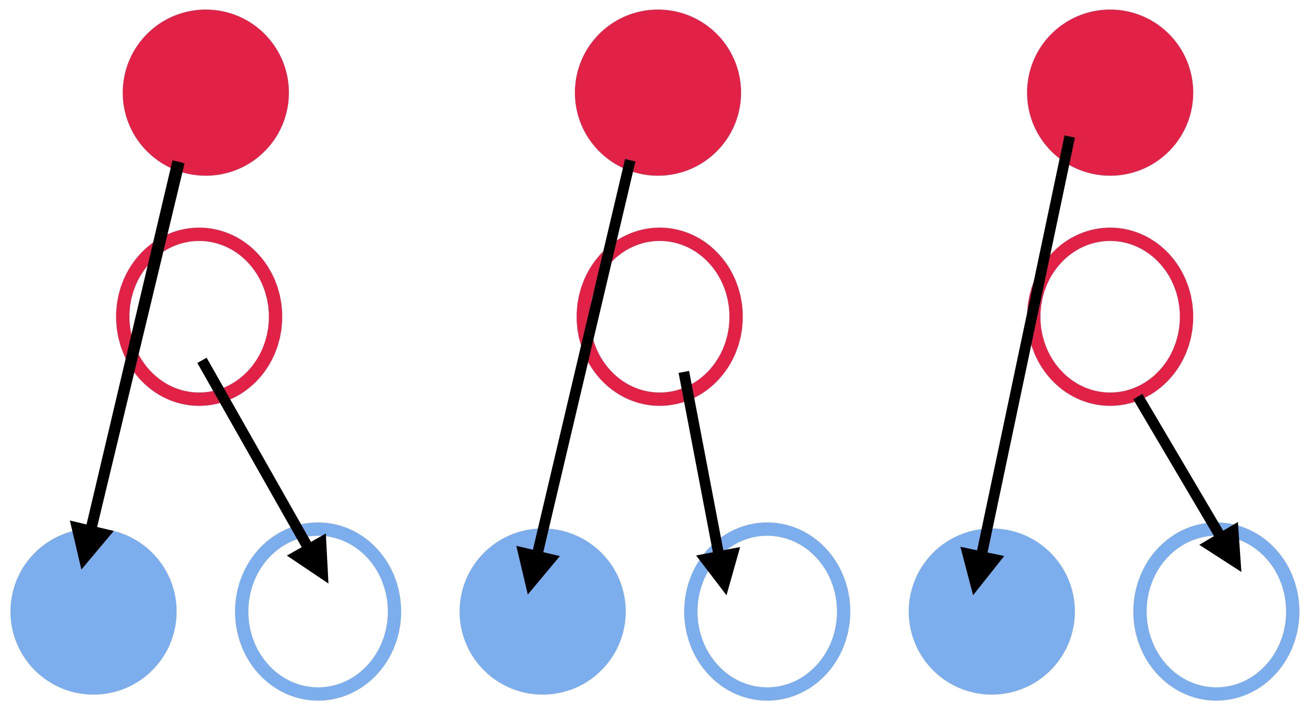
- Map QFT to qubits+gates system
- Prepare the external hadronic state $| h \rangle$
- Evaluate the real-time dynamical correlation function

Simulate hadron partonic structure on quantum computer

✦ Quantum field to qubits+gates

$$\mathscr{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + g(\bar{\psi}\psi)^2$$

- Discretization: staggered fermion, put different fermion components, flavors on different sites



$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \to \begin{pmatrix} \phi_{2n} \\ \phi_{2n+1} \end{pmatrix}$$

- Jordan-Wigner transformation

$$\phi_n = \prod_{i<n} Z_i (X + iY)_n$$

- Discretized PDF:

$$f(x) \to \sum_{i,j} \sum_{z} \frac{1}{4\pi} e^{-ixM_h z} \langle h | e^{iHz} \phi^\dagger_{-2z+i} e^{-iHz} \phi_j | h \rangle$$

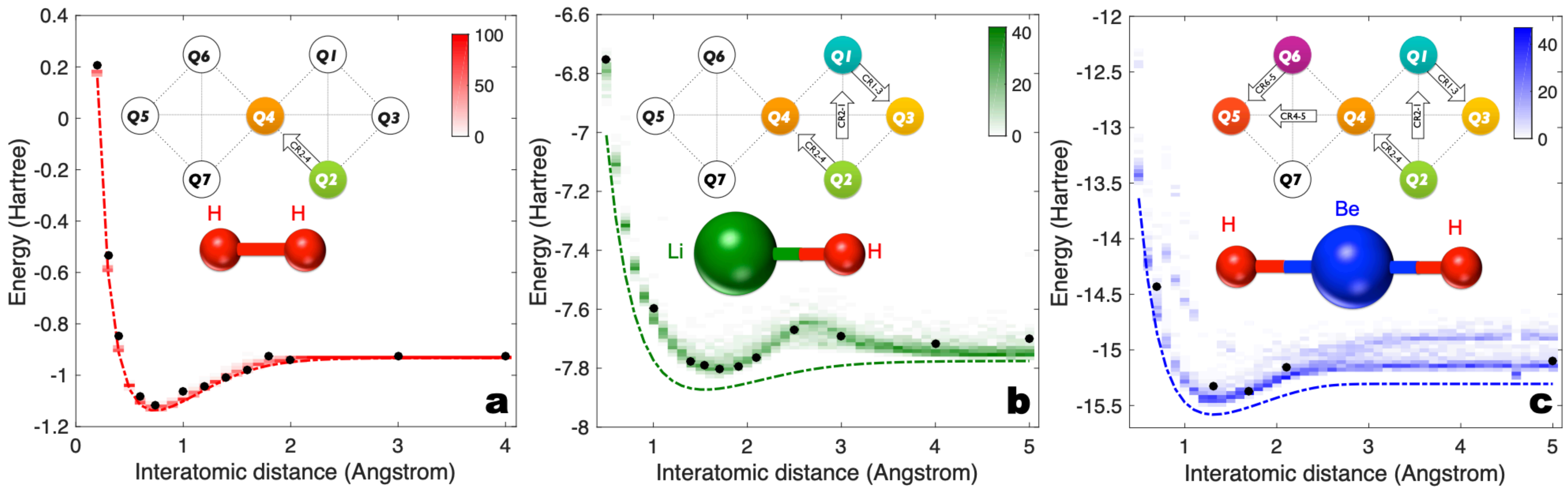
$$H = H_1 + H_2 + H_3 + H_4 \qquad H_1 = \sum_{n=\text{even}} \frac{1}{4} \left[X_n Y_{n+1} - Y_n X_{n+1} \right]$$

Simulate hadron partonic structure on quantum computer

✦ Hadron state preparation - VQE

- Hadron states are the eigenstates of the Hamiltonian with certain quantum numbers.
- Prepare the state by variational quantum eigensolver (VQE) 2103.08505 + …
- VQE is a hybrid method involves both classical and quantum computers

Potential energy surfaces Nature 549, 242 (2017)



show its power in quantum chemistry

Simulate hadron partonic structure on quantum computer

✦ Hadron state preparation - VQE

Li et al (QuNu), PRD (letter, 2022)

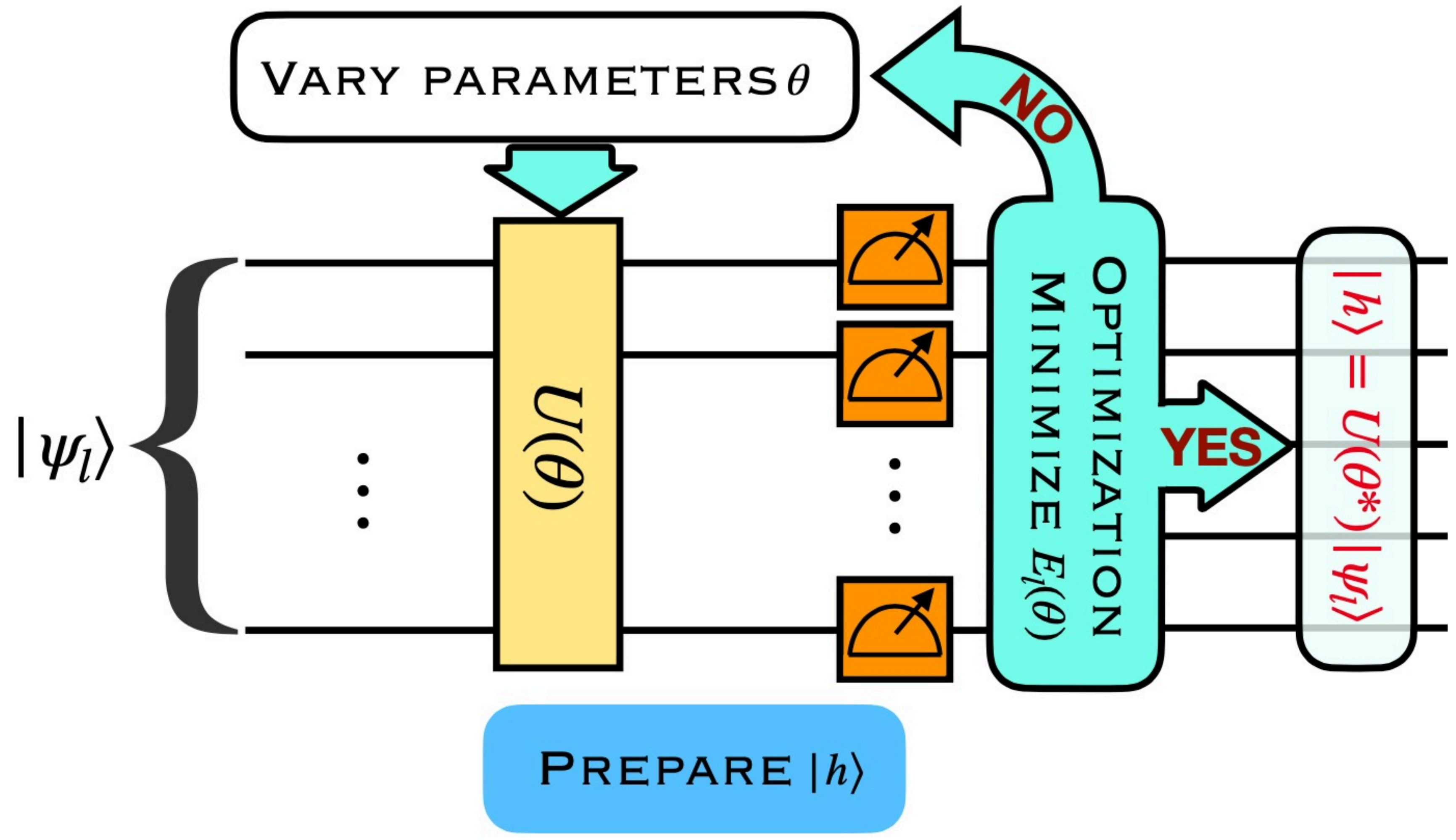
I. Construct a trial hadronic state $|\psi_{lk}\rangle$, and a symmetry-preserving unitary operator $U(\theta)$

II. The k-th state with quantum number l
$|\psi_{lk}(\theta)\rangle = U(\theta)|\psi_{lk}\rangle$

III. Optimization for hadronic state, minimize the cost function (PRL 113, 020505)

$$E_l(\theta) = \sum_{i=1}^{k} w_{li}\langle\psi_{li}(\theta)|H|\psi_{li}(\theta)\rangle$$

IV. $|h\rangle = U(\theta^*)|\psi_{lk}\rangle$, θ^* is the optimized parameter set



Step II is carried out on quantum computer, all the others are computed on a classical one

Simulate hadron partonic structure on quantum computer

✦ Hadron state preparation

- Construct $U(\theta)$: quantum alternating operator ansatz (QAOA)

I. Divide the hamiltonian, each term inherits the symmetries of H, $H = H_1 + H_2 + H_3 + H_4$

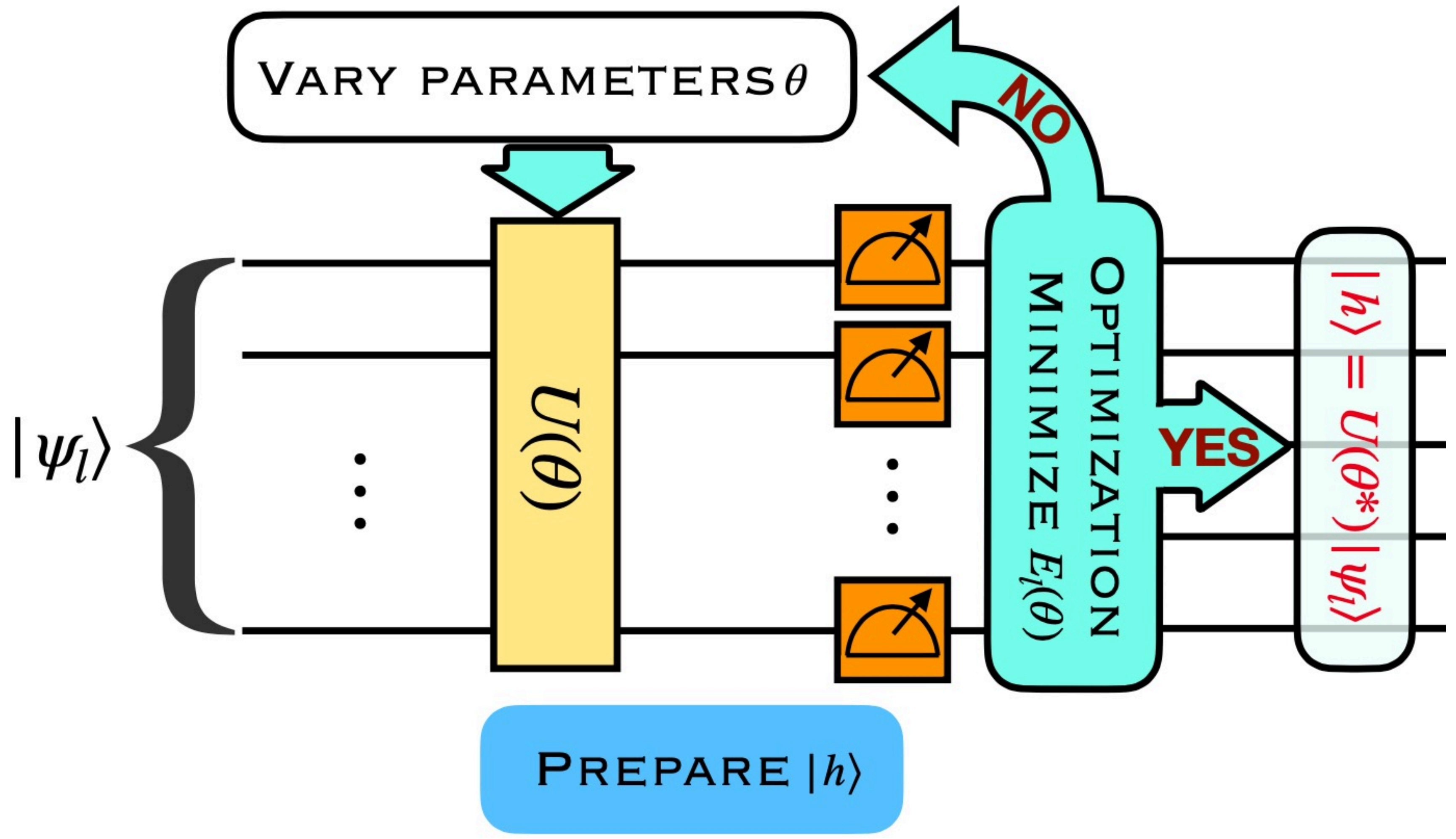
II. $U(\theta)$ consists p layers, each layer evolve H_j with time duration θ_{ij}, $U(\theta) \equiv \prod_{i=1}^{p} \prod_{j=1}^{n} \exp(i\,\theta_{ij} H_j)$

III. Prepare the input reference states for QAOA

$$|\psi_{\Omega,1}\rangle = |010101\ldots01\rangle$$ → Naive vacuum

$$|\psi_{\Omega,2}\rangle = \frac{1}{\sqrt{N/2}}\Big(|1001,\ldots,01\rangle + |0110,\ldots,01\rangle + \ldots + |0101,\ldots,10\rangle \Big)$$

"quark pair" excitation



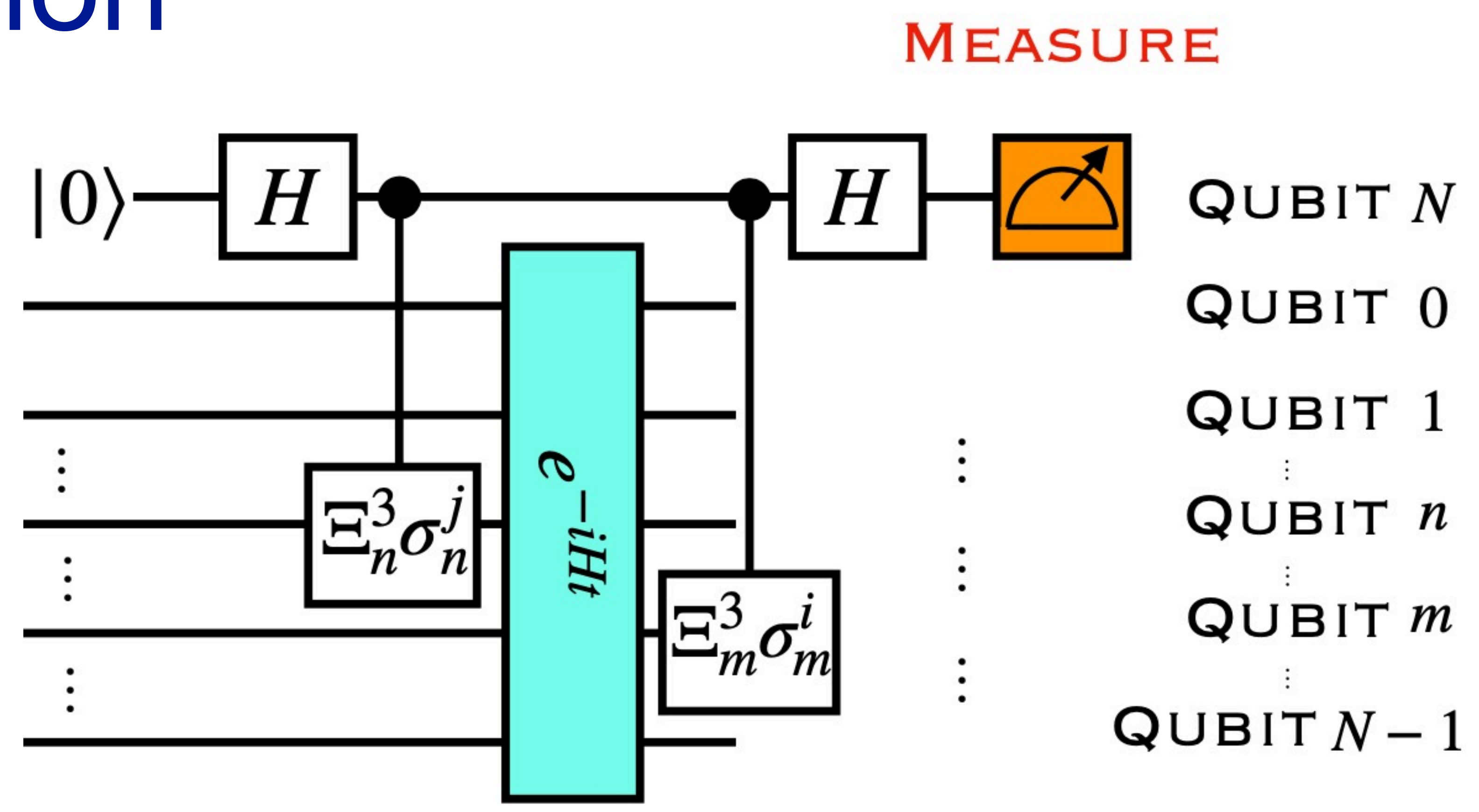
Simulate hadron partonic structure on quantum computer

Pedernales et al, PRL. 113, 020505 (2014)

✦ Evaluate the real-time dynamical correlation function

$$S_{mn}(t) = \langle h | e^{iHt} \Xi_m^3 \sigma_m^i e^{-iHt} \Xi_n^3 \sigma_n^j | h \rangle$$

PDFs can be written as a sum of such correlation functions



QUANTUM CIRCUIT FOR $S_{nm}(t)$

✦ Measure the observable with one auxiliary qubit

Measure the ancillary qubit on X (Y) basis to get the real (imaginary) part of $S_{mn}(t)$

- $|\alpha\rangle_a |0\rangle_b \rightarrow \frac{\sqrt{2}}{2} |\alpha\rangle_a (|0\rangle_b + |1\rangle_b) \rightarrow |\phi\rangle \equiv \frac{\sqrt{2}}{2} (|\alpha\rangle_a |0\rangle_b + \hat{O} |\alpha\rangle_a |1\rangle_b)$
- $\langle \phi | I_a \otimes X_b | \phi \rangle = \frac{1}{2} + Re(\langle \alpha | \hat{O} | \alpha \rangle)$
- $\langle \phi | I_a \otimes Y_b | \phi \rangle = \frac{1}{2} - Im(\langle \alpha | \hat{O} | \alpha \rangle)$

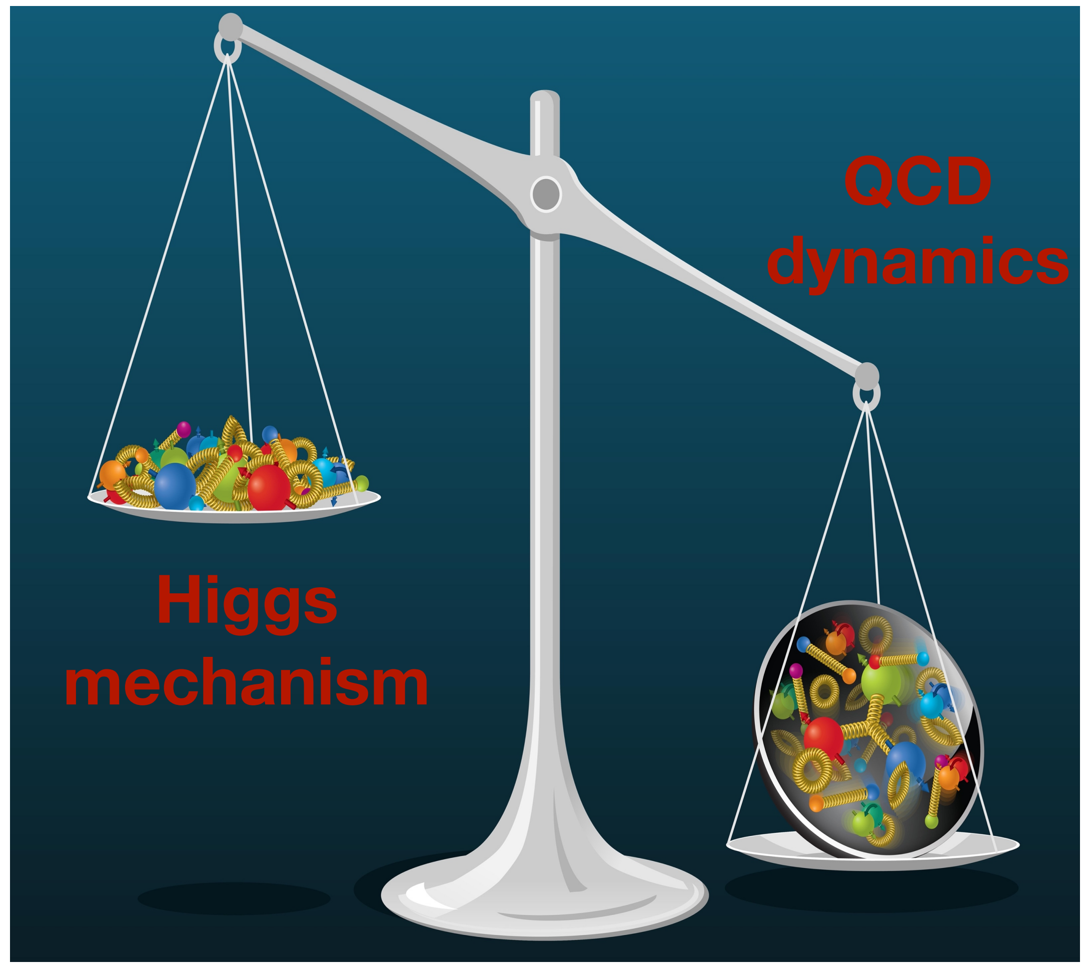
Numerical results from quantum computing

✦ Measurement of hadron mass $M_h = \langle h|H|h\rangle - \langle \Omega|H|\Omega\rangle$

g	0.2	0.4	0.6	0.8	1.0
$M_{h,\mathrm{QC}}a$	1.002	1.810	2.674	3.534	4.352
$M_{h,\mathrm{NUM}}a$	1.001	1.801	2.659	3.509	4.342

$N = 12$

$ma = 0.2$



- Considering the current limitations of using real quantum devices, the results are generated using a classical simulation of the quantum circuit

- Measure the mass of the lowest-lying ud-like hadron in NJL model with 2 flavors, QAOA has good accuracy

- For small quark mass, the dominant contribution comes from the interaction rather than the quark masses

- For $ma = 0.8$, the quark masses are dominant

Numerical results from quantum computing

✦ quark PDF of the lowest-lying zero-charge hadron

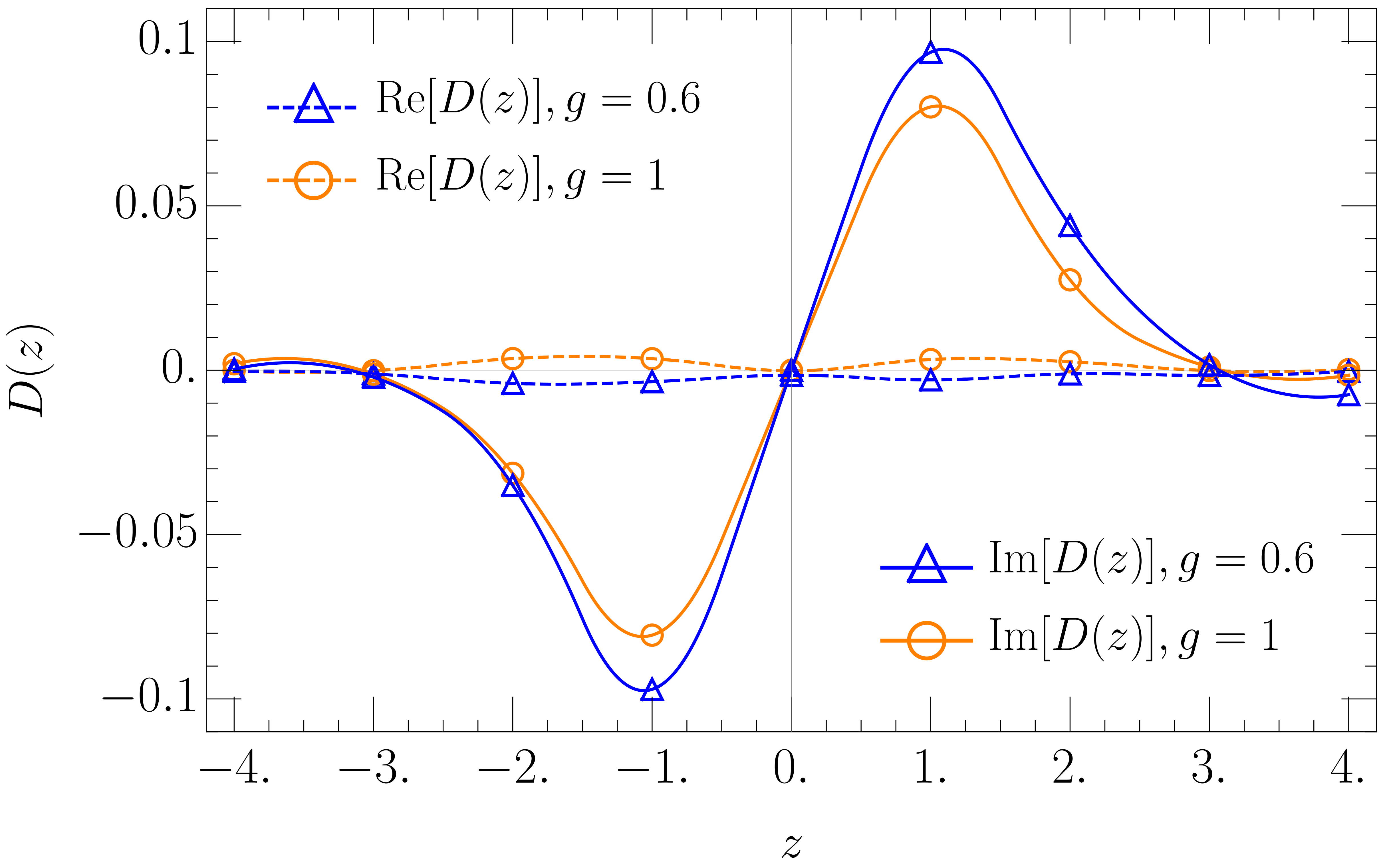
- quark PDF in position space

$$ma = 0.8 \qquad N = 18 \qquad n_f = 1$$

- The real part is consistent with 0

$$f_q(x) = f_{\bar{q}}(x) = -f_q(-x)$$

- The bound state behavior

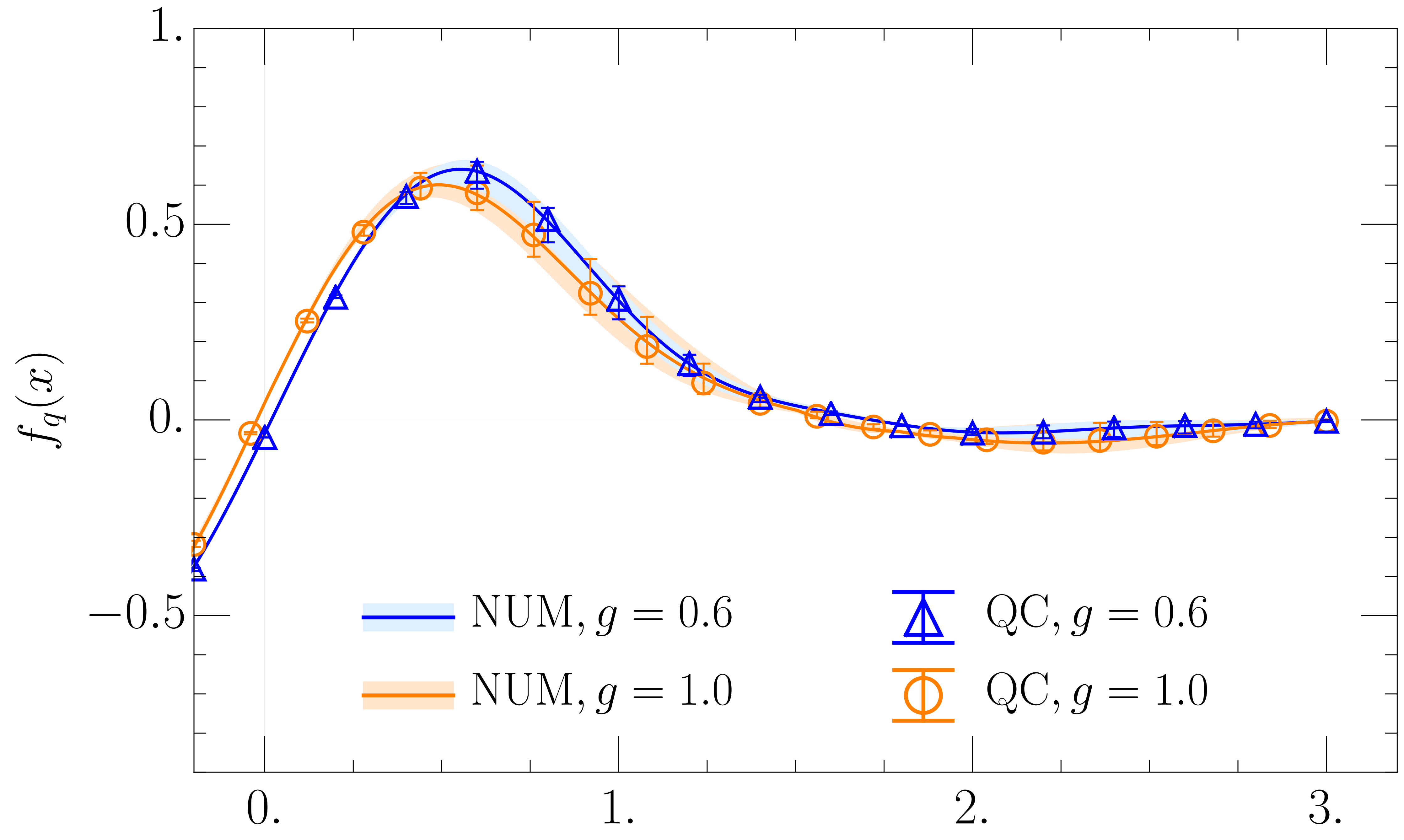


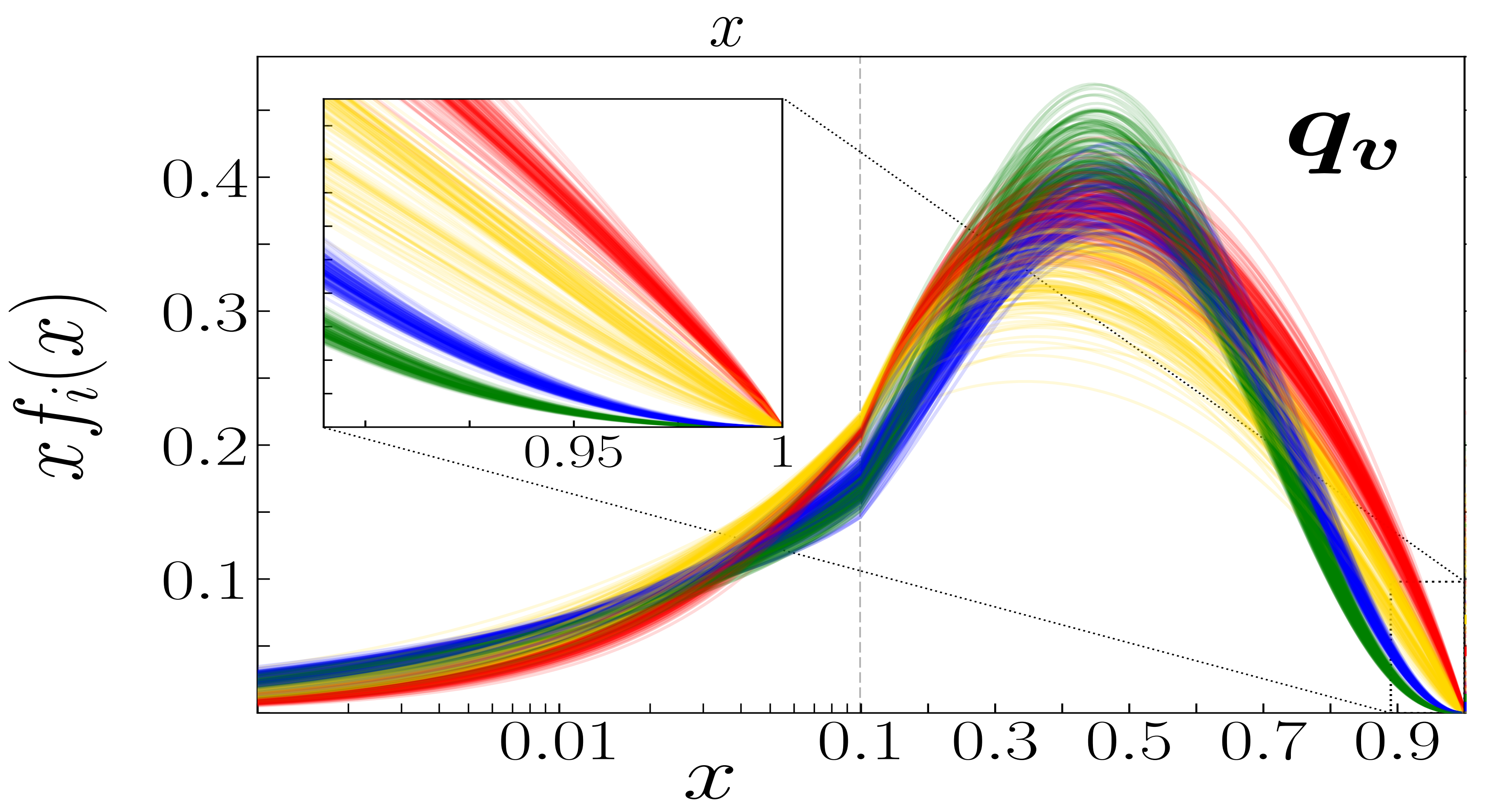
Numerical results from quantum computing

Li et al (QuNu), PRD (letter, 2022)

✦ quark PDF of the lowest-lying zero-charge hadron

- Good agreement between quantum computing and numerical diagonalization
- The non-vanishing contributions in the $x > 1$ are partly due to the finite volume effect
- We observe the expected peak around $x = 0.5$ and qualitative agreement with pion PDFs





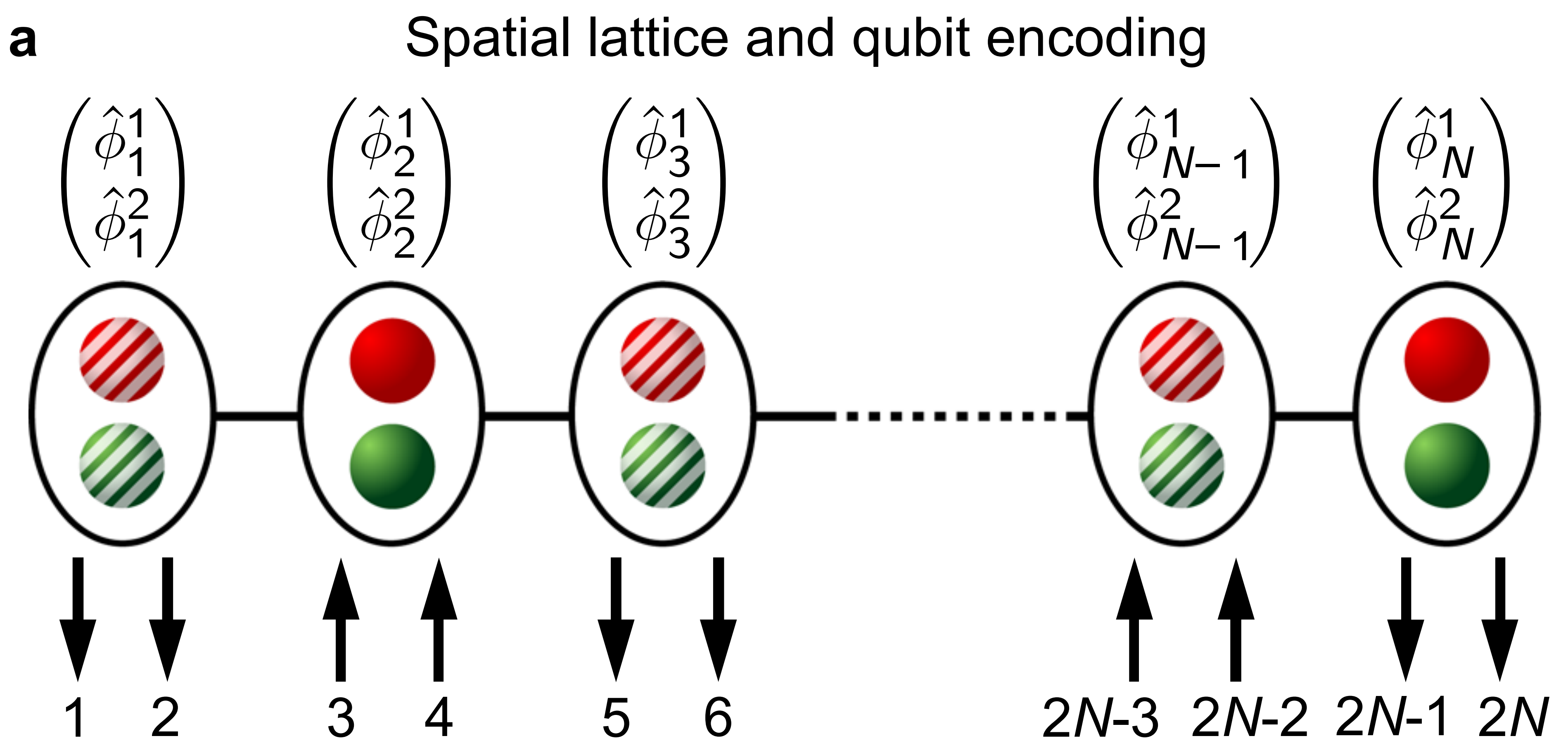
JAM Collaboration, PRL, 2021

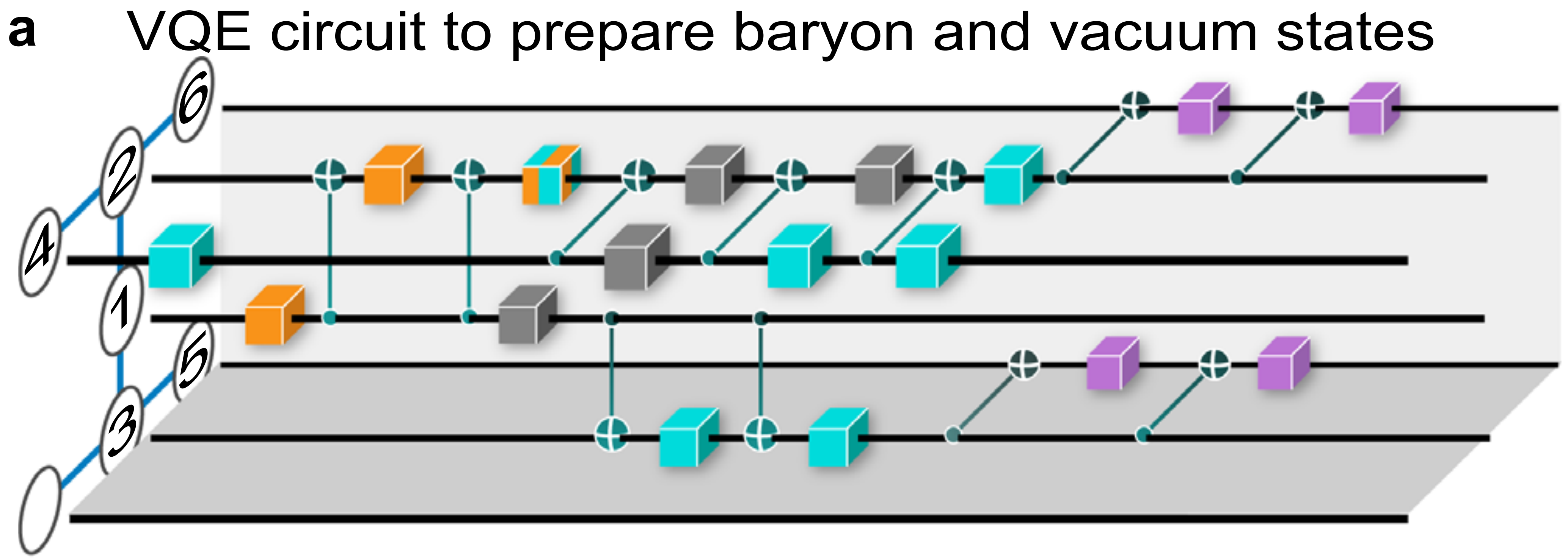
Simulate SU(2) hadron on quantum computer

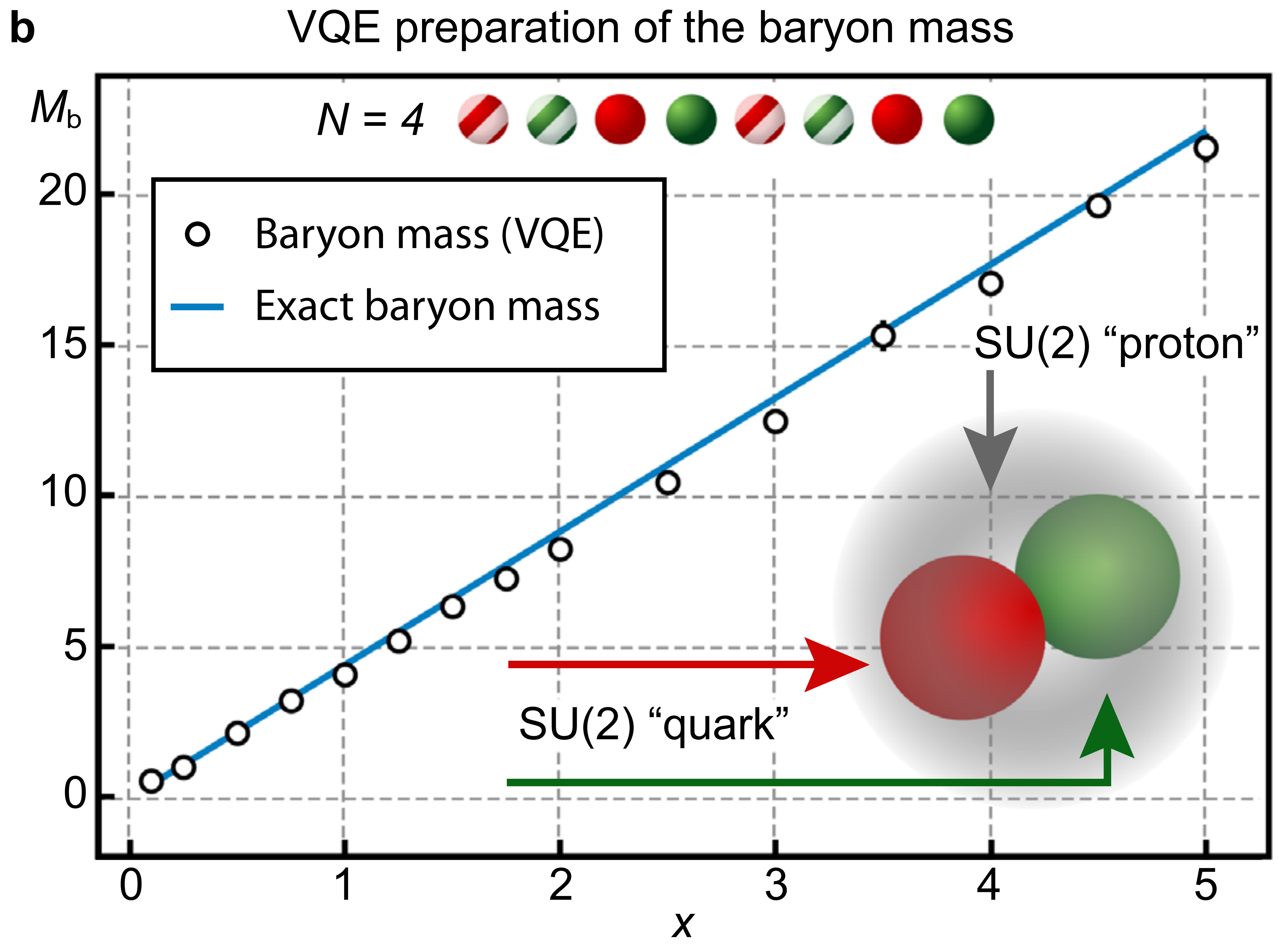
- Global fitting with quantum circuit at initial scale

Atas et al, Nature Commun. 2021

SU(2) Hamiltonian: $$\hat{H}_l = \frac{1}{2a_l}\sum_{n=1}^{N-1}\left(\hat{\phi}_n^\dagger \hat{U}_n \hat{\phi}_{n+1} + \mathrm{H.C.}\right) + m\sum_{n=1}^{N}(-1)^n \hat{\phi}_n^\dagger \hat{\phi}_n + \frac{a_l g^2}{2}\sum_{n=1}^{N-1}\hat{\boldsymbol{L}}_n^2$$







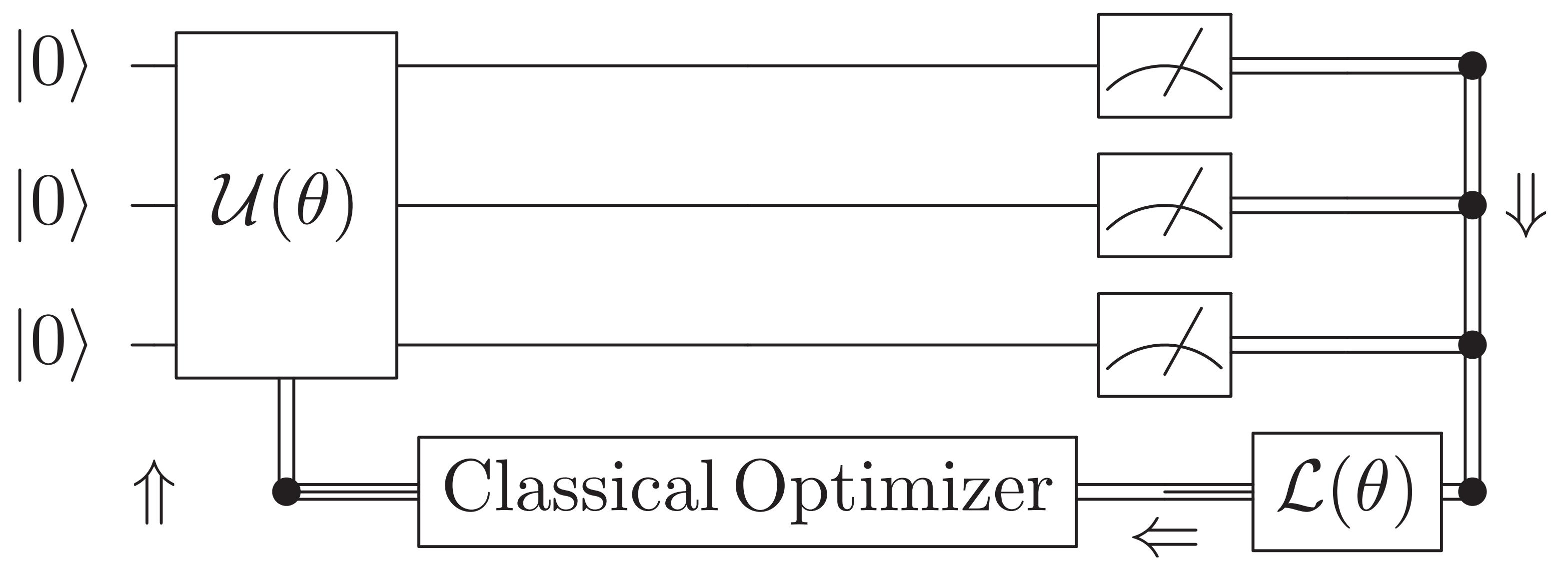
Alternative approaches

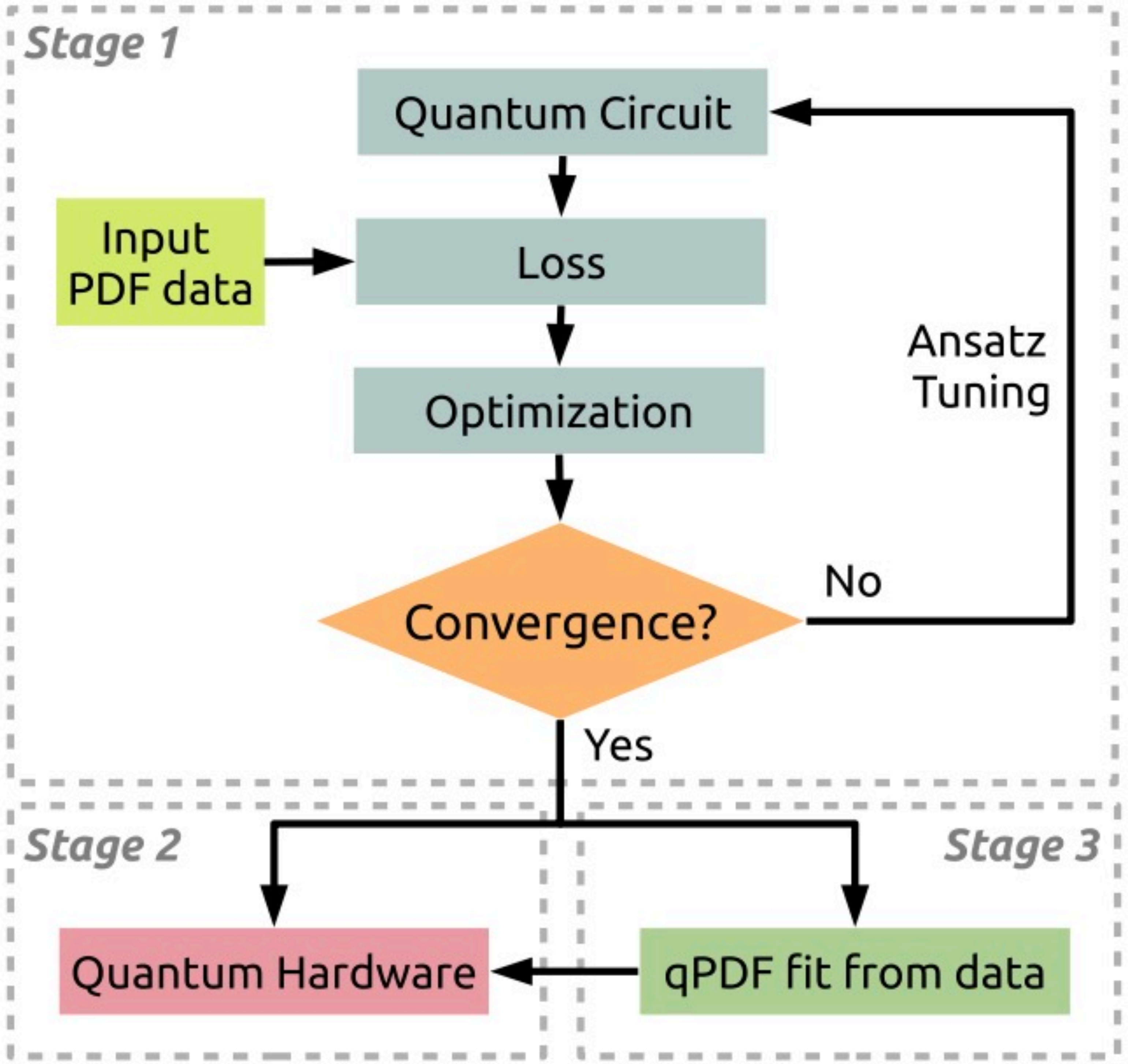
- Global fitting with quantum circuit at initial scale

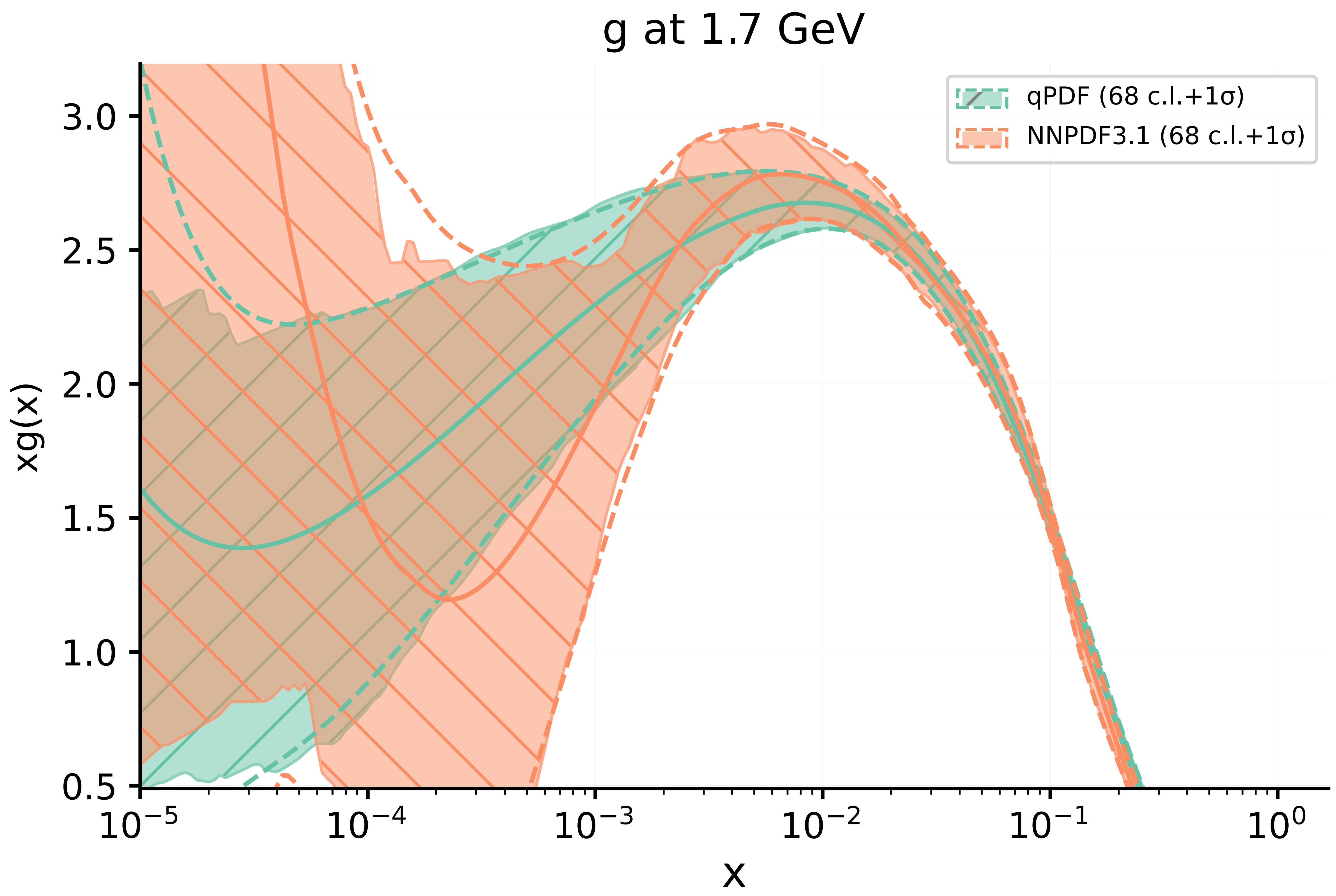
quantum parametrization: $\mathrm{qPDF}_i(x, Q_0, \theta) = \dfrac{1 - z_i(\theta, x)}{1 + z_i(\theta, x)}$

variational quantum circuit: $z_i(\theta, x) = \langle \psi(\theta, x) | Z_i | \psi(\theta, x) \rangle$

$$\mathcal{U}(\theta, x)|0\rangle^{\otimes n} = |\psi(\theta, x)\rangle$$







Salinas et al, PRD 2021

Alternative approaches

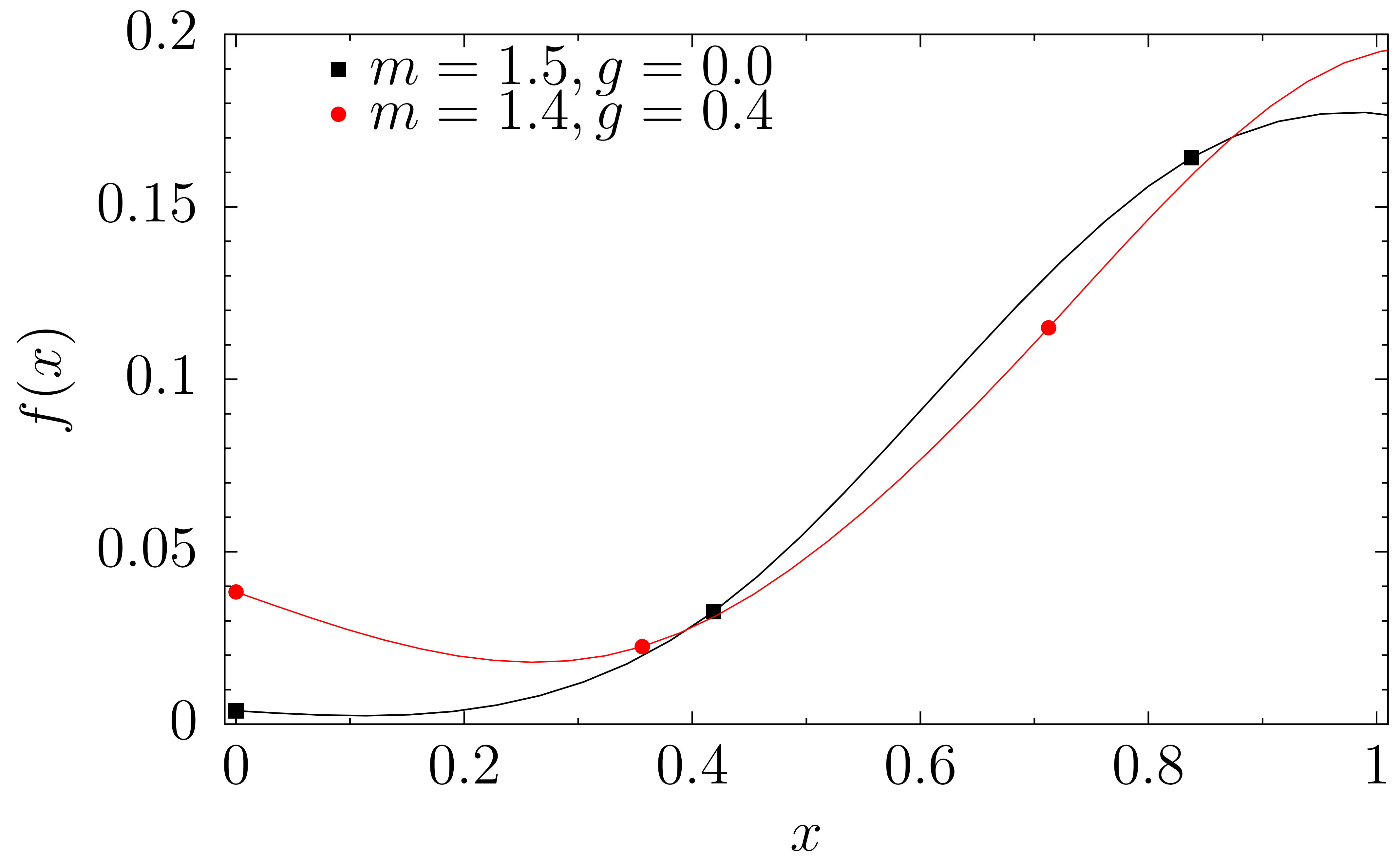
- Global fitting based hadronic tensor

NuQS, PRR 2020

Hadronic tensor: $$W^{\mu\nu}(q) = \mathrm{Re}\int d^d x\, e^{iqx}\langle P|T\{J^\mu(x)J^\nu(0)\}|P\rangle$$

Collinear factorization: $$W^{\mu\nu} = \sum_{i,j} f_i \otimes P_{i\to j} \otimes \hat{W}^{\mu\nu}$$

- A test from exact diagonalization of Hamiltonian in Thirring model



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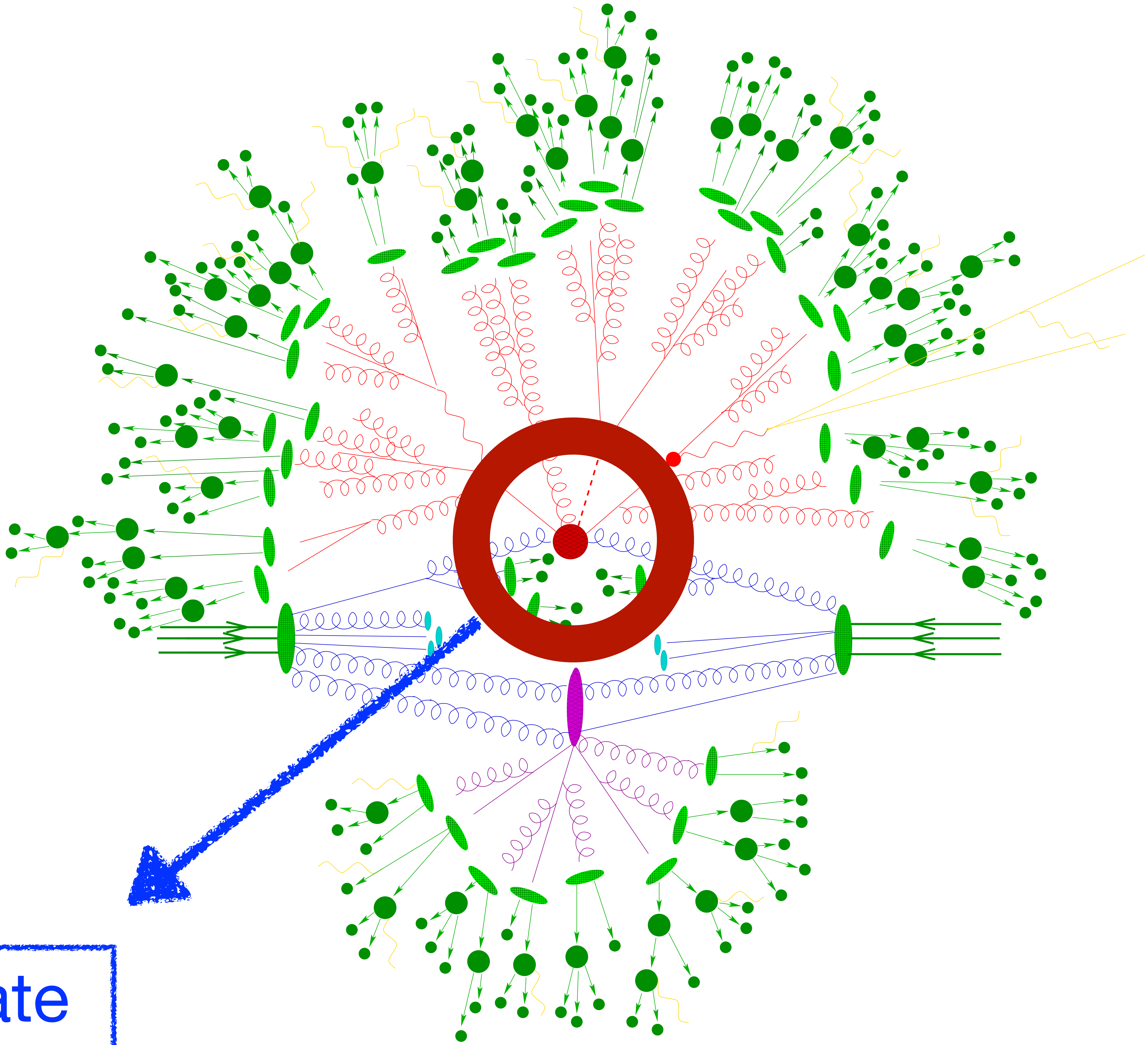

Intermediate state partonic scatterings

Quantum computing for scattering amplitudes

✦ Computing scattering amplitudes for strongly-coupled QFT

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Quantum Algorithms for Quantum Field Theories

STEPHEN P. JORDAN, KEITH S. M. LEE, AND JOHN PRESKILL Authors Info & Affiliations

SCIENCE • 1 Jun 2012 • Vol 336, Issue 6085 • pp. 1130-1133 • DOI: 10.1126/science.1217069

1,061 251

Quantum Leap?

Quantum computers are expected to be able to solve some of the most difficult problems in mathematics and physics. It is not known, however, whether quantum field theories (QFTs) can be simulated efficiently with a quantum computer. QFTs are used in particle and condensed matter physics and have an infinite number of degrees of freedom; discretization is necessary to simulate them digitally. **Jordan *et al.*** (p. 1130; see the Perspective by **Hauke *et al.***) present an algorithm for the efficient simulation of a particular kind of QFT (with quartic interactions) and estimate the error caused by discretization. Even for the most difficult case of strong interactions, the run time of the algorithm was polynomial (rather than exponential) in parameters such as the number of particles, their energy, and the prescribed precision, making it much more efficient than the best classical algorithms.

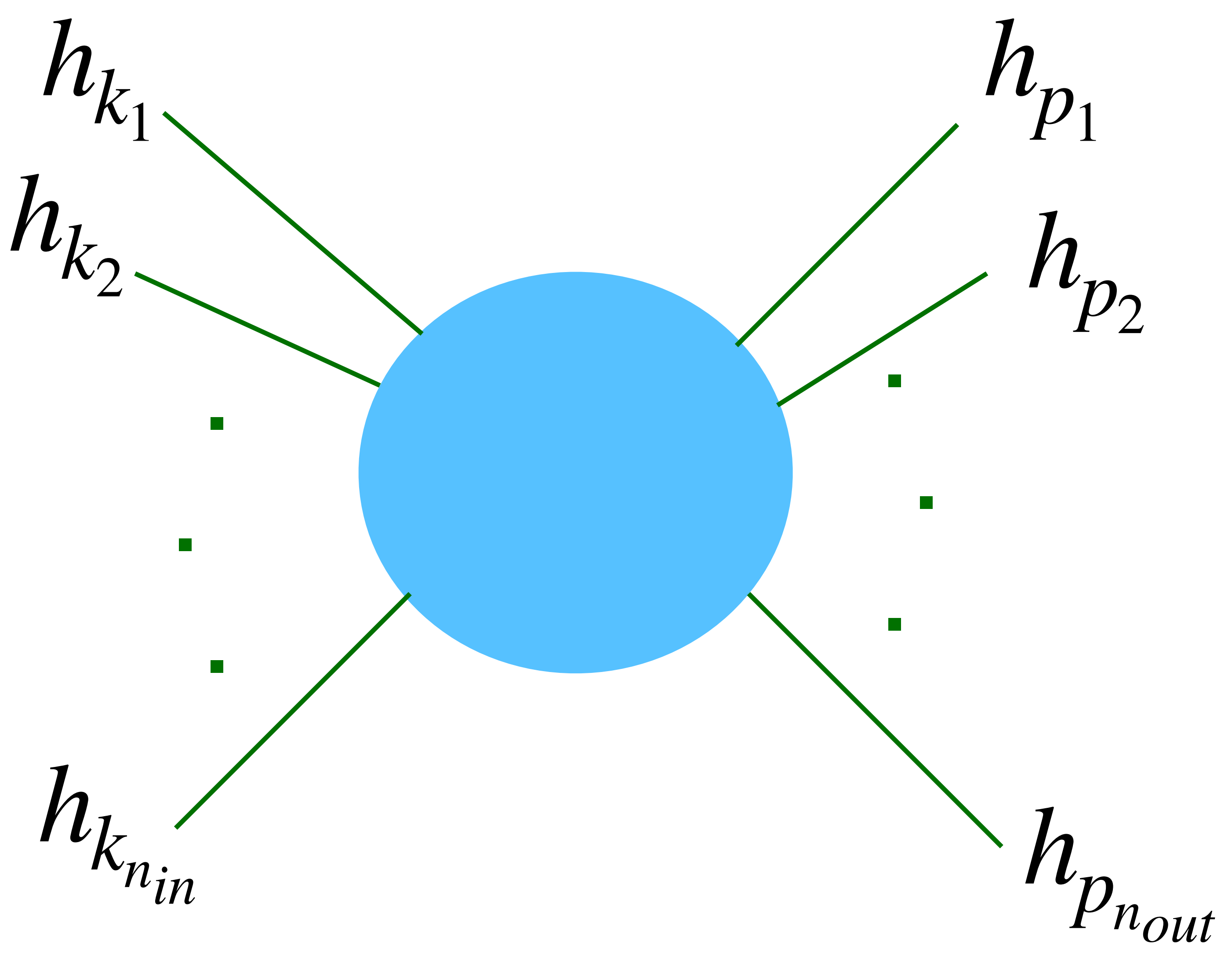
- No reliable way on classical computers (real time dynamics, exponentially costly)
- Quantum computing offers a possible way, complexity scaling polynomially in energies and number of particles.

1. Incoming particles are widely separated wave packets
   $L \gg d_{ij} \gg 1/|p_i|$ —> requires large lattice
2. Adiabatically turn on coupling, interactions happen
   Long time span of evolution, broadening of wave packet
3. Adiabatically turned off coupling, measure final states

Quantum computing for scattering amplitudes

✦ A new proposal - LSZ reduction formula

Li et al (QuNu), arXiv: 2207.13258



- Lehmann-Symanzik-Zimmermann (LSZ) reduction formula

$$i\mathcal{M} = R^{n/2} \lim_{\substack{p_i^2 \to m^2 \\ k_j^2 \to m^2}} G(\{p_i\},\{k_j\}) \left(\prod_{r=1}^{n_{\rm out}} K^{-1}(p_r)\right) \left(\prod_{s=1}^{n_{\rm in}} K^{-1}(k_s)\right)$$

- connected n-point function in momentum space

$$G(\{p_i\},\{k_j\}) = \left(\prod_{i=1}^{n_{\rm out}} \int d^4x_i\, e^{ip_i\cdot x_i}\right) \left(\prod_{j=1}^{n_{\rm in}-1} \int d^4y_j\, e^{-ik_j\cdot y_j}\right)$$
$$\times \langle\Omega| T\left\{\phi(x_1)\cdots\phi(x_{n_{\rm out}})\phi^\dagger(y_1)\cdots\phi^\dagger(y_{n_{\rm in}-1})\phi^\dagger(0)\right\}|\Omega\rangle_{\rm con}$$

- two-point function in momentum space (propagator)

$$K(p) = \int d^4x\, e^{ip\cdot x}\langle\Omega|T\{\phi(x)\phi^\dagger(0)\}|\Omega\rangle$$

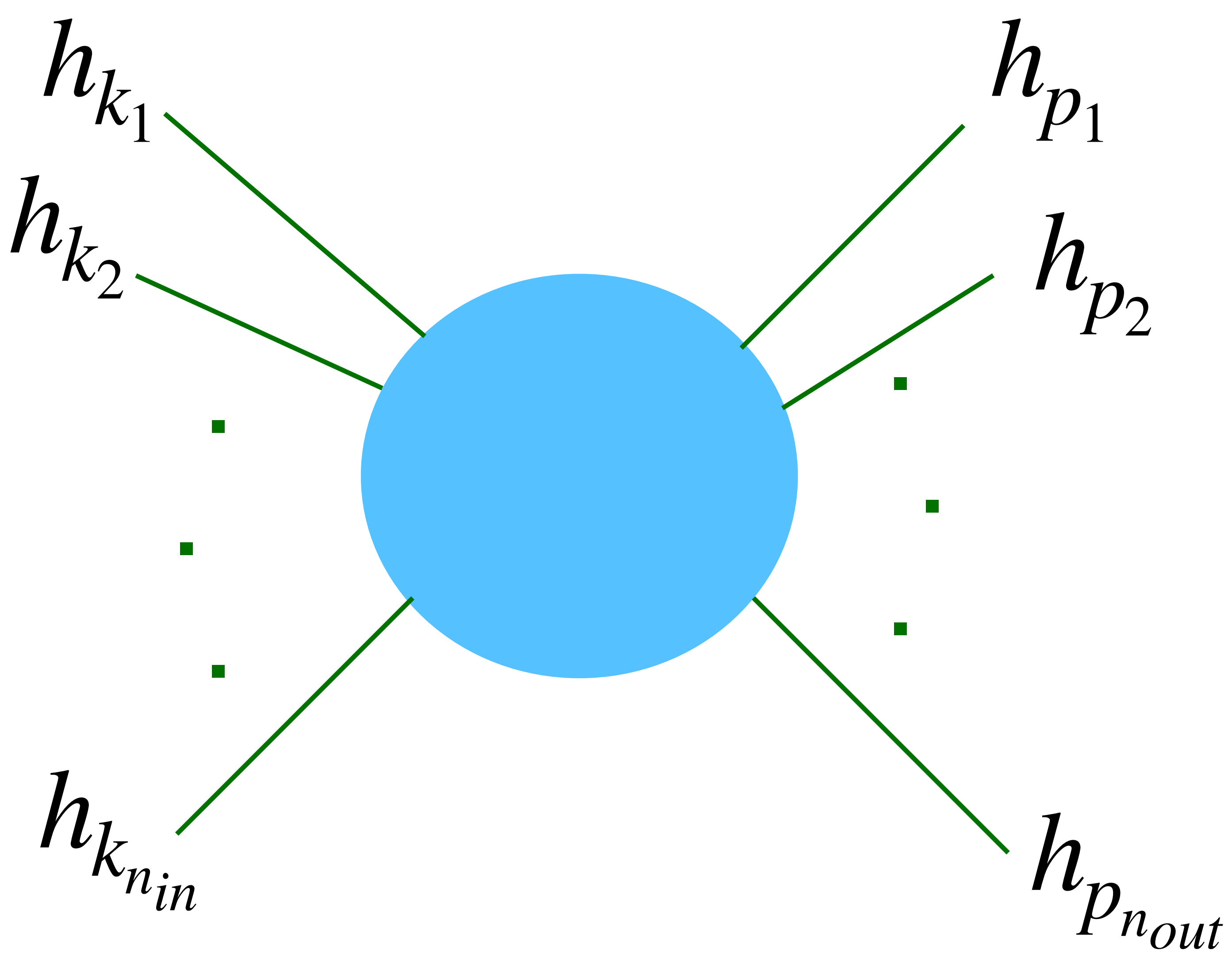
- field normalization

$$R = |\langle\Omega|\phi(0)|h(\boldsymbol{p}=0)\rangle|^2$$

Quantum computing for scattering amplitudes

✦ A new proposal - LSZ reduction formula

Li et al (QuNu), arXiv: 2207.13258



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- two-point function in momentum space (propagator)

$$K(p) = \int d^4x\, e^{ip\cdot x}\langle\Omega|T\{\phi(x)\phi^\dagger(0)\}|\Omega\rangle$$

pole singularities cancel on mass-shell, giving finite scattering amplitude

- field normalization

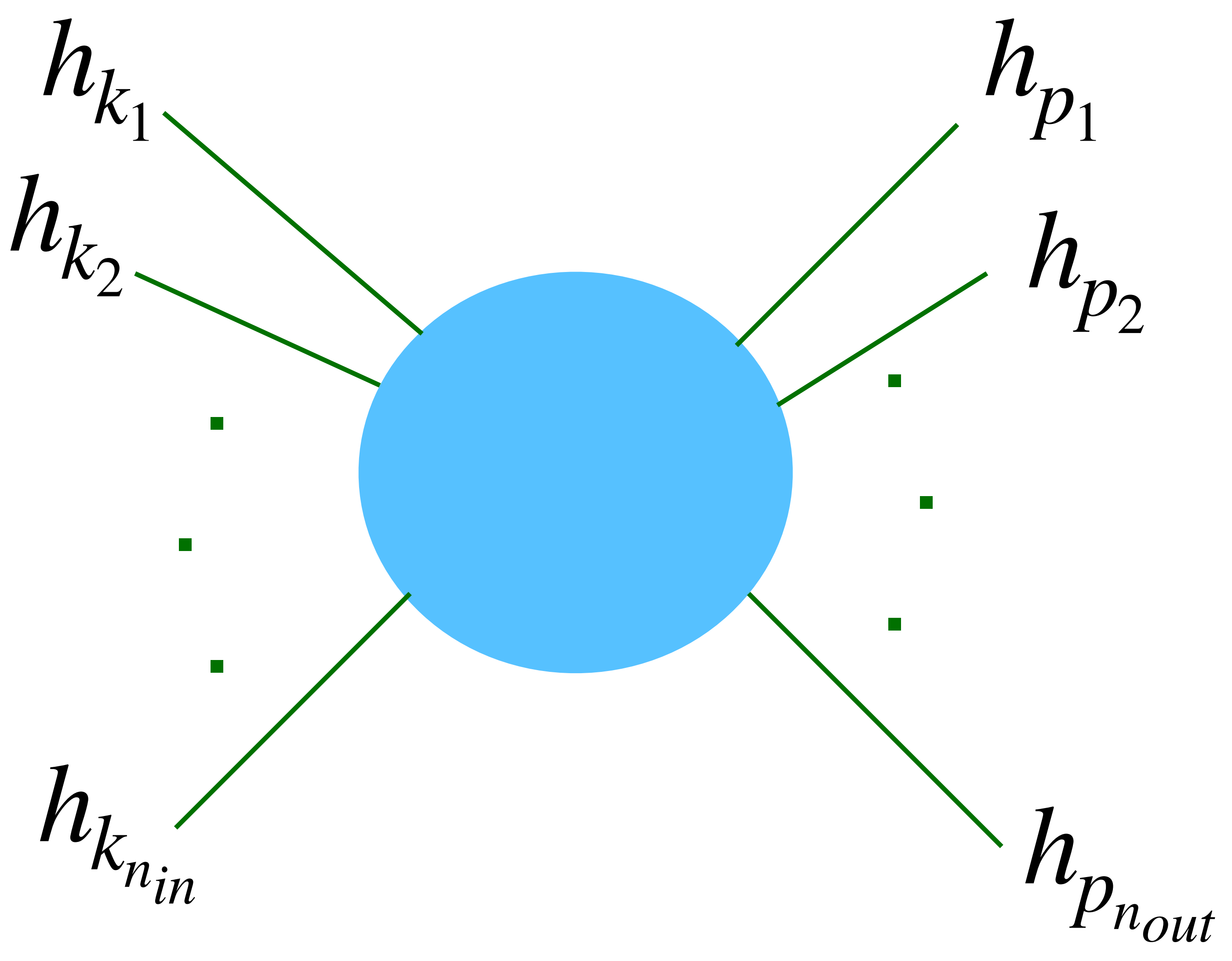
$$R = |\langle\Omega|\phi(0)|h(\boldsymbol{p}=0)\rangle|^2$$

→ QAOA for $|\Omega\rangle$ and $|h\rangle$

Quantum computing for scattering amplitudes

✦ A new proposal - LSZ reduction formula

Li et al (QuNu), arXiv: 2207.13258



- Lehmann-Symanzik-Zimmermann (LSZ) reduction formula

$$i\mathcal{M} = R^{n/2} \lim_{\substack{p_i^2 \to m^2 \\ k_j^2 \to m^2}} G(\{p_i\},\{k_j\}) \left(\prod_{r=1}^{n_{\rm out}} K^{-1}(p_r)\right)\left(\prod_{s=1}^{n_{\rm in}} K^{-1}(k_s)\right)$$

- No preparation of incoming wave packets, smaller lattice is allowed.

- No adiabatic turn on and turn off of coupling constants, no associated extra time evolution

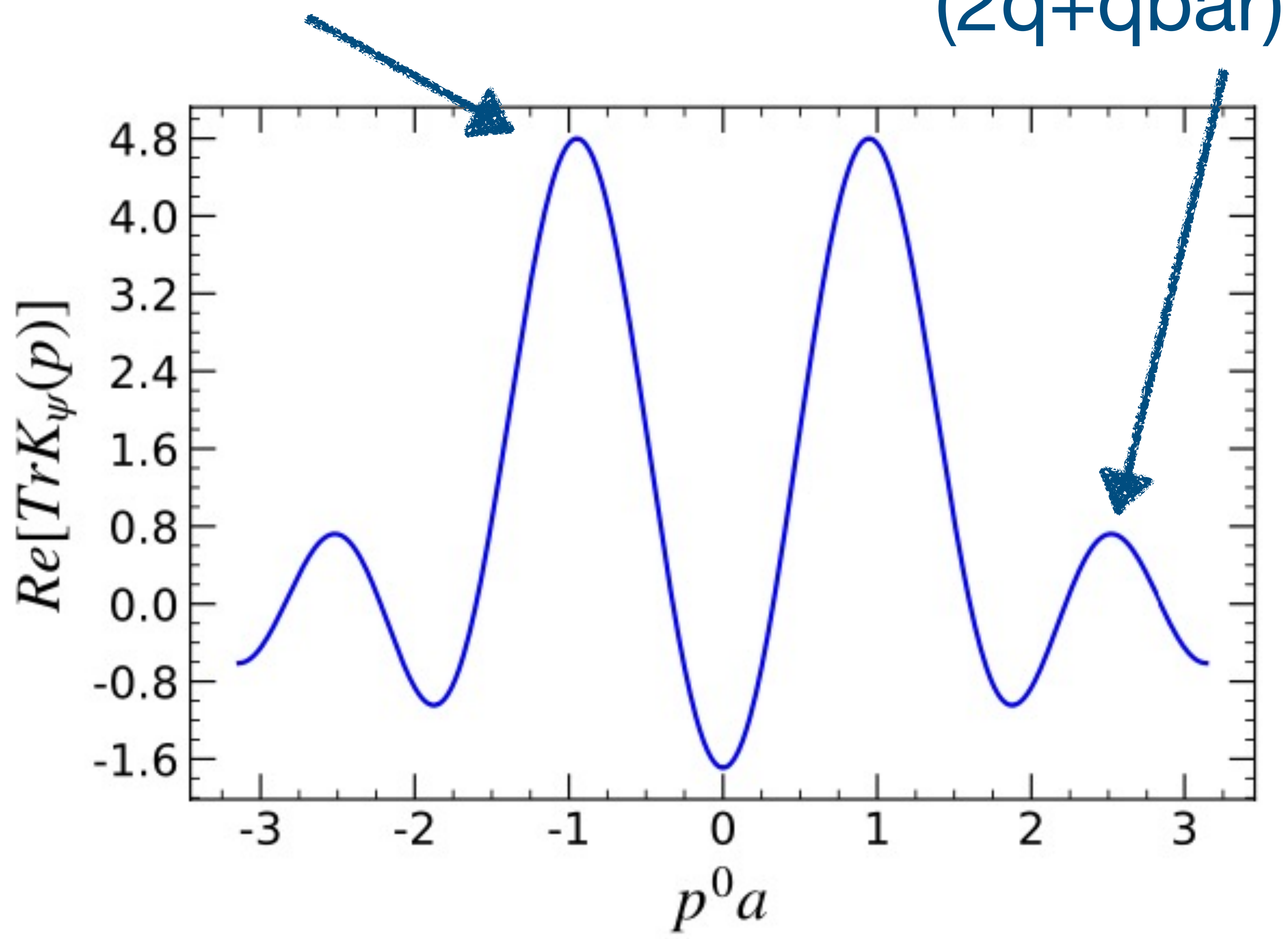
- Bound-states are allowed as incoming and outgoing particles

- Complexity scales exponentially in particle number n, ideal for exclusive scattering process, e.g. $2 \to 2$ scattering. JLP formalism scales polynomially with n.

Quantum computing for scattering amplitudes

✦ LSZ reduction formula - 1+1 NJL

- Fermion propagator $K_\psi(p) = \int d^2x \, e^{ip\cdot x} \langle \Omega | T\{\psi(x)\bar{\psi}(0)\} | \Omega \rangle$

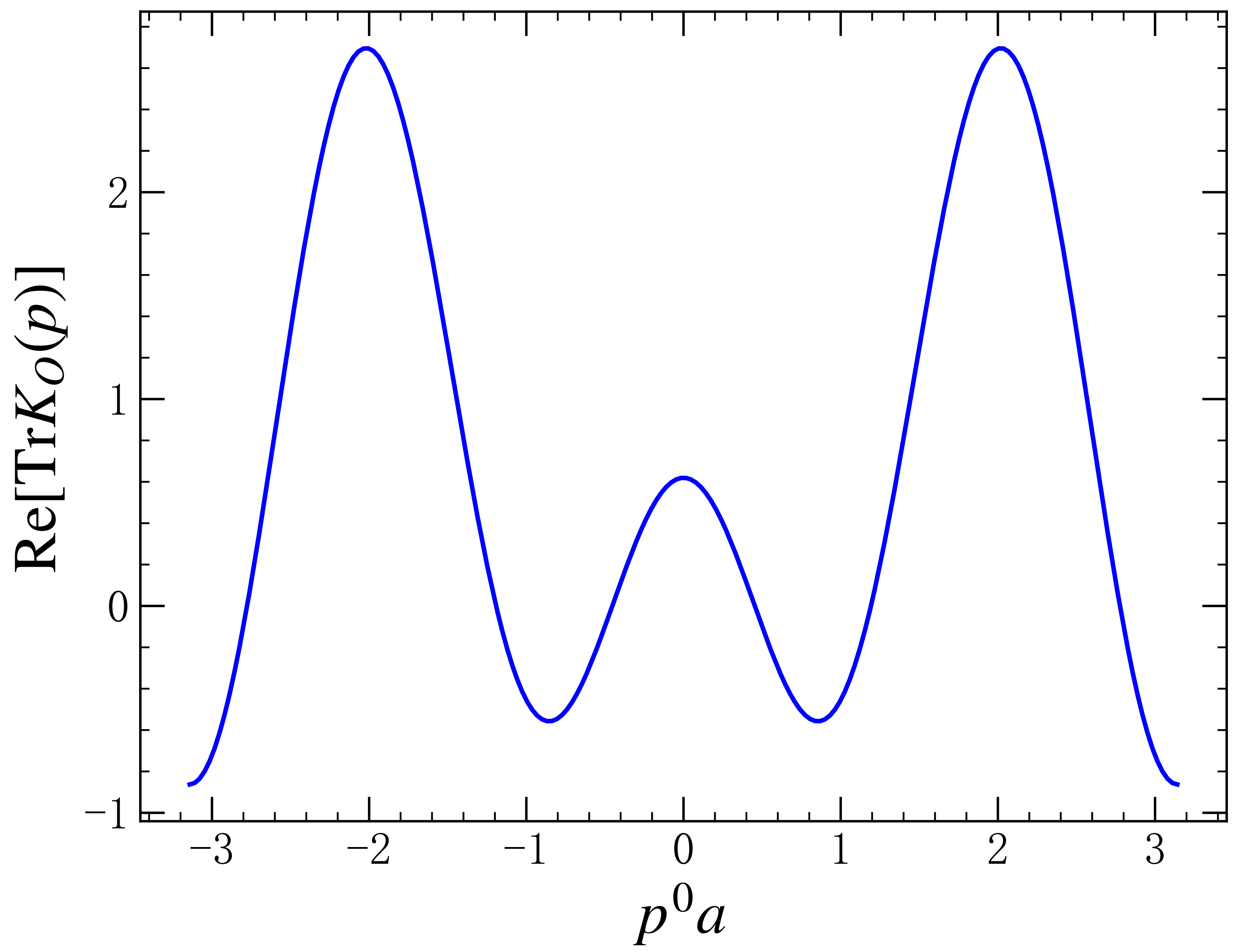


Real part of $\mathrm{Tr} K_\psi(p)$ a function of $p^0 a$ with $p^1 = 0$.

- propagator of composite operator

$$K_O(p) = \int d^2x \, e^{ip\cdot x} \langle \Omega | T\{O(x)O(0)\} | \Omega \rangle_{\mathrm{con}}$$

$$O(x) = \bar{\psi}(x)\psi(x)$$

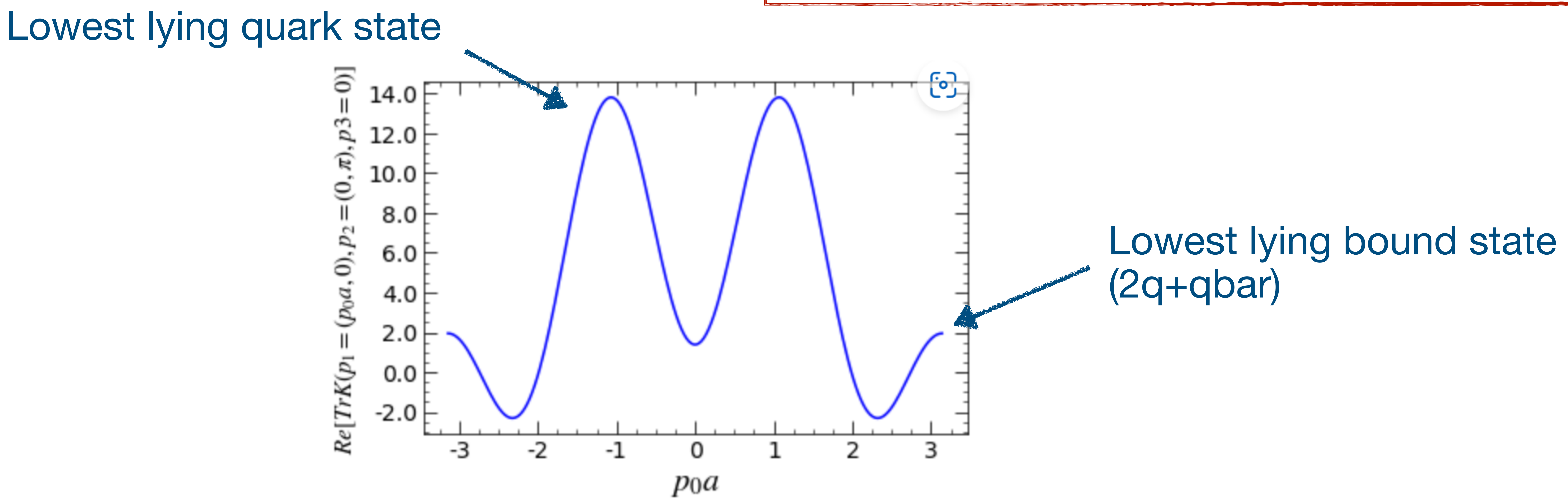


Quantum computing for scattering amplitudes

✦ LSZ reduction formula - 1+1 NJL

- Four point correlation function

Our quantum algorithm succeeds in recovering the expected pole structure, which is crucial to the implementation of LSZ formula.



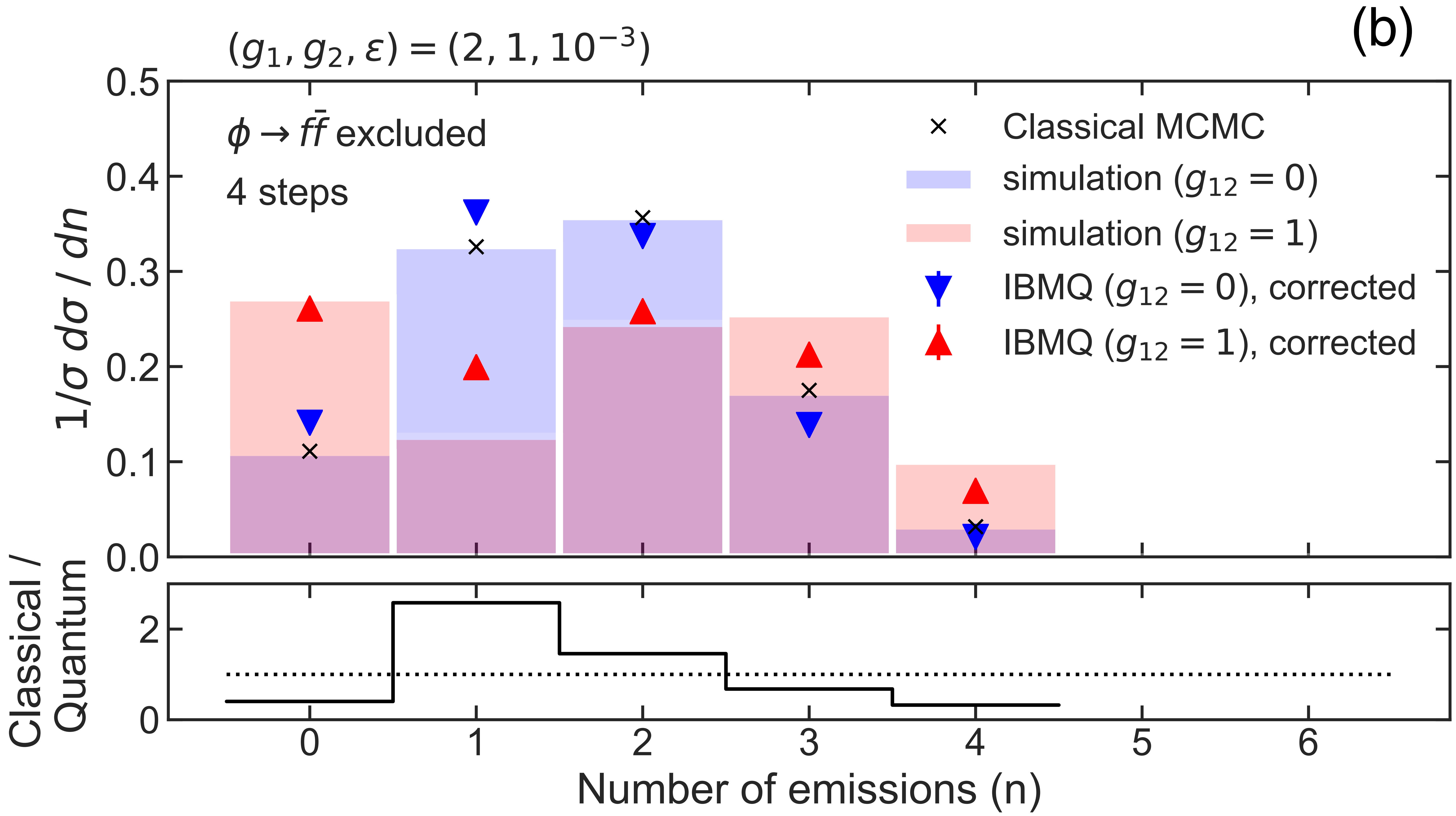
Real part of $G_{\psi}^{\alpha\beta\alpha\beta}(p_1, p_2, k_1, 0)$ as a function of $p_1^0 a$, with $p_1 = (p_1^0, 0)$, $p_2 = (0, \pi/a)$, $k_1 = (0, 0)$.

Quantum parton shower

✦ Simulate the quantum interference effect

Nachman et al, PRL 2021

$$\mathcal{L} = \bar{f}_1(i\partial\!\!\!/ + m_1)f_1 + \bar{f}_2(i\partial\!\!\!/ + m_2)f_2 + (\partial_\mu\phi)^2$$
$$+ g_1\bar{f}_1 f_1\phi + g_2\bar{f}_2 f_2\phi + g_{12}[\bar{f}_1 f_2 + \bar{f}_2 f_1]\phi$$



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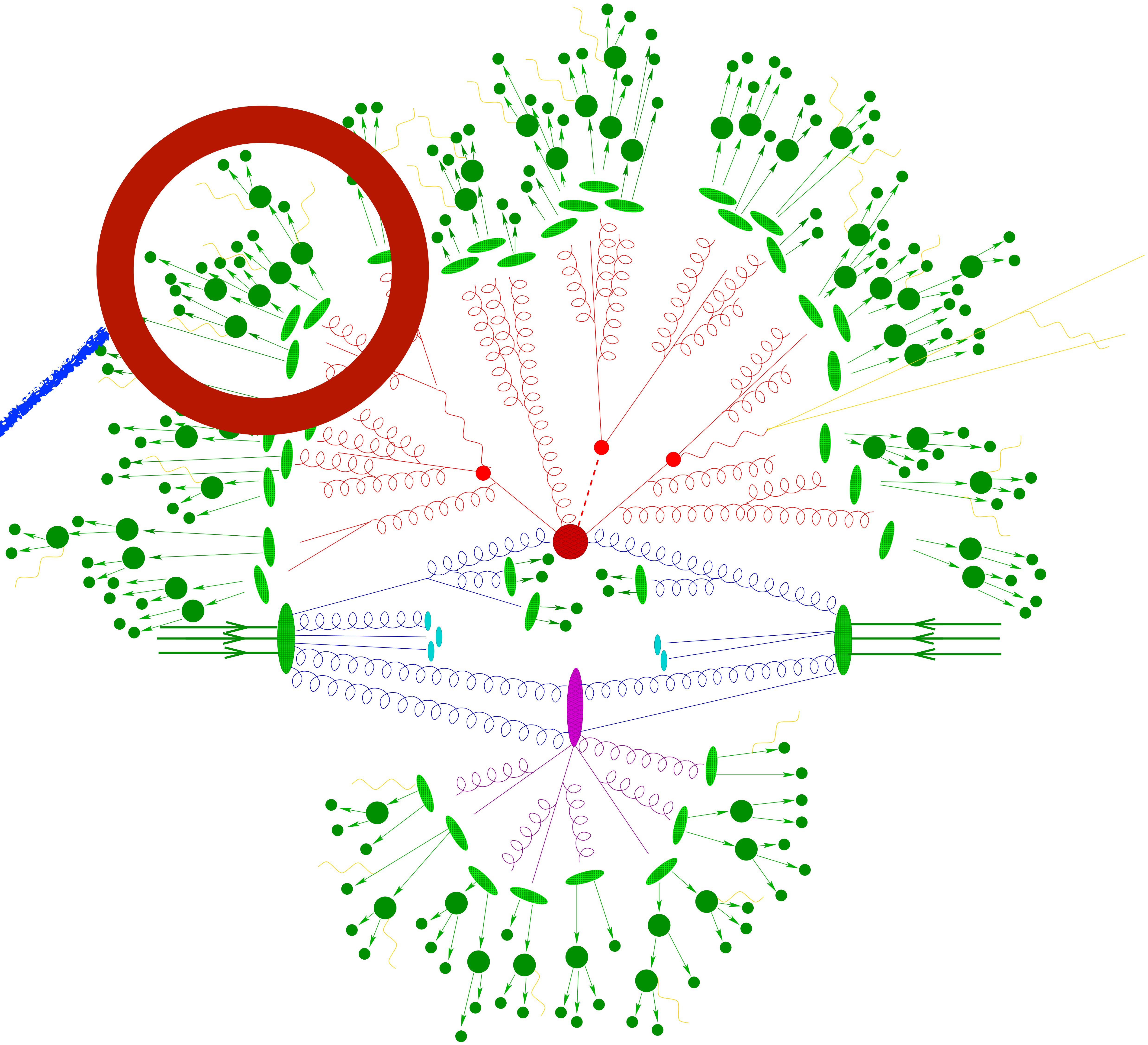


Final state

hadron fragmentation function $D_{q \to h}$

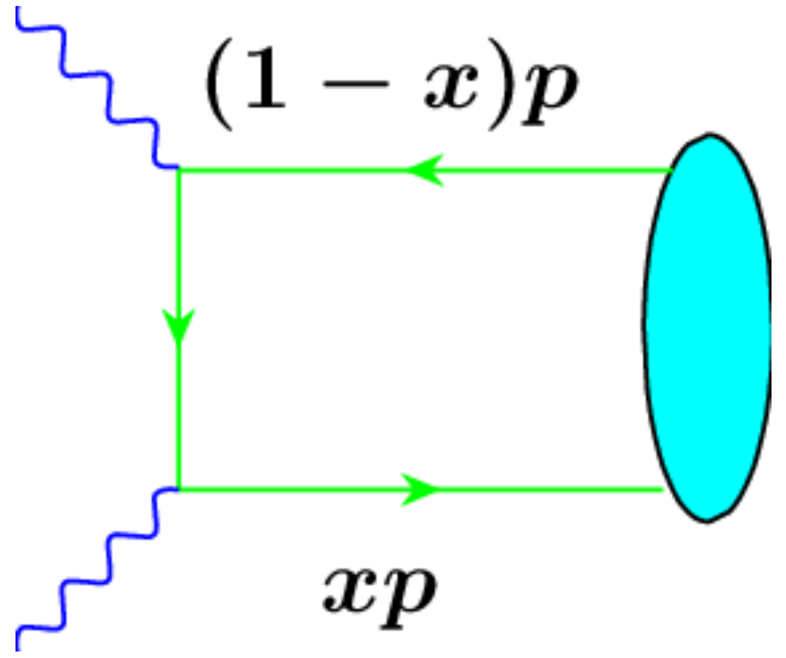
Quantum computing for exclusive hadronization

✦ LCDA - light cone distribution amplitude, describes the formation/decay of a hadron

✦ LCDA is an essential ingredient in exclusive high-energy QCD processes, e.g. form factor in the process $\gamma^*\gamma \to \pi^0$

$$F(Q^2) = f_\pi \int_0^1 dx\, T_H(x, Q^2; \mu)\phi_\pi(x; \mu) + \mathcal{O}(\Lambda^2_{\mathrm{QCD}}/Q^2)$$

$$\phi(x) = \frac{1}{f}\int dz e^{-i(x-1)n\cdot Pz}\langle\Omega\,|\,\bar{\psi}(zn)\gamma^+\psi(0)\,|\,h(P)\rangle$$



✦ The current knowledge on LCDA is limited, mainly on models and lattice calculations

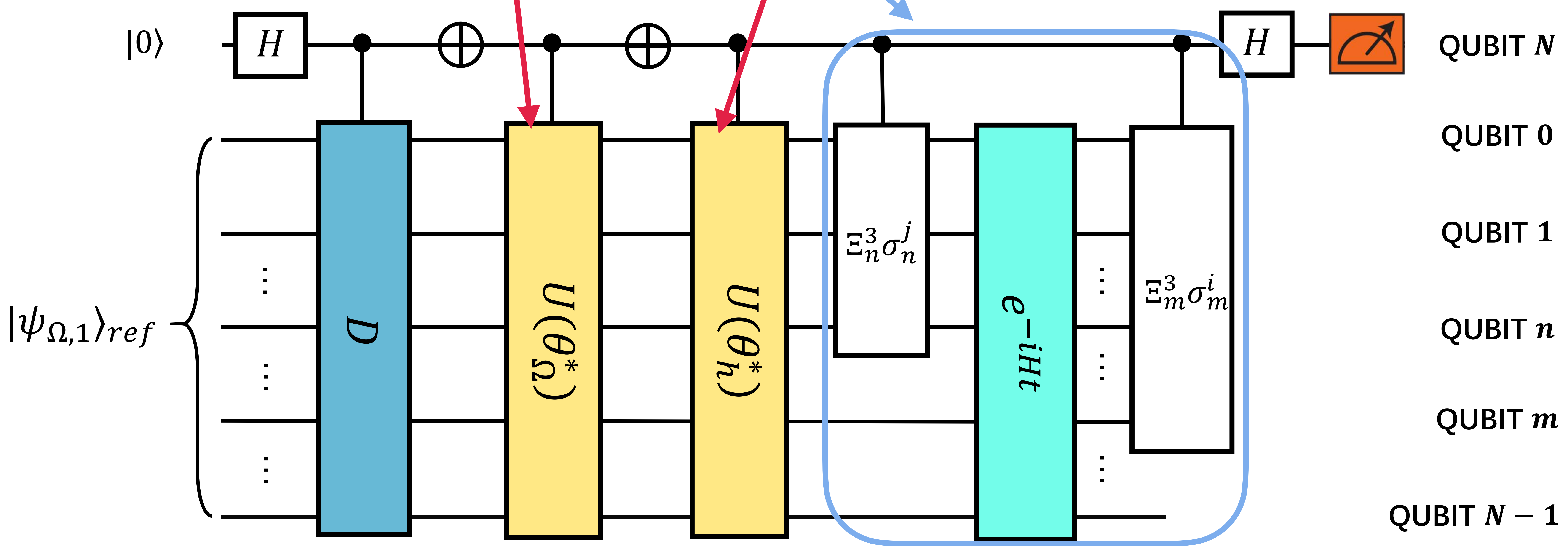
✦ First try using quantum computing

LCDA on quantum computer

✦ Quantum circuit

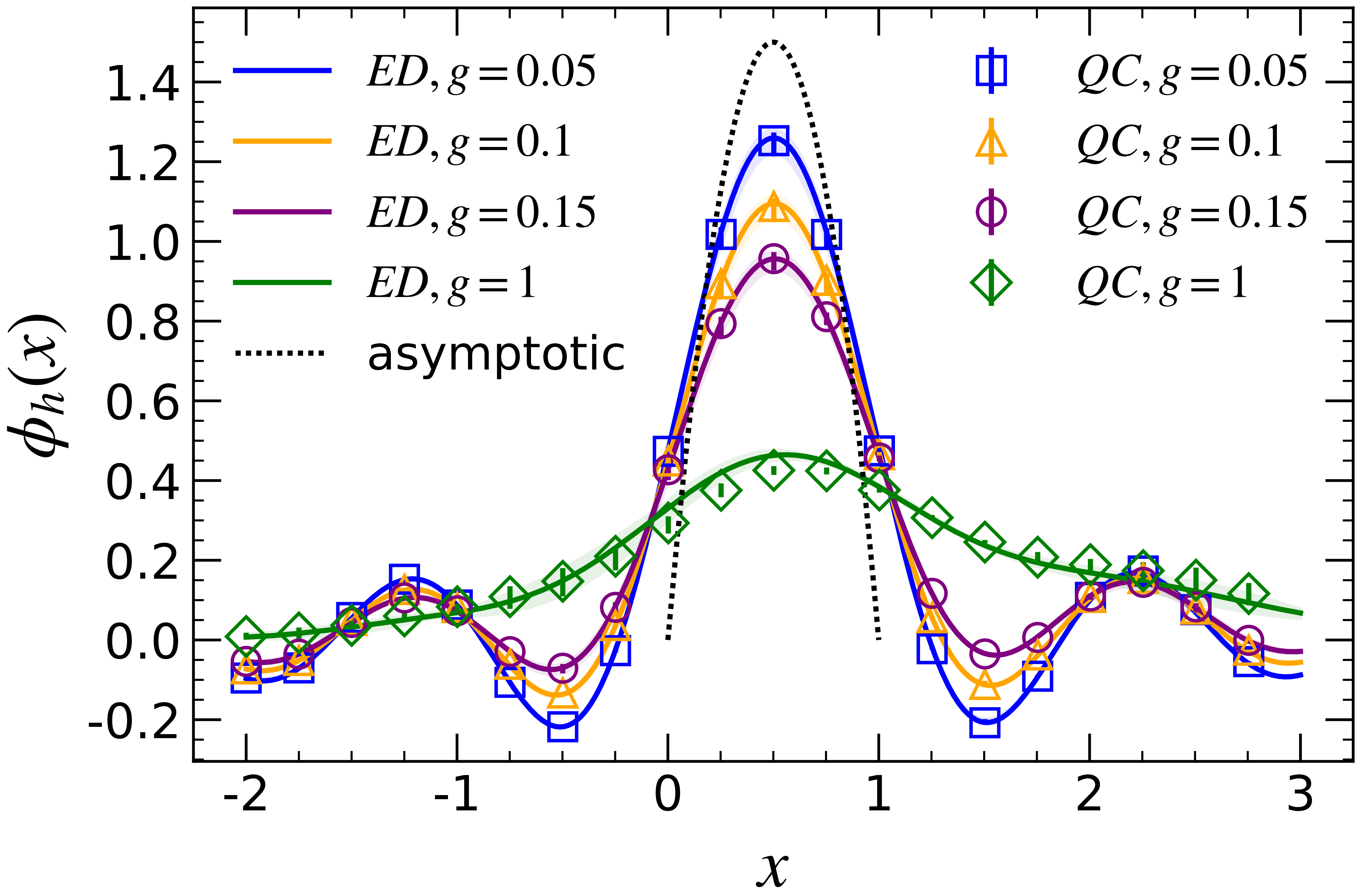
Li et al (QuNu), SCPMA (2023)

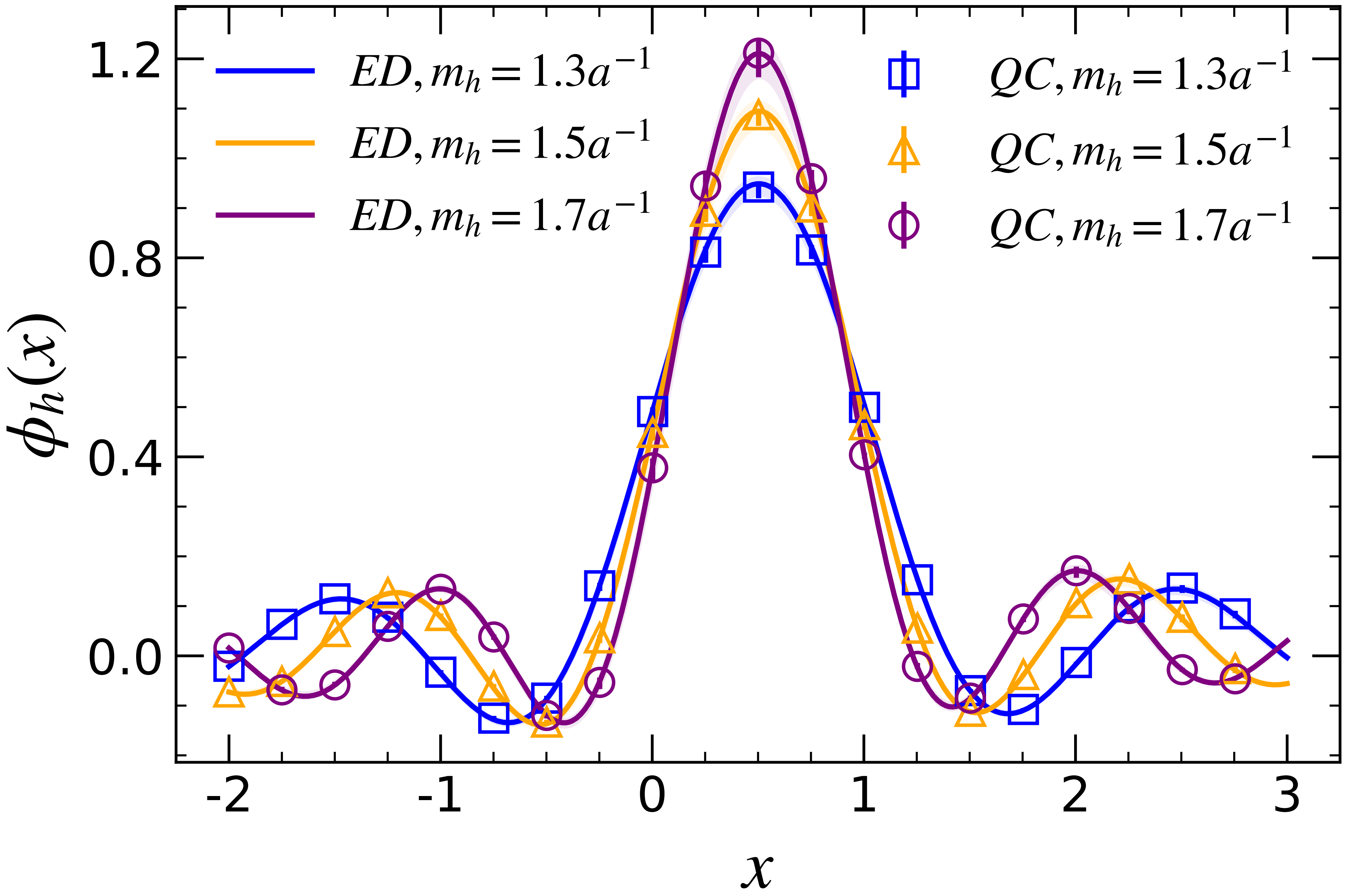
$$|\phi'\rangle = \frac{1}{\sqrt{2}}(|\Omega\rangle|0\rangle + \hat{O}|h\rangle|1\rangle)$$



LCDA on quantum computer

✦ Numerical results

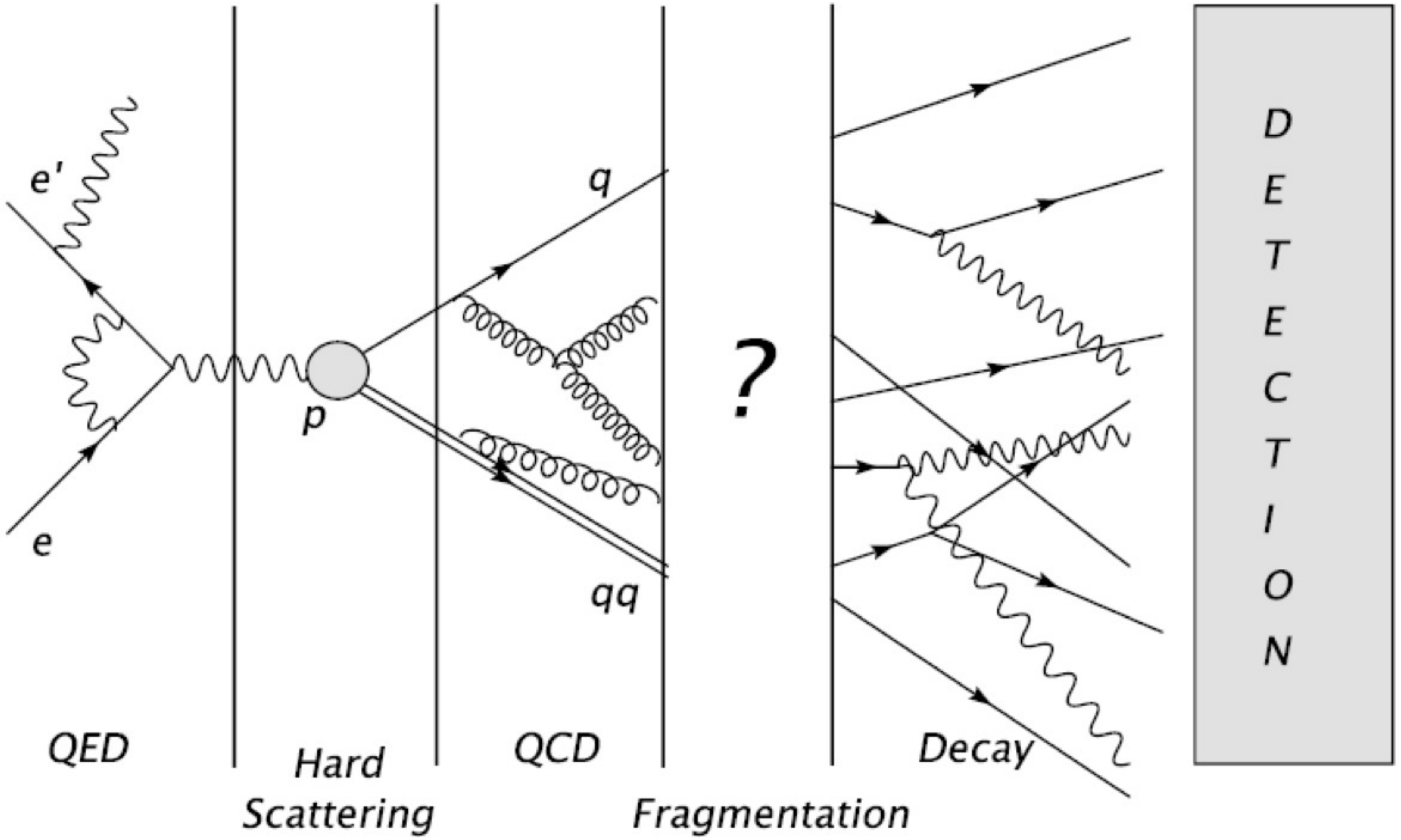


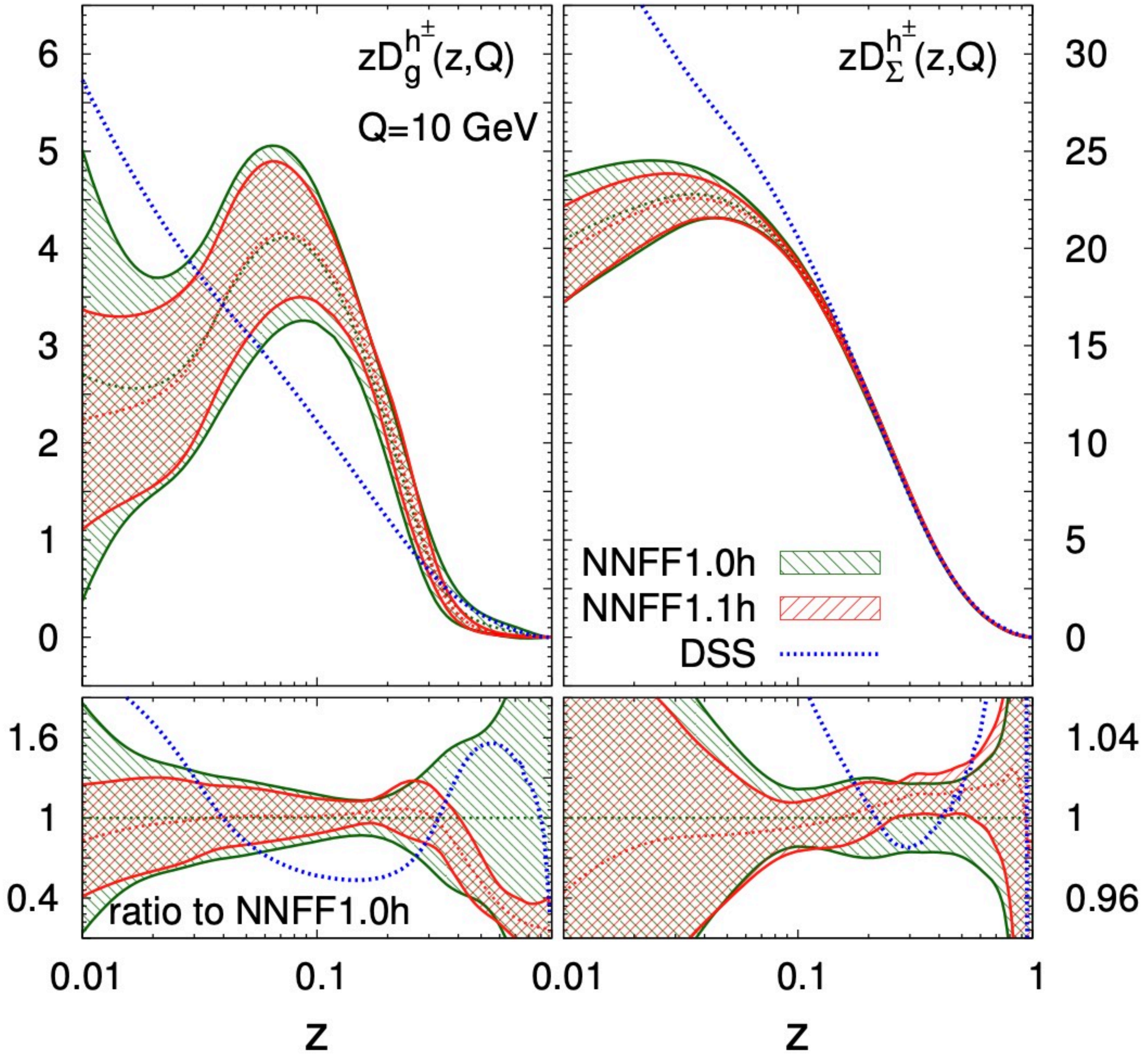


- peak gets narrower with decreasing coupling constant or increasing hadron mass
- Converges to asymptotic result in weak coupling limit

Quantum computing for inclusive hadron fragmentation functions

- ✦ Global fitting - the only reliable way to extract hadron fragmentation functions





Quantum computing for inclusive hadron fragmentation functions

✦ The first attempt using quantum computing

Li et al (QuNu), in preparation

$$D_{q\to h}(z) = z\sum_X \int \frac{dy^-}{2\pi} e^{ik^+y^-} Tr\left[\langle 0|\psi_q(y^-)|h,X\rangle\langle h,X|\bar{\psi}(0)|0\rangle\gamma^+\right] \qquad y^- = \frac{1}{\sqrt{2}}(y_0 - y_3)$$

- Challenge in lattice QCD: FF is a real time dynamical function; can not define $|h,X\rangle$
- Quantum computing:

1. Using VQE to construct multi-hadron state $|h,X\rangle$

$$|\Omega\rangle = U|I_0^0, I_1^0, \ldots, I_{M-1}^0\rangle, \qquad |h^\alpha(i)\rangle = U|I_0^0, \ldots, I_{i-1}^0, I_i^\alpha, I_{i+1}^0, \ldots, I_{M-1}^0\rangle = U|\tilde{h}^\alpha\rangle,$$

$$|h^\alpha(i), h^\beta(j)\rangle = U|I_0^0, \ldots, I_{i-1}^0, I_i^\alpha, I_{i+1}^0, \ldots, I_{j-1}^0, I_j^\beta, I_{j+1}^0, \ldots, I_{M-1}^0\rangle, \qquad \cdots$$

2. construct hadron projector

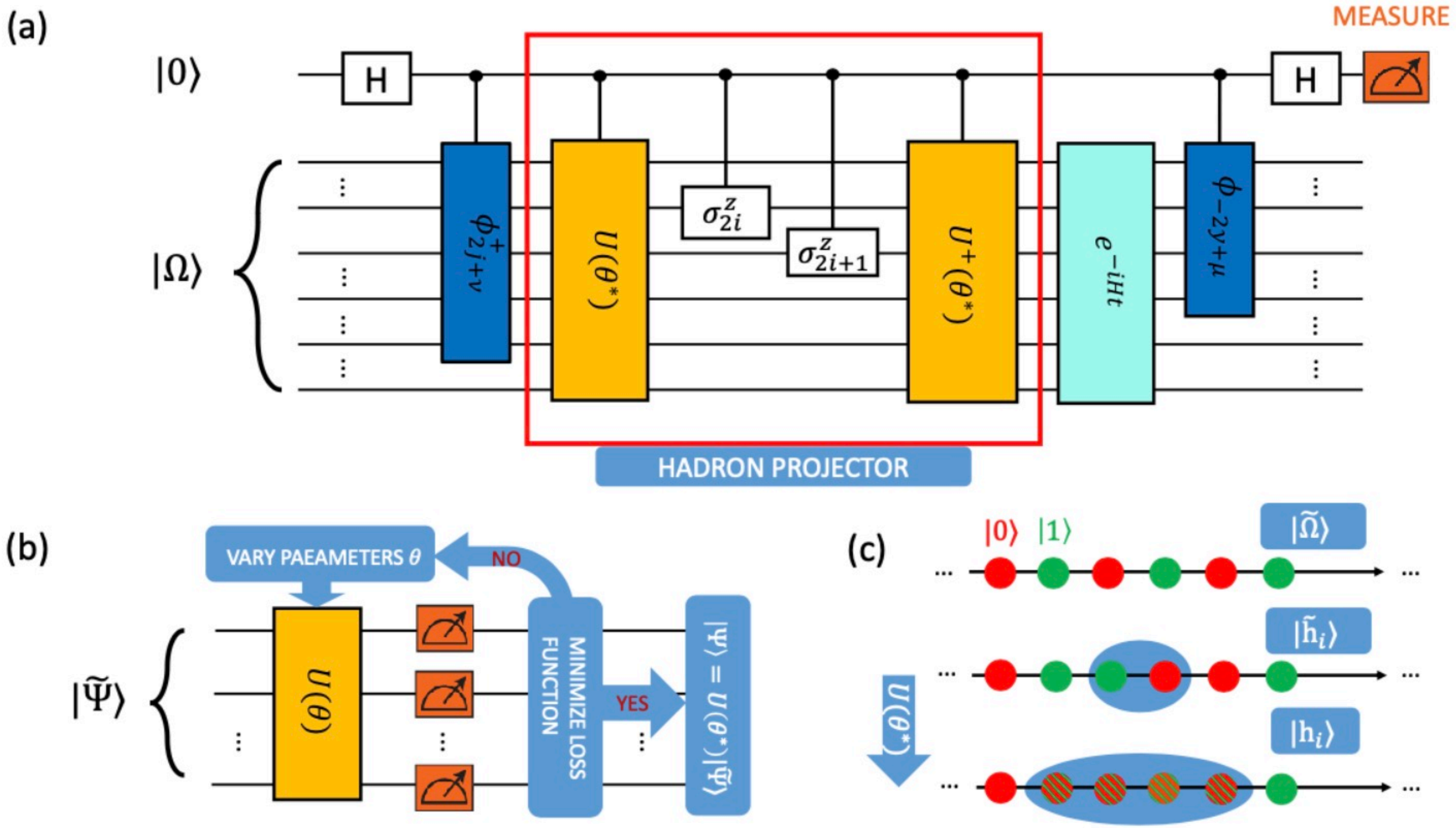
$$P_h = \sum_X |h,X\rangle\langle h,X| = \frac{1}{\sqrt{M}}\sum_{j=0}^{M-1}\sum_{i=0}^{M-1} U|I_i^h\rangle\langle I_i^h|U^\dagger T^j$$

T^j is translational operator on lattice

Quantum computing for inclusive hadron fragmentation functions

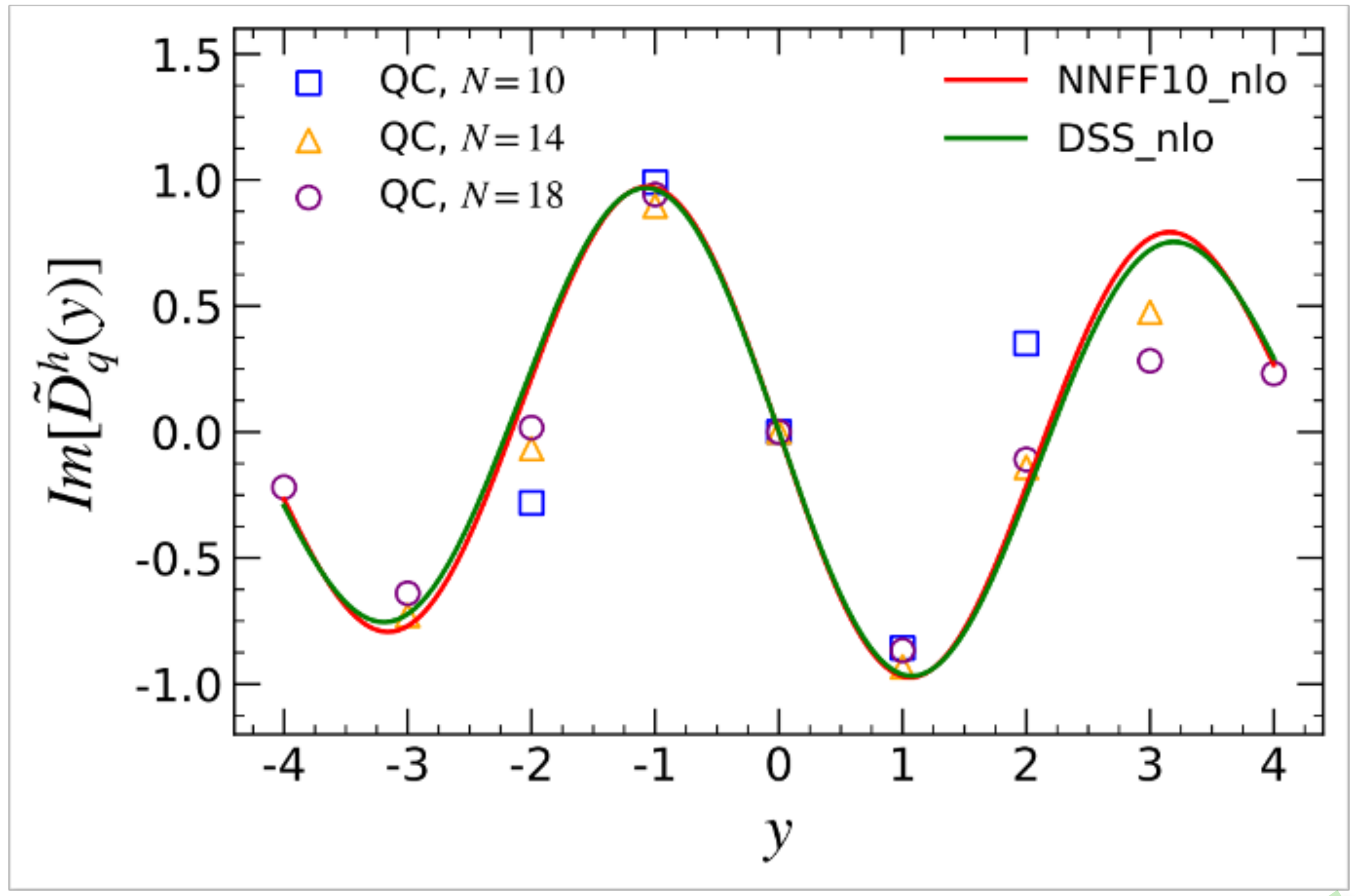
- ✦ quantum circuits for fragmentation functions

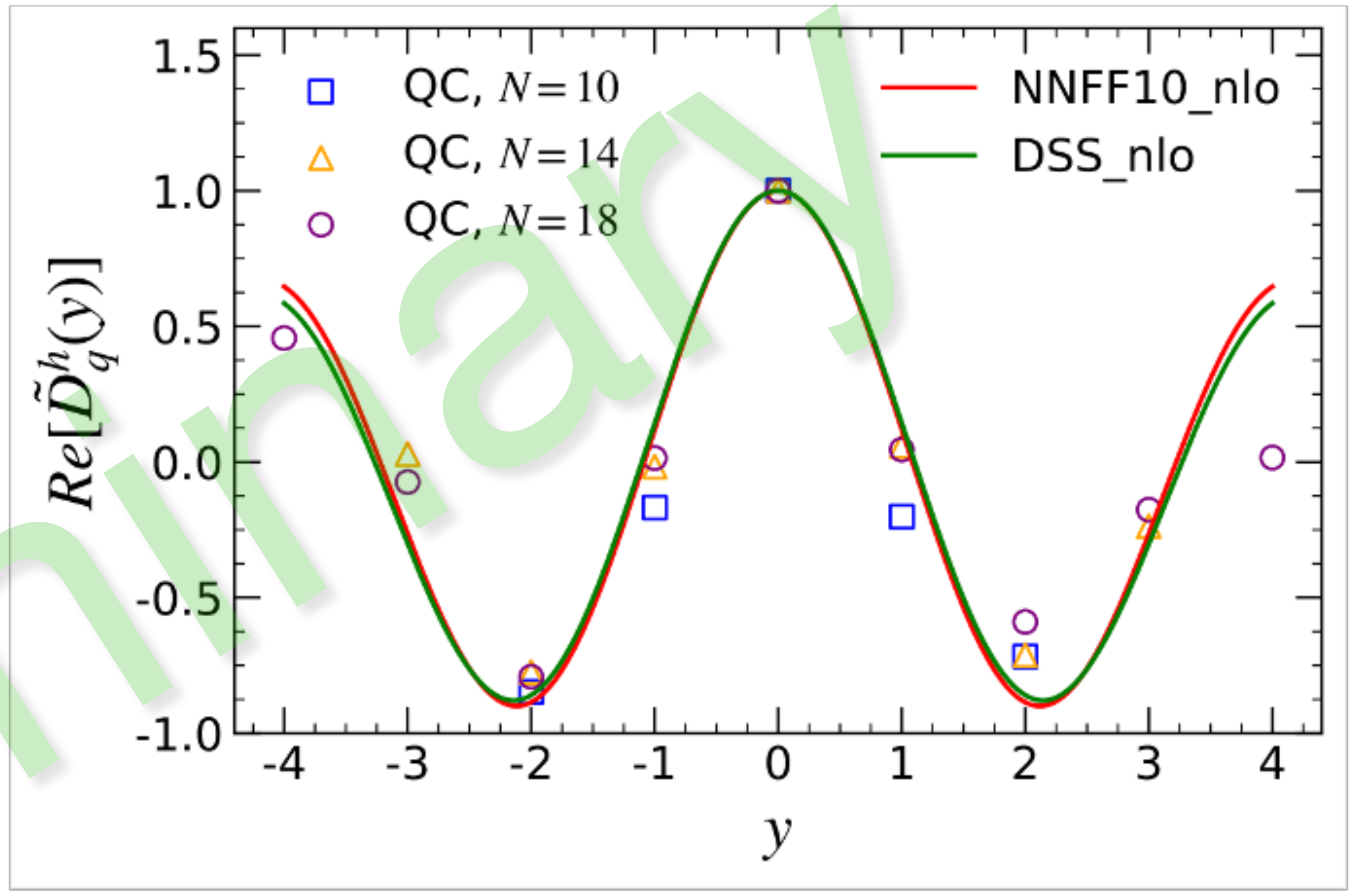
Li et al (QuNu), in preparation

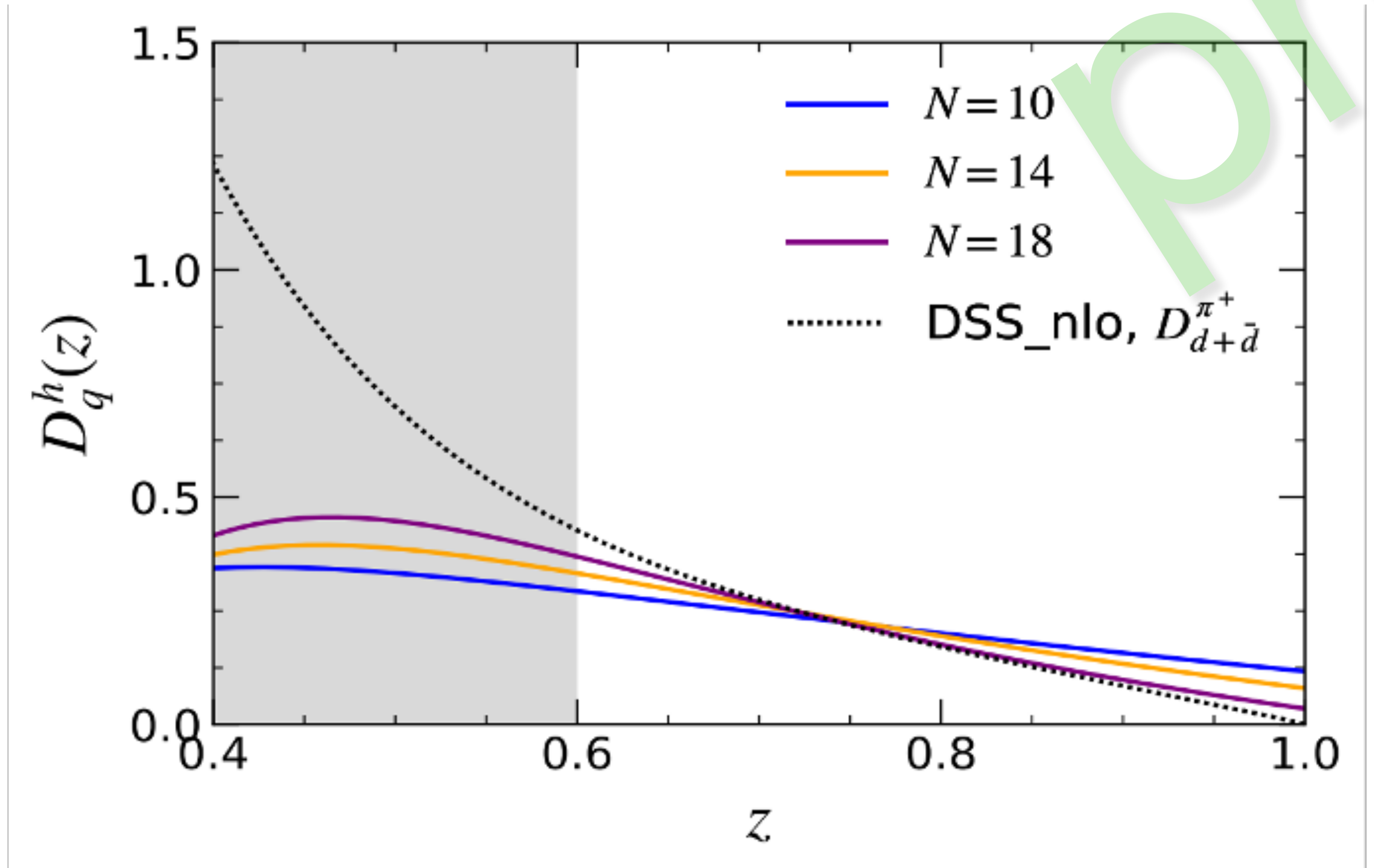


Quantum computing for inclusive hadron fragmentation functions

✦ hadron fragmentation functions from quantum computing







- Quantum simulation of FFs using NJL, $m_q = 0.4, m_h = 0.6.$
- Qualitative agreement with global fitting
- Finite volume effect significantly affect the small-z behavior

Summary and outlook

- Systematic computing of hadronic scatterings

  1. Use NJL model as a proof of concept study
  2. Include both parton distribution function, scattering amplitude and fragmentation functions

- Many topics are not covered, such as phase transition, jet quenching, quantum machine learning for data analysis …

- The field is still at its infant age, many more need to be done

  1. Consider gauge field
  2. Extend to higher dimensions for TMDs and spin dependent processes
  3. Consider noises

Thanks for your attention!