Toward the understanding of big bang singularity from loop quantum gravity

Anzhong Wang

Institute for Theoretical Physics and Cosmology, Zhejiang University of Technology
GCAP-CASPER, Physics Department, Baylor University

[G. Cleaver, K. Kirsten, B.-F. Li, Q. Sheng, P. Singh, T. Zhu]

(arXiv:1607.06329; 1705.07544; 1709.07479; 1801.07313; 1807.05236; 1906.01001)

交叉学科理论研究中心
The Interdisciplinary Center for Theoretical Study

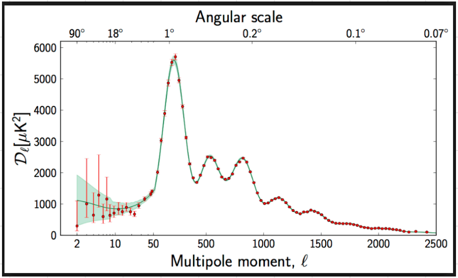
June 11, 2019

Table of Contents

Table of Contents

1.1 Inflation

- Inflation is a paradigm to solve several Big Bang Puzzles, such as the Cauchy, horizon, monopole and flatness problems.
- All observations carried so far are consistent with Inflation.



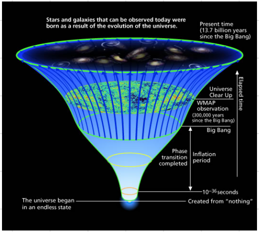
(Planck 2015, arXiv:1502.02114)

(Simmilar results were obtained in Planck 2018, arXiv:1807.06209)

1.2 Challenging (Theoretical) Problems

Inflation also faces various (theoretical) problems:

- Initial singularity problem: General relativity (GR) inevitably leads inflation to an initial singularity [1].
- Trans-Plancian problem: The current observational size of the universe was smaller than the Planck one at the onset of the inflation for models with e-folds $N \gtrsim 72$.



- ...
- Any physical understanding of them requires quantum gravity [2].

---

[1] A. Borde and A. Vilenkin, PRL72 (1994) 3305.

[2] D. Baumann, L. McAllister, Inflation and String Theory (Cambridge, 2015).

1.3 Loop Quantum Cosmology

- Loop quantum cosmology (LQC) is one of few theories that offer resolutions of these problems.
- In particular, the big bang singularity is replaced by a quantum bounce [3].
- By now, a large number of cosmological models have been studied in detail in LQC [4], including
  - the closed FLRW model
  - FLRW models with Λ with any signs
  - the Bianchi models
  - the Gowdy model
  - $f(R)$ universe
  - ...
- In all cases, the singularity is resolved!

[3] A. Ashtekar, P. Singh, CQG28 (2011) 213001.

[4] I. Agullo, P. Singh, arXiv:1612.01236; P. Singh, arXiv:1809.01747.

1.3 Loop Quantum Cosmology (Cont.)

- It was also found that: the probability for the desired — i.e. in agreement with CMB measurements — slow roll inflation not to occur in an LQC solution is less than about one part in a million [5],

$$\lesssim 1.2 \times 10^{-6}$$

— Slow-roll inflation is an attractor in LQC!

[5] P. Singh, K. Vandersloot and G. V. Vereshchagin, PRD74 (2006) 043510;
X. Zhang and Y. Ling, JCAP08 (2007) 012;
A. Ashtekar A and D. Sloan, GRG43 (2011) 3619;
A. Corichi and A. Karami PRD83 (2011) 104006;
L. Linsefors and A. Barrau, PRD87 (2013) 123509;
L. Chen and J.-Y. Zhu, PRD92 (2015) 084063.

1.3 Loop Quantum Cosmology (Cont.)

- However, ambiguities rise, as depending on the ways how to carry out the quantizations, different effective field equations can be resulted [6].

[6] A. Ashtekar, T. Pawlowski, P. Singh, PRL96 (2006) 141301;
J. Yang, Y. Ding, Y. Ma, PLB682 (2009) 1;
A. Dapor and K. Liegener, PLB785 (2018) 506;
E. Alesci et al., arXiv:1808.10225;
J. Bilski, A. Marciano, arXiv:1905.00001.

Table of Contents

2.1 A Brief Introduction to LQC

- LQC is symmetry reduced quantization of cosmology by mimicking the constructions used in LQG [7].
- In the spatially flat FLRW universe,

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2,$$

the relevant constraint is the gravitational Hamiltonian constraint, which is a sum of the Euclidean and Lorentz terms,

$$\begin{aligned}
\mathcal{H}_{\rm grav} &= \mathcal{H}^{(E)}_{\rm grav} - (1+\gamma^2)\mathcal{H}^{(L)}_{\rm grav} \\
\mathcal{H}^{(E)}_{\rm grav} &= \frac{1}{16\pi G}\int d^3x\, \epsilon_{i\,jk} F^i_{ab}\frac{E^{aj}E^{bk}}{|\det(q)|} \\
\mathcal{H}^{(L)}_{\rm grav} &= \frac{1}{8\pi G}\int d^3x\, K^j_{[a}K^k_{b]}\frac{E^{aj}E^{bk}}{|\det(q)|}
\end{aligned}$$

F_{ab}: the field strength of connection A^i_a

K^i_a: the extrinsic curvature; q_{ab}: the spatial metric

γ: the Barbero-Immirzi parameter

[7] A. Ashtekar, T. Pawlowski, P. Singh, PRD74 (2006) 084003.

2.1 A Brief Introduction to LQC (Cont.)

- For spatially flat FLRW universe, we have

$$\mathcal{H}^{(\mathrm{E})}_{\mathrm{grav}} = 2\gamma^2 \mathcal{H}^{(\mathrm{L})}_{\mathrm{grav}}$$

- Upon quantization, ambiguities can arise resulting from different treatments of these two terms.
- In LQC, using the above relation, instead of quantizing the Euclidean and Lorentz terms separately, only the Euclidean term $\mathcal{H}^{(\mathrm{E})}_{\mathrm{grav}}$ is quantized, and the resulted effective Hamiltonian (for sharply peaked states) is given by [8],

$$\mathcal{H}_{\mathrm{LQC}} = -\frac{3\mathrm{v}\sin^2(\lambda \mathrm{b})}{8\pi G\gamma^2\lambda^2} + \mathcal{H}_{\mathrm{M}}, \quad \left(\mathrm{v} \propto \mathrm{a}^3;\ \mathrm{b} \propto \mathrm{H}\right) \tag{1}$$

$$\lambda^2 \equiv 4\sqrt{3}\pi\gamma\ell^2_{\mathrm{PL}}$$

[8] A. Ashtekar, T. Pawlowski, P. Singh, PRD74 (2006) 084003.

2.1 A Brief Introduction to LQC (Cont.)

- Then, we obtain the following Hamilton equations

$$
\begin{aligned}
\dot{\mathrm{v}} &= \left\{\mathrm{v}, \mathcal{H}_{\mathrm{LQC}}\right\} = \frac{3\mathrm{v}}{2\lambda\gamma}\sin(2\lambda \mathrm{b}) \\
\dot{\mathrm{b}} &= \left\{\mathrm{b}, \mathcal{H}_{\mathrm{LQC}}\right\} = -\frac{3\sin^2(\lambda \mathrm{b})}{2\gamma\lambda^2} - 4\pi G\gamma P \\
P &\equiv -\frac{\partial \mathcal{H}_{\mathcal{M}}}{\partial \mathrm{v}}, \quad \rho \equiv \frac{\mathcal{H}_{\mathcal{M}}}{\mathrm{v}} = \rho_c \sin^2(\lambda \mathrm{b}) \\
\rho_c &\equiv \frac{\sqrt{3}\rho_{\mathrm{Pl}}}{32 G\pi^2\gamma^3\ell_{\mathrm{Pl}}^2} \approx 0.41\rho_{\mathrm{Pl}}
\end{aligned}
$$

- The corresponding Friedmann-Raychaudhuri (FR) equations for a scalar field read,

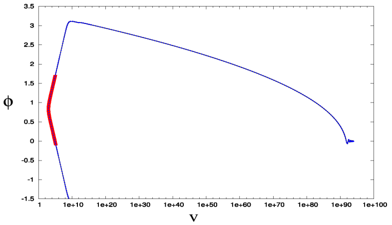
2.1 A Brief Introduction to LQC (Cont.)

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \tag{2}$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0 \tag{3}$$

$$\rho \equiv \frac{H_M}{v} = \left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \quad P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

- The big bang singularity is replaced by a quantum bounce at $\rho = \rho_c$ [9].



$(v \propto a^3)$

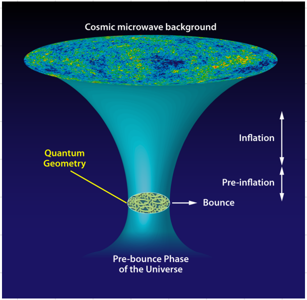
[9] A. Ashtekar, P. Singh, CQG28 (2011) 213001.

2.2 Universality of the Background

- In the framework of LQC, the background evolution can be divided into two classes:

$$\frac{1}{2}\dot{\phi}^2(t_B) - V(\phi(t_B)) = \begin{cases} > 0, & \text{KE Dominated} \\ < 0, & \text{PE Dominated} \end{cases}$$

- A potential dominated bounce is either not able to produce the desired slow-roll inflation or leads to a large amount of e-folds of expansion [10]. In the latter, all the physics was washed out, and no new physics will be present.



[10] A. Ashtekar and A. Barrau, CQG32 (2015) 234001.

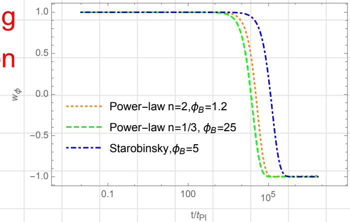
2.2 Universality of the Background (Cont.)

- In the kinetic energy initially (well) dominated case,

$$\frac{1}{2}\dot{\phi}^2(t_B) \gg V(\phi(t_B))$$

the evolution can **always** be divided into three phases [11]:

$$w(\phi) \equiv \frac{P_\phi}{\rho_\phi} = \begin{cases} +1, & \text{bouncing} \\ \in (-1,1), & \text{transition} \\ -1, & \text{inflation} \end{cases}$$

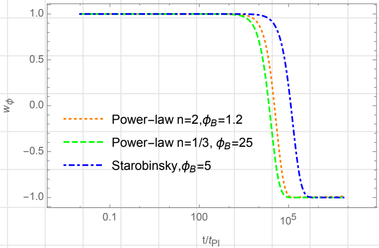


[11] Zhu, AW, G. Cleaver, K. Kirsten, Q. Sheng, PLB773 (2017) 196; PRD96 (2017) 083520; Shahalam, Sharma, Wu, AW, PRD96 (2017) 123533; M. Shahalam, M. Sami, AW, PRD98 (2018) 043524; M. Sharma, M. Shahalam, W. Qiang, AW, JCAP11 (2018) 003; M. Sharma, T. Zhu, AW, arXiv:1903.07382; B.-F. Li, P. Singh, AW, arXiv:1906.01001.

2.2 Universality of the Background (Cont.)

- The transition phase is short, during which the kinetic energy decreases dramatically:

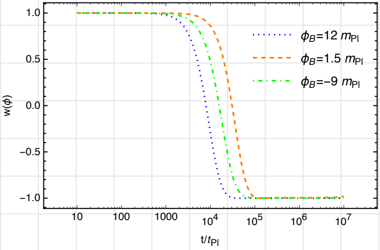
$$\frac{1}{2}\dot{\phi}^2 \simeq \rho_{\mathrm{B}} \to 10^{-12}\rho_{\mathrm{B}} \leq V(\phi)$$

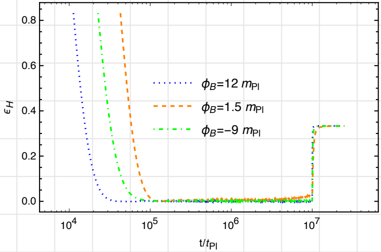


2.2 Universality of the Background (Cont.)

The three-phase division is universal:

- Quadratic Potential $V(\phi) = \lambda_0 \phi^2$:

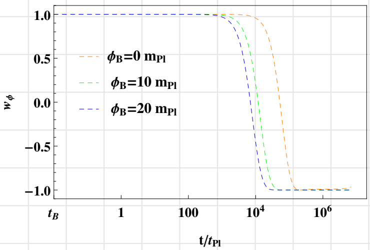


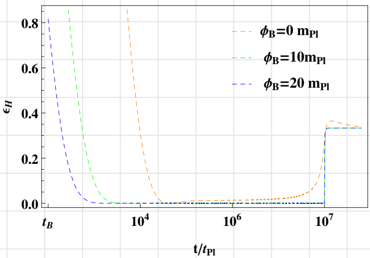


$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \begin{cases} \simeq 0, \text{ inflation} \\ \gtrsim 1, \text{ non-inflation} \end{cases} \qquad \left(H \equiv \frac{\dot{a}}{a}\right)$$

2.2 Universality of the Background (Cont.)

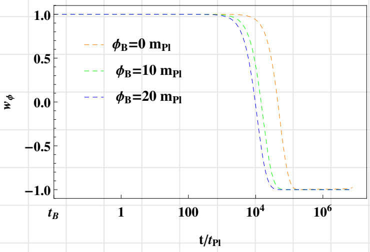
- Power-law Potential $V(\phi) = \lambda_0 \phi^{7/4}$:

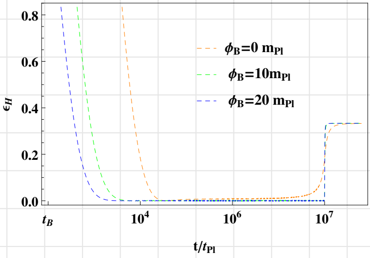




2.2 Universality of the Background (Cont.)

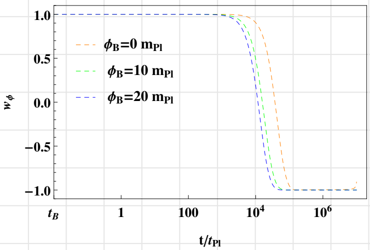
- Power-law Potential $V(\phi) = \lambda_0 \phi^{4/3}$:

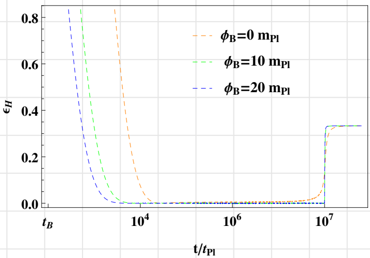




2.2 Universality of the Background (Cont.)

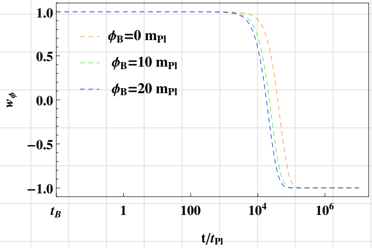
- Power-law Potential $V(\phi) = \lambda_0 \phi$:

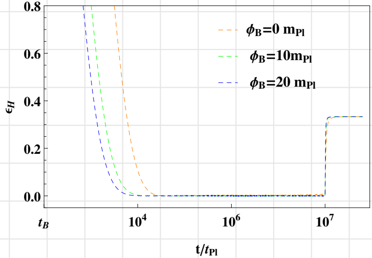




2.2 Universality of the Background (Cont.)

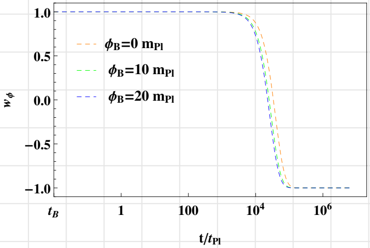
- Power-law Potential $V(\phi) = \lambda_0 \phi^{2/3}$:

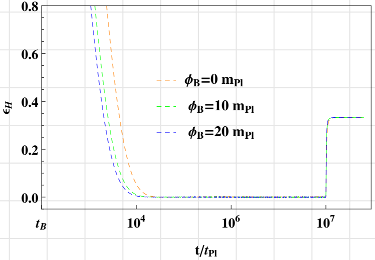




2.2 Universality of the Background (Cont.)

- Power-law Potential $V(\phi) = \lambda_0 \phi^{1/3}$:

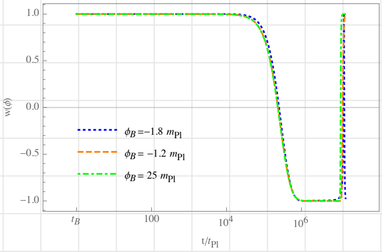


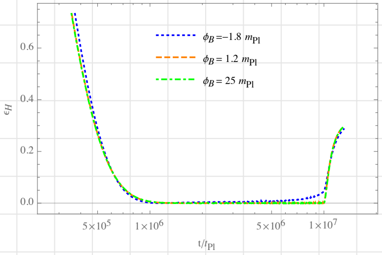


2.2 Universality of the Background (Cont.)

- Starobinsky Potential

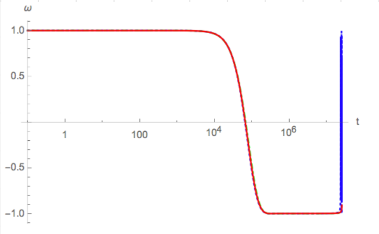
$$V(\phi) = \frac{3}{32\pi} M^2 m_{\rm Pl}^2 \times \left(1 - e^{-\sqrt{\frac{16\pi}{3}}\frac{\phi}{m_{\rm Pl}}}\right)^2$$

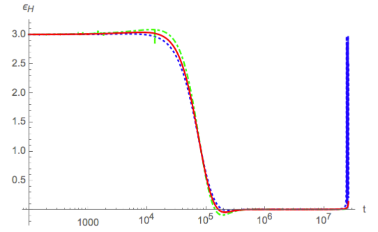




2.2 Universality of the Background (Cont.)

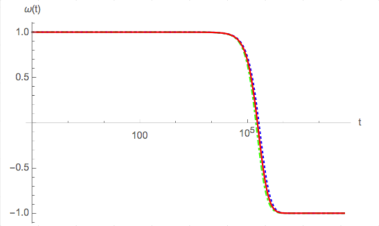
- Monodromy Potential $V(\phi) = V_0 \left|\frac{\phi}{m_{pl}}\right|^p$:

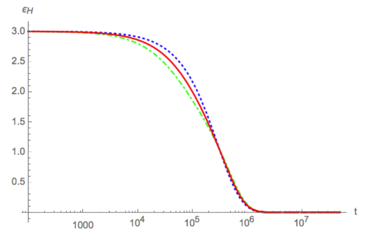




2.2 Universality of the Background (Cont.)

- Higgs Potential $V(\phi) = V_0\left(\phi^2 - \phi_0^2\right)^2$:

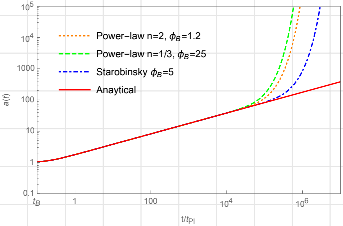




2.2 Universality of the Background (Cont.)

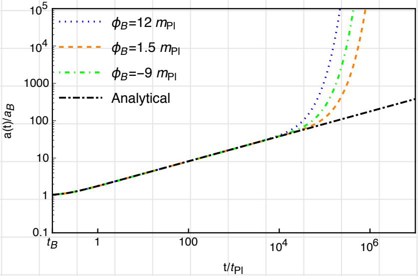
- During the bouncing phase, the evolution of $a(t)$ can be well approximated analytically by

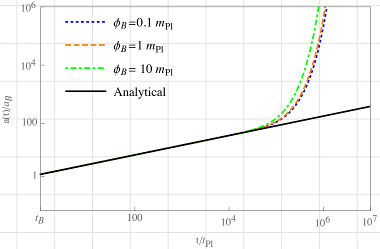
$$a(t) \simeq a_B \left(1 + \gamma_B \frac{t^2}{t_{Pl}^2}\right)^{1/6}, \quad \gamma_B \equiv 24\pi\rho_c/m_{Pl}^4$$



2.2 Universality of the Background (Cont.)

- Evolution of $a(t)$ for different potentials:



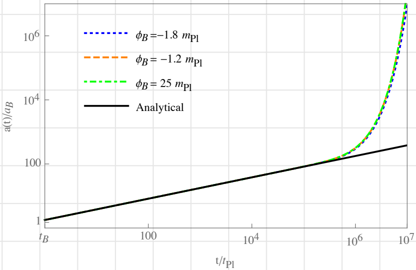


$(V = V_0\phi^2)$

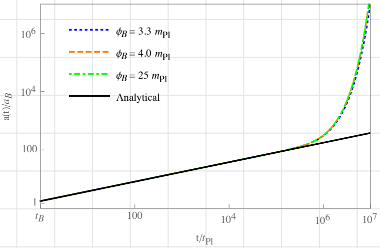
$(V = V_0\phi^{1/2})$

2.2 Universality of the Background (Cont.)

- Evolution of $a(t)$ for the Starobinsky Potential:



$(\dot{\phi}_{\rm B} > 0)$



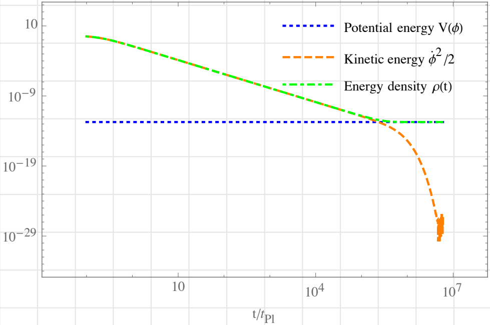
$(\dot{\phi}_{\rm B} < 0)$

2.2 Universality of the Background (Cont.)

- The main reason is that

$$\frac{1}{2}\dot{\phi}_B^2 \gg V(\phi_B) \quad \Rightarrow \quad \frac{1}{2}\dot{\phi}^2 \gg V(\phi),$$

holds in the whole bouncing phase, once it holds at the bounce $t = t_B$.



2.2 Universality of the Background (Cont.)

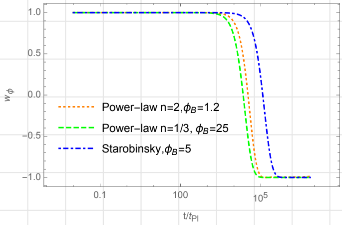
- The evolution during the transition phase is given by,

$$\phi(t) = \phi_c + t_c\dot{\phi}_c \ln\frac{t}{t_c}, \qquad a(t) = a_c\left(1 + t_c H_c \ln\frac{t}{t_c}\right),$$

H_c, a_c, ϕ_c: integration constants

- During the slow-roll inflation, we have

$$a(t) = a_i e^{H_{\text{inf.}} t}, \qquad \phi \simeq \phi_0 \tag{5}$$



2.3 Universality of Linear Perturbations

- The scalar and tensor perturbations are given by [12],

$$\mu_k'' + \left(k^2 - \frac{a''}{a} + U(\eta)\right)\mu_k = 0 \tag{6}$$

where $a' \equiv da/d\eta,\ d\eta = dt/a(t)$, and

$$U(\eta) = \begin{cases} a^2\left(\mathfrak{f}^2 V(\phi) + 2\mathfrak{f} V_{,\phi}(\phi) + V_{,\phi\phi}(\phi)\right), & \text{scalar} \\ 0, & \text{tensor} \end{cases}$$

$$\mathfrak{f} \equiv \sqrt{24\pi G}\dot{\phi}/\sqrt{\rho}.$$

[12] A. Ashtekar and A. Barrau, CQG32 (2015) 234001.

2.3 Universality of Linear Perturbations (Cont.)

- Both of the scalar and tensor perturbations are universal and independent of the slow-roll inflationary models during the bouncing phase
- This is because the potential $U(\eta)$ is very small in comparing with a''/a, so we have

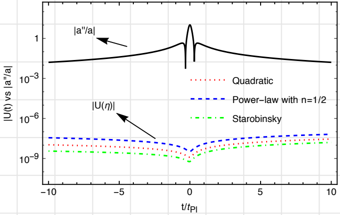
$$\Omega_k^2 = k^2 - \frac{a''}{a} + U(\eta) \simeq k^2 - \frac{a''}{a}$$

during the whole bouncing phase.

- Since $a(t)$ is universal during this phase, clearly the mode functions $\mu_k^{(s,t)}$,

$$\mu_k^{(s,t)''} - \frac{a''}{a}\mu_k^{(s,t)} = 0$$

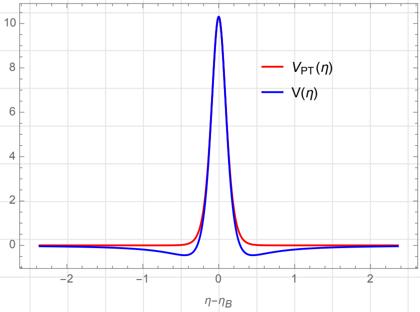
are also universal.



2.3 Universality of Linear Perturbations (Cont.)

- More interestingly, the term a''/a can be replaced by a Pöschl-Teller (PT) potential,

$$V_{PT}(\eta) = \frac{\mathscr{V}_0}{\cosh^2 \alpha(\eta - \eta_B)}, \quad \mathscr{V}_0 = k_B^2 = \frac{\alpha^2}{6}, \qquad V(\eta) \equiv \frac{a''}{a}$$



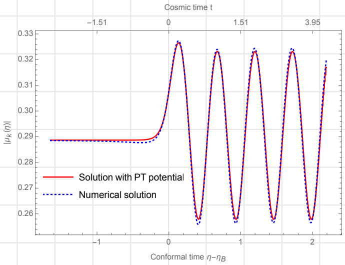
2.3 Universality of Linear Perturbations (Cont.)

- Then, the mode function has the analytical solution,

$$\begin{aligned}\mu_k^{\text{(PT)}}(\eta) &= a_k x^{ik/(2\alpha)}(1-x)^{-ik/(2\alpha)} \\ &\times {}_2F_1(a_1-a_3+1, a_2-a_3+1, 2-a_3, x) \\ &+ b_k[x(1-x)]^{-ik/(2\alpha)}\ {}_2F_1(a_1, a_2, a_3, x).\end{aligned}$$

a_k, b_k: integration constants, to be determined by initial conditions. ${}_2F_1(a, b, c, x)$: the hypergeometric function

$$\begin{aligned}a_1 &\equiv \frac{1}{2}\left(1+\frac{1}{\sqrt{3}}\right) - \frac{ik}{\sqrt{6}\,k_B}, \\ a_2 &\equiv \frac{1}{2}\left(1-\frac{1}{\sqrt{3}}\right) - \frac{ik}{\sqrt{6}\,k_B}, \\ a_3 &\equiv 1 - \frac{ik}{\sqrt{6}\,k_B}.\end{aligned}$$



2.3 Universality of Linear Perturbations (Cont.)

- In the transition phase, the mode functions are given by,

$$\mu_k(\eta) = \frac{1}{\sqrt{2k}}\left(\tilde{\alpha}_k e^{-ik\eta} + \tilde{\beta}_k e^{ik\eta}\right)$$

$\tilde{\alpha}_k, \tilde{\beta}_k$: integration constants

- In the slow-roll inflation phase, the mode functions are given by the standard forms,

$$\mu_k^{(s,t)}(\eta) \simeq \frac{\sqrt{-\pi\eta}}{2}\left[\alpha_k H^{(1)}_{\nu_{s,t}}(-k\eta) + \beta_k H^{(2)}_{\nu_{s,t}}(-k\eta)\right],$$

α_k, β_k: integration constants.

- Three sets of integration constants:

1) Bouncing: (a_k, b_k)

2) Transition: $\left(\tilde{\alpha}_k, \tilde{\beta}_k\right)$

3) Slow-roll Inflation: (α_k, β_k)

2.3 Universality of Linear Perturbations (Cont.)

- Matching them together, we find that the Bogoliubov coefficients, $\alpha_k,\ \beta_k$, are given by

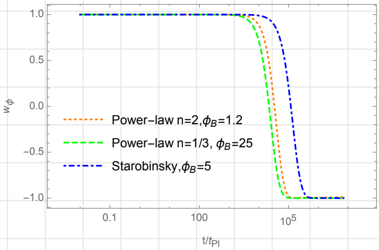
$$
\begin{aligned}
\alpha_k &= \sqrt{2k}\left[a_k \frac{\Gamma(2-a_3)\Gamma(a_1+a_2-a_3)}{\Gamma(a_1-a_3+1)\Gamma(a_2-a_3+1)}\right. \\
&\qquad \left. + b_k \frac{\Gamma(a_3)\Gamma(a_1+a_2-a_3)}{\Gamma(a_1)\Gamma(a_2)}\right] e^{ik\eta_B}, \\
\beta_k &= \sqrt{2k}\left[a_k \frac{\Gamma(2-a_3)\Gamma(a_3-a_1-a_2)}{\Gamma(1-a_1)\Gamma(1-a_2)}\right. \\
&\qquad \left. + b_k \frac{\Gamma(a_3)\Gamma(a_3-a_1-a_2)}{\Gamma(a_3-a_1)\Gamma(a_3-a_2)}\right] e^{-ik\eta_B}.
\end{aligned}
$$

- Since $a_i = a_i(k)$, so $\alpha_k,\ \beta_k$ are in general k-dependent.

2.3 Universality of Linear Perturbations (Cont.)

- In general $|\beta_k|^2 \neq 0$, so particles are *generically* created at the onset of inflation.
- In GR, we normally impose the BD vacuum at the onset of the inflation,

$$\alpha_k^{GR} = 1, \quad \beta_k^{GR} = 0$$



2.3 Universality of Linear Perturbations (Cont.)

- Then, the scalar and tensor power spectra are given by,

$$\mathcal{P}_{\mathcal{R}}(\mathrm{k}) = |\alpha_{\mathrm{k}} + \beta_{\mathrm{k}}|^2 \mathcal{P}_{\mathcal{R}}^{\mathsf{GR}}(\mathrm{k}),$$
$$\mathcal{P}_{\mathrm{h}}(\mathrm{k}) = |\alpha_{\mathrm{k}} + \beta_{\mathrm{k}}|^2 \mathcal{P}_{\mathrm{h}}^{\mathsf{GR}}(\mathrm{k}),$$

with

$$\mathcal{P}_{\mathcal{R}}^{\mathsf{GR}}(\mathrm{k}) \equiv \frac{\mathrm{k}^2}{4\pi^3}\left(\frac{\mathrm{H}}{\mathrm{a}\dot{\phi}}\right)^2 \Gamma^2(\nu_{\mathrm{s}})\left(\frac{-\mathrm{k}\eta}{2}\right)^{1-2\nu_{\mathrm{s}}},$$
$$\mathcal{P}_{\mathrm{h}}^{\mathsf{GR}}(\mathrm{k}) \equiv \frac{\mathrm{k}^2}{\pi^3 \mathrm{M}_{\mathsf{Pl}}^2}\frac{1}{\mathrm{a}^2}\Gamma^2(\nu_{\mathrm{t}})\left(\frac{-\mathrm{k}\eta}{2}\right)^{1-2\nu_{\mathrm{t}}}$$

2.3 Universality of Linear Perturbations (Cont.)

- Note that, as mentioned above, $\alpha_{\rm k}, \beta_{\rm k}$ are usually ${\rm k}$-dependent, so the quantities $\mathcal{P}_{\mathcal{R}}({\rm k})$ and $\mathcal{P}_{\rm h}({\rm k})$ now also become ${\rm k}$-dependent.

- This provides an excellent opportunity to test LQC.

- In addition, sine $\mu_{\rm k}^{\rm S} = \mu_{\rm k}^{\rm T}$ during the bouncing phase, so if we choose

$$
\begin{aligned}
\left({\rm a}_{\rm k}^{\rm S}, {\rm b}_{\rm k}^{\rm S}\right) &= \left({\rm a}_{\rm k}^{\rm T}, {\rm b}_{\rm k}^{\rm T}\right) \quad \Rightarrow \quad \left(\alpha_{\rm k}^{\rm S}, \beta_{\rm k}^{\rm S}\right) = \left(\alpha_{\rm k}^{\rm T}, \beta_{\rm k}^{\rm T}\right) \\
\Rightarrow &\quad {\rm r}_{\rm LQC} \equiv \frac{\mathcal{P}_{\rm h}({\rm k})}{\mathcal{P}_{\mathcal{R}}({\rm k})} = \frac{\mathcal{P}_{\rm h}^{\rm GR}({\rm k})}{\mathcal{P}_{\mathcal{R}}^{\rm GR}({\rm k})} \\
&= {\rm r}_{\rm GR}
\end{aligned}
$$

2.3 Universality of Linear Perturbations (Cont.)

- Clearly, such dependence cannot be strong. Otherwise, it will not be consistent with current observations, which show that the power spectra are almost scale-invariant [13].

- To fix $(\alpha_{\rm k}, \beta_{\rm k})$ or $({\rm a}_{\rm k}, {\rm b}_{\rm k})$, one needs to impose the initial conditions, which is still a challenging question in LQC.

- In the framework of LQC, various sets of initial conditions have been investigated. However, this is a subtle issue, because in general there is not a preferred initial state for a quantum field in arbitrarily curved space-times [14].

---

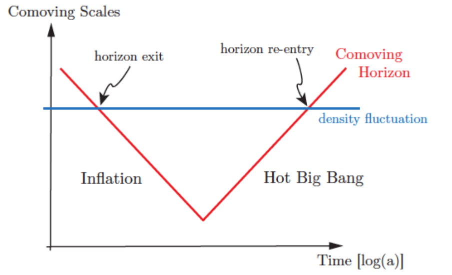
[13] P. Collaboration et al., Planck 2015. XX. Constraints on inflation, arXiv:1502.02114.

[14] A. Ashtekar, B. Gupt, CQG34 (2017) 035004.

2.3 Universality of Linear Perturbations (Cont.)

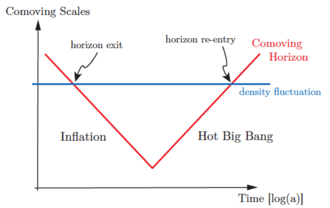
- If all the modes are inside the Hubble horizons, as in the inflationary case, the initial state can be chosen as the Bunch-Davies vacuum:

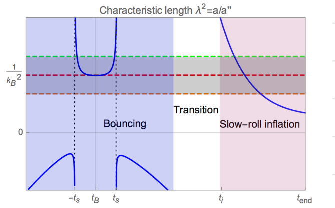
$$\alpha_k^{GR} = 1, \quad \beta_k^{GR} = 0$$



2.3 Universality of Linear Perturbations (Cont.)

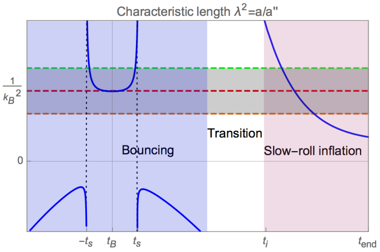
- However, in the pre-inflationary phases, especially near the bounce, the wavelengths can be larger, equal, or smaller than the corresponding characteristic scale. Thus, it is in general impossible to assume that the universe is in the Bunch-Davies vacuum at the bounce.





2.3 Universality of Linear Perturbations (Cont.)

- Recently, we considered two different kinds of initial conditions [15]
  - The fourth-order adiabatic vacuum at the bounce [16]
  - The BD vacuum in contracting phase [17]



---

[15] Zhu AW, K. Kirsten, G. Cleaver, Q. Sheng,, PRD96, 083520 (2017).

[16] I. Agullo, A. Ashtekar, W. Nelson, PRD87 (2013) 043507.

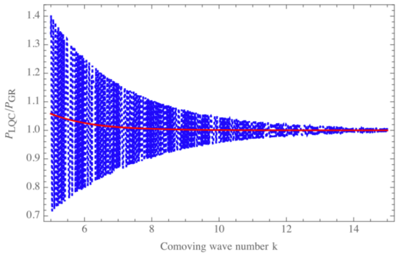
[17] A. Barrau, B. Bolliet, Int. J. Mod. Phys. D25, 1642008 (2016).

2.3 Universality of Linear Perturbations (Cont.)

- Surprisingly, both of them lead to the same results:

$$a_k^{(S,T)} = 0, \quad b_k^{(S,T)} = \frac{e^{ik\eta_B}}{\sqrt{2k}} \quad \Rightarrow \quad r_{LQC} = r_{GR}.$$

- Then, we found that the power spectra for both scalar and tensor perturbations are consistent with the numerical ones [18].



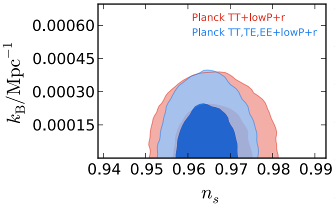
[18] I. Agullo, N. Morris, PRD92 (2015) 124040.

2.3 Universality of Linear Perturbations (Cont.)

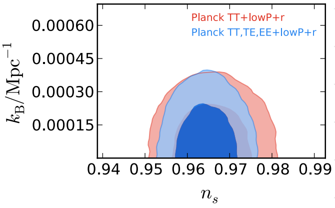
- Fitting with the Planck 2018 data, we find that the total e-folds from the bounce until now must be,

$$\mathrm{N}_{\mathrm{tot.}} \gtrsim 142,$$

in order to wash out the k-dependent of the power spectra during the pre-inflationary phase.



(Scalar spectrum)



(Scalar + tensor spectra)

2.3 Universality of Linear Perturbations (Cont.)

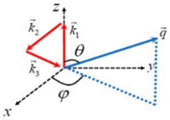
- Recently, the non-Gaussianity was studied [19], and shown that strong correlation between observable scales and modes with longer (super-horizon) wavelength arise, which can induce a dipole-dominated modulation on large angular scales in the CMB, in agreement with observations.
- Lately, we also studied the non-Gaussianity using our analytical solutions of the mode functions, and confirmed the above results [20].

[19] I. Agullo, PRD92 (2015) 064038;
I. Agullo, B. Bolliet, V. Sreenath, PRD97, 066021 (2018).

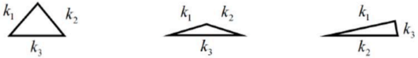
[20] T. Zhu, AW, K. Kirsten, G. Cleaver, Q. Sheng, PRD97 (2018) 043501.

2.3 Universality of Linear Perturbations (Cont.)

- In particular, the non-Gaussianity in the squeezed limit can be enhanced at superhorizon scales, which can yield a large statistical anisotropy on the power spectrum.



$$\boldsymbol{k}_1 = k\hat{z}, \quad \boldsymbol{k}_2 = k\frac{\sqrt{4\alpha^2 - 1}}{2\alpha^2}\hat{x} + k\frac{(1 - 2\alpha^2)}{2\alpha^2}\hat{z}, \quad \boldsymbol{k}_3 = -\boldsymbol{k}_1 - \boldsymbol{k}_2$$



The Equilateral Limit
$k_1 = k_2 = k_3 : \alpha = 1$

The Folded Limit
$k_1 = k_2 = 2k_3 : \alpha = 1/2$

The Squeezed Limit
$k_1 \simeq k_2 \gg k_3 : \alpha \gg 1$

Table of Contents

3.1 mLQC - I

- In the process of the quantization of LQC, using the relation,

$$\mathcal{H}_{\rm grav}^{(E)} = 2\gamma^2 \mathcal{H}_{\rm grav}^{(L)}$$

only the Euclidean term $\mathcal{H}_{\rm grav}^{(E)}$ was quantized. However, in LQG the Lorentz term $\mathcal{H}_{\rm grav}^{(L)}$ is normally quantized using a different procedure.

- In particular, following the process of regularizations proposed in [21], Yongge and his collaborators found the effective Hamiltonian takes the form [22],

$$\mathcal{H}_{\rm mLQC-I} = \frac{3v}{8\pi G\lambda^2}\left\{\sin^2(\lambda b) - \frac{(\gamma^2+1)\sin^2(2\lambda b)}{4\gamma^2}\right\} + \mathcal{H}_M \quad (7)$$

---

[21] T. Thiemann, CQG24(1998) 839; 875.

[22] J. Yang, Y. Ding, Y. Ma, PLB682 (2009) 1.

3.1 mLQC - I (Cont.)

- In [23], we referred this model was as mLQC-I (modified LQC-I model).
- The same effective Hamiltonian was obtained recently by Andrea & Klaus [24], by the complexifier coherent states with the μ_0-scheme.
- Note that Yongge et al actually used the so-called $\bar{\mu}$-scheme.
- In addition, treating the Lorentz and Euclidean terms separately was first considered by Bojowald in 2002 [25], and also derived the mLQC-I model by a μ_0-like scheme.

---

[23] B.-F. Li, P. Singh, AW, PRD98 (2018) 066016

[24] A. Dapor and K. Liegener, PLB785 (2018) 506

[25] M. Bojowald, CQG19 (2002) 2717.

3.1 mLQC - I (Cont.)

- Then, Hamilton's equations read,

$$\begin{aligned}
\dot{\mathrm{v}} &= \left\{\mathrm{v}, \mathcal{H}_{\mathrm{mLQC-I}}\right\} = \frac{3\mathrm{v}\sin(2\lambda \mathrm{b})}{2\gamma\lambda}\Big\{(\gamma^2+1)\cos(2\lambda\mathrm{b}) - \gamma^2\Big\}, \\
\dot{\mathrm{b}} &= \left\{\mathrm{b}, \mathcal{H}_{\mathrm{mLQC-I}}\right\} = \frac{3\sin^2(\lambda\mathrm{b})}{2\gamma\lambda^2}\Big\{\gamma^2\sin^2(\lambda\mathrm{b}) - \cos^2(\lambda\mathrm{b})\Big\} \\
&\quad - 4\pi G\gamma P
\end{aligned}$$

with the constraint $\mathcal{H}_{\mathrm{mLQC-I}} = 0$, or

$$\rho - \frac{3}{8\pi G\lambda^2}\left\{\frac{(\gamma^2+1)\sin^2(2\lambda\mathrm{b})}{4\gamma^2} - \sin^2(\lambda\mathrm{b})\right\} = 0.$$

3.1 mLQC - I (Cont.)

- To write them in the form of the FR equations, we first note that the Hubble parameter $H \equiv \dot{a}/a = \dot{v}/3v$ and ρ are given by,

$$\begin{aligned} H^2 &= \frac{\sin^2(2\lambda b)}{4\lambda^2\gamma^2}\Big\{\gamma^2 - (\gamma^2+1)\cos(2\lambda b)\Big\}^2, \\ \rho &= \frac{3}{8\pi G\lambda^2}\left(-\sin^2(\lambda b) + \frac{(\gamma^2+1)\sin^2(2\lambda b)}{4\gamma^2}\right) \end{aligned}$$

or inversely,

$$\sin^2(\lambda b_\pm) = \frac{1 \pm \sqrt{1-\rho/\rho_c^I}}{2(\gamma^2+1)}, \quad \rho_c^I \equiv \frac{\rho_c}{4(1+\gamma^2)}$$

— Two branches!

3.1 mLQC - I (Cont.)

- For a scalar field, we have

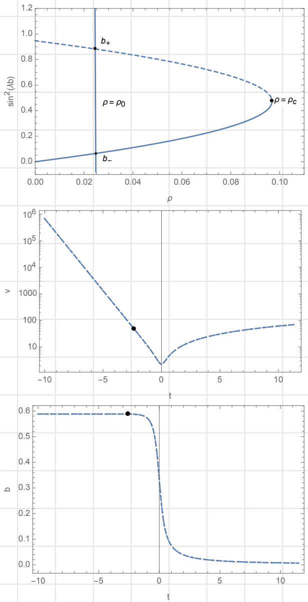
  $\dot{b} = -4\pi G\gamma\dot{\phi}^2 \leq 0$

- Since ϕ is continuous across the bounce, so is b.

- Thus, if the universe before the bounce is described by the b_+ branch, it must be described by the b_- branch after the bounce, or vice versa.

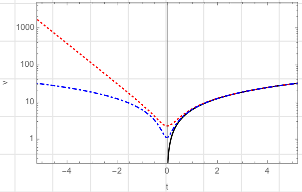
  — asymmetric bounce!

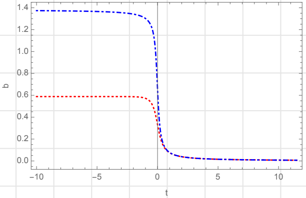
- This is in contrast to LQC, in which the evolution is symmetric with respect to the bounce.



3.1 mLQC - I (Cont.)

- In the figures, we compare results from three different models:
  - the black solid straight line is the result from GR
  - the blue dot-dashed curve is from LQC
  - the red dotted line is from the modified LQC, in which the big bang singularity is also replaced by a quantum bounce.





  - This was further confirmed later by numerical simulations [26].

---

[26] M. Assanioussi, A. Dapor, K. Liegener, T. Pawlowski, PRL121 (2018) 081303.

3.1 mLQC - I (Cont.)

- In the b_- branch, we have

$$H^2 = \frac{8\pi G\rho}{3}\left(1 - \frac{\rho}{\rho_c^I}\right)\left[1 + \frac{\gamma^2}{\gamma^2+1}\left(\frac{\sqrt{\rho/\rho_c^I}}{1+\sqrt{1-\rho/\rho_c^I}}\right)^2\right]$$

$$\dot{\rho} + 3H(\rho + P) = 0$$

- In the limit $\rho/\rho_c^I \ll 1$, we find

$$H^2 \approx \frac{8\pi G}{3}\rho, \qquad \frac{\ddot{a}}{a} \approx -\frac{4\pi G}{3}(\rho + 3P)$$

— GR limit

3.1 mLQC - I (Cont.)

- In the b_+ branch, we have

$$H^2 = \frac{8\pi G_\alpha \rho_\Lambda}{3}\left(1-\frac{\rho}{\rho_c^I}\right)\left[1+\left(\frac{1-2\gamma^2+\sqrt{1-\rho/\rho_c^I}}{4\gamma^2\left(1+\sqrt{1-\rho/\rho_c^I}\right)}\right)\frac{\rho}{\rho_c^I}\right]$$

$$\dot{\rho}+3H(\rho+P)=0$$

$$G_\alpha \equiv \alpha G = (1-5\gamma^2)G/(1+\gamma^2)$$

$$\rho_\Lambda \equiv 3/[8\pi G\alpha\Delta(1+\gamma^2)^2] \simeq \mathcal{O}\left(\rho_{Pl}\right).$$

- In the limit $\rho/\rho_c^I \ll 1$, we obtain

$$H^2 \approx \frac{8\pi G_\alpha}{3}\left(\rho+\rho_\Lambda\right), \quad \frac{\ddot{a}}{a} \approx -\frac{4\pi G_\alpha}{3}\left(\rho+3P-2\rho_\Lambda\right)$$

— GR limit but with a modified G_α and a cosmological constant ρ_Λ

3.1 mLQC - I (Cont.)

If this branch lies in the post-bounce universe where we live, then there are several phenomenological problems:

- First, for $\gamma \approx 0.2375$, we have

$$\rho_\Lambda \approx 0.03\rho_{\rm Pl} \simeq 10^{120}\rho_{\rm ob},$$

which leads to the well-known cosmological problem.

- Another problem is related to the primordial ^{4}He abundance, due to the modification of the effective Newtonian constant G_α [27],

$$H^2 = \frac{8\pi G_\alpha}{3}\rho_\Lambda\left(1-\frac{\rho}{\rho_c^{\rm I}}\right)\left[1+\left(\frac{1-2\gamma^2+\sqrt{1-\rho/\rho_c^{\rm I}}}{4\gamma^2\left(1+\sqrt{1-\rho/\rho_c^{\rm I}}\right)}\right)\frac{\rho}{\rho_c^{\rm I}}\right]$$

$G_\alpha = (1-5\gamma^2)G/(1+\gamma^2)$.

[27] S. M. Carroll and E. A. Lim, Phys. Rev. D70, 123525 (2004).

3.1 mLQC - I (Cont.)

- The current constraint is [28],

$$\left|\frac{G_\alpha}{G} - 1\right| \leq \frac{1}{8}$$

- For $\gamma \approx 0.2375$, we find that

$$\left|\frac{G_\alpha}{G} - 1\right| \simeq 0.32 > \frac{1}{8}$$

- Thus, the b_+ branch is unsuitable to describe an expanding universe such as ours if we use the observational constraints from either the cosmological constant or BBN [29].

---

[28] C. Patrignani et al. [Particle Data Group], Chin. Phys. C40, 100001 (2016).

[29] B.-F. Li, P. Singh, AW, PRD97 (2018) 084029.

3.2 mLQC - II

- On the other hand, due to the spatial homogeneity and isotropy, one can also set the spin connection Γ^i_a to zero, so the connection A^i_a is proportional to the extrinsic curvature K^i_a,

$$A^i_a = \Gamma^i_a + \gamma K^i_a = \gamma K^i_a.$$

  Then, a symmetry-reduced classical Hamiltonian is obtained.

- Although it also consistent of two terms, Euclidean and Lorentz, now the Lorentz term takes a different form.

3.2 mLQC - II

- Up on the quantization, by following the same procedure of Thiemann, the effective Hamiltonian now is given by [30],

$$\mathcal{H}_{\mathrm{mLQC-II}} = -\frac{3\mathrm{v}}{2\pi G\lambda^2\gamma^2}\sin^2\left(\frac{\lambda b}{2}\right)\left\{1+\gamma^2\sin^2\left(\frac{\lambda b}{2}\right)\right\} + \mathcal{H}_{\mathrm{M}} \quad (8)$$

- In [31], we referred this model was as mLQC-II (modified LQC-II model).
- Then, Hamilton's equations read

$$\begin{aligned}
\dot{\mathrm{v}} &= \left\{\mathrm{v}, \mathcal{H}_{\mathrm{mLQC-II}}\right\} = \frac{3\mathrm{v}\sin(\lambda b)}{\gamma\lambda}\left\{1+\gamma^2-\gamma^2\cos\left(\lambda b\right)\right\},\\
\dot{b} &= \left\{b, \mathcal{H}_{\mathrm{mLQC-II}}\right\} = -\frac{6\sin^2\left(\frac{\lambda b}{2}\right)}{\gamma\lambda^2}\left\{1+\gamma^2\sin^2\left(\frac{\lambda b}{2}\right)\right\}\\
&\quad - 4\pi G\gamma P
\end{aligned}$$

[30] J. Yang, Y. Ding, Y. Ma, PLB682 (2009) 1.

[31] B.-F. Li, P. Singh, AW, PRD98 (2018) 066016.

3.2 mLQC - II (Cont.)

- In terms of ρ, P, the FR equations read [32],

$$\mathrm{H}^2 = \frac{8\pi \mathrm{G}\rho}{3}\left(1+\gamma^2\frac{\rho}{\rho_\mathrm{c}}\right)\left(1-\frac{(\gamma^2+1)\rho/\rho_\mathrm{c}}{(1+\sqrt{\gamma^2\rho/\rho_\mathrm{c}+1})^2}\right),$$

$$\dot{\rho}+3\mathrm{H}(\rho+\mathrm{P})=0$$

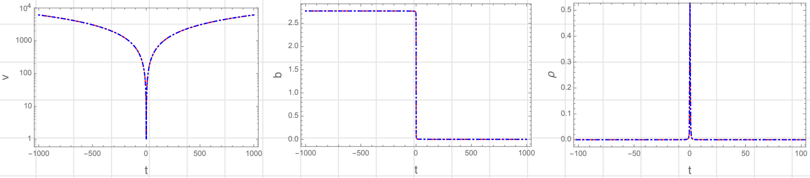
- Similar to LQC and mLQC-I, in this model the big bang singularity is also replaced by a quantum bounce but now at

$$\rho=\rho_\mathrm{c}^{\mathrm{II}}\equiv 4(1+\gamma^2)\rho_\mathrm{c}.$$

[32] B. -F. Li, P. Singh, AW, PRD97 (2018) 084029..

3.2 mLQC - II (Cont.)

- In addition, the bounce now is symmetric.



3.2 mLQC - II (Cont.)

- Then, several natural questions rise:
  - Is the big bang singularity still resolved?
  - Is the slow-roll inflation still an attractor?
  - Are the resulted power spectra and non-Gaussianities still consistent with observations?
  - Are there any observational signatures?
  - ...

Table of Contents

4.1 Dynamical Systems

- Qualitative dynamics plays an important role [33], whenever analytical solutions are hard to obtain, as in the present cases, and reveals details of the existence of attractors and asymptotic behavior.

- Studying phase space portraits of dynamical variables, one can easily identify the slow-roll inflationary separatrices before entering the reheating phase.

[33] Oleg I. Bogoyavlensky, Methods in the Qualitative Theory of Dynamical Systems in Astrophysics and Gas Dynamics, Springer-Verlag Berlin Heidelberg, 1985.

4.1 Dynamical Systems (Cont.)

- In LQC, qualitative dynamics has been already studied for several potentials, including power-law, Starobinsky, and α-attractor [34].

[34] P. Singh and K. Vandersloot, PRD72, 084004 (2005);
E. Ranken, P. Singh, PRD85, 104002 (2012);
A. Ashtekar and D. Sloan, GRG43, 3619 (2011);
A. Corichi and A. Karami, PRD83, 104006 (2011);
I. Agullo, A. Ashtekar and W. Nelson, CQG30, 085014 (2013);
B. Bonga and B. Gupt, PRD93, 063513 (2016);
T. Zhu, AW, G. Cleaver, K. Kirsten and Q. Sheng, PLB773, 196 (2017); PRD96, 083520 (2017);
M. Shahalam, M. Sharma, Q. Wu, AW, PRD96, 123533 (2017);
M. Shahalam, M. Sami, AW, PRD98 (2018) 043524.

4.1 Dynamical Systems (Cont.)

- Recently, we considered several popular potentials: chaotic, Starobinsky, monodromy, non-minimal Higgs, and exponential, in all three models, LQC, mLQC-I, and mLQC-II [35].

- In the following I shall report only the results for the fractional monodromy potential, as in other cases similar conclusions are obtained.

[35] B.-F. Li, P. Singh, AW, PRD97 (2018) 084029; PRD98 (2018) 066016; arXiv:1906.01001.

4.1 Dynamical Systems (Cont.)

- Starting from the Klein-Gordon equation of the scalar field,

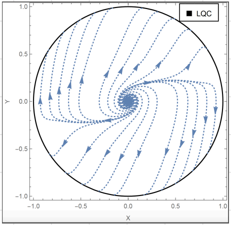
$$\ddot{\phi} + 3\mathrm{H}^{\mathrm{I}}\dot{\phi} + \mathrm{V}_{,\phi} = 0,$$

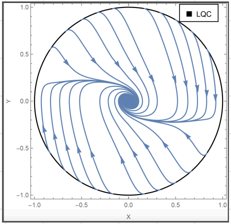
H^{I}: the Hubble parameter for LQC, mLQC-I or mLQC-II, we define two dimensionless variables,

$$\mathrm{X} \equiv \epsilon_\phi \sqrt{\frac{\mathrm{V}}{\rho_{\mathrm{c}}^{\mathrm{I}}}}, \qquad \mathrm{Y} \equiv \frac{\dot{\phi}}{\sqrt{2\rho_{\mathrm{c}}^{\mathrm{I}}}}, \quad \epsilon_\phi = \pm 1$$

which satisfy,

$$\mathrm{X}^2 + \mathrm{Y}^2 = \frac{\rho}{\rho_{\mathrm{c}}^{\mathrm{I}}} \leq 1.$$





4.1 Dynamical Systems (Cont.)

- With the help of the Friedmann equation, the Klein-Gordon equation can be written in an autonomous system,

$$\dot{X} = \frac{\epsilon_\phi V_{,\phi} Y}{\sqrt{2V}}, \qquad \dot{Y} = -3H^I Y - \frac{V_{,\phi}}{\sqrt{2\rho_c^I}}.$$

$V_{,\phi}$, V: functions of X, Y.

- The equation of state,

$$w_\phi \equiv \frac{\rho}{P} = 1 - \frac{2}{1+(Y/X)^2} = \begin{cases} +1, & Y/X \gg 1, \\ -1, & Y/X \simeq 0. \end{cases}$$

4.2 Dynamics of Monodromy Potential

- The monodromy potential

$$V(\phi) = V_0 \left|\frac{\phi}{m_{Pl}}\right|^p, \ (0 < p \le 1)$$

inspired by string/M-Theory and supergravity, has received lots of attention lately, as it fits the CMB data quite well [36].

- A modification of the potential has been proposed [37],

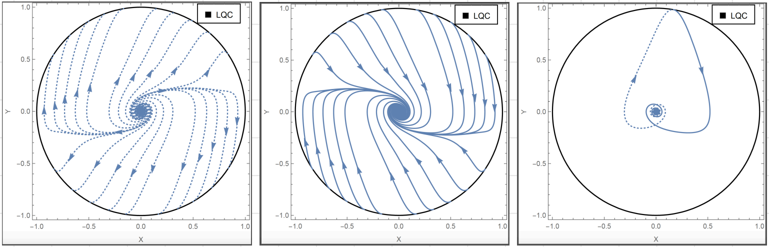
$$V = V_1 \left|\frac{\phi}{\phi_0}\right|^p \left[1 + \left(\frac{\phi_0}{\phi}\right)^n\right]^{\frac{p-2}{n}} = V_1 \times \begin{cases} \left|\frac{\phi}{\phi_0}\right|^p, & \left|\frac{\phi}{\phi_0}\right| \gg 1, \\ \left|\frac{\phi}{\phi_0}\right|^2, & \left|\frac{\phi}{\phi_0}\right| \ll 1. \end{cases}$$

which alleviates the discontinuity problem of the original monodromy potential in the reheating phase ($\phi \simeq 0$).

[36] P. A. R. Ade et al., Astron. Astrophys. 594, A20 (2016).

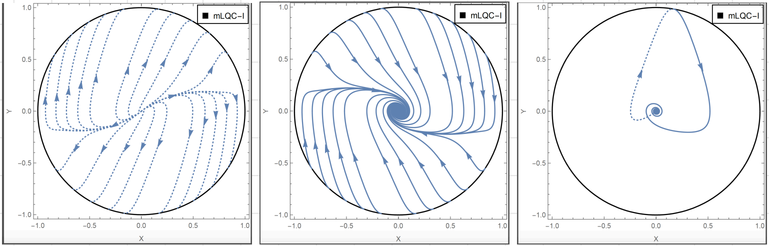
[37] S. S. Mishra, V. Sahni, A. V. Toporensky, PRD98, 083538 (2018).

4.2 Dynamics of Monodromy Potential (Cont.)



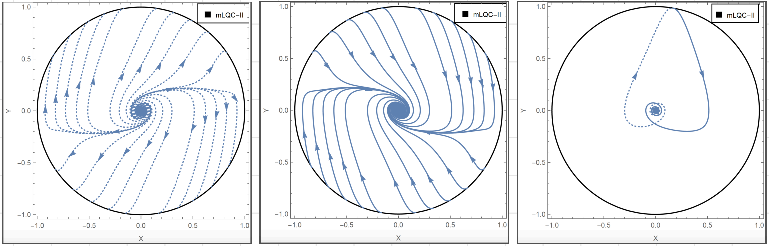
- In the pre-bounce phase, the origin is a repeller
- In the post-bounce phase, the origin is an attractor
- Inflation happens when $|Y/X| \ll 1$.

4.2 Dynamics of Monodromy Potential (Cont.)



- In the pre-bounce phase, the origin is a repelling node
- In the post-bounce phase, the origin is an attractor
- Inflation happens when $|Y/X| \ll 1$.

4.2 Dynamics of Monodromy Potential (Cont.)



- In the pre-bounce phase, the origin is a repeller
- In the post-bounce phase, the origin is an attractor
- Inflation happens when $|Y/X| \ll 1$.

4.3 Probability of Slow-Roll Inflation in mLQCs

- Let us first consider the phase space $\mathbb{S}$ of the modified FR equations, which consists of four variables, v, b and ϕ, p_ϕ. Then, the symplectic form on the 4D phase space is

$$\Omega = \mathrm{dp}_\phi \wedge \mathrm{d}\phi + \frac{\mathrm{dv} \wedge \mathrm{db}}{4\pi \mathrm{G}\gamma}.$$

- On the other hand, the phase space $\mathbb{S}$ is isomorphic to a 2D gauge-fixed surface $\hat{\Gamma}$ of $\bar{\Gamma}$, which is intersected by each dynamical trajectory once and only once.
- Since b satisfies,

$$\dot{\mathrm{b}} = -4\pi \mathrm{G}(\rho + \mathrm{P}),$$

it is monotonically decreasing, as long as the matter field satisfies the weak energy condition $\rho + \mathrm{P} \geq 0$. Thus, a natural parameterization of this 2D surface is $\mathrm{b} = \mathrm{b}_0$.

4.3 Probability of Slow-Roll Inflation in mLQCs (Cont.)

- The Hamiltonian constraint $\mathcal{C}$ reduces the 4D phase space to the hypersurface $\bar{\Gamma}$,

$$\begin{aligned} p_\phi^A &= v\left\{-2\left[\hat{\mathcal{H}}_{grav}^A + V(\phi)\right]\right\}^{1/2}, \\ dp_\phi^A\Big|_{\hat{\Gamma}} &= \frac{p_\phi^A}{v}dv - \frac{v^2 V_{,\phi}}{p_\phi}d\phi. \end{aligned}$$

$A = I, II,\ \hat{\mathcal{H}}_{grav}^A \equiv v^{-1}\mathcal{H}_{grav}^A(v, b_0).$

- Then, the pulled-back symplectic structure $\hat{\Omega}$ reads

$$\hat{\Omega}^A\Big|_{\hat{\Gamma}} = \left\{-2\left[\hat{\mathcal{H}}_{grav}^A(b_0) + V(\phi)\right]\right\}^{1/2} d\phi \wedge dv.$$

- The Liouville measure $d\hat{\mu}_L$ on $\hat{\Gamma}$ is given by

$$d\hat{\mu}_L^A = \left\{-2\left[\hat{\mathcal{H}}_{grav}^A(b_0) + V(\phi)\right]\right\}^{1/2} d\phi dv.$$

4.3 Probability of Slow-Roll Inflation in mLQCs (Cont.)

- The key observation is that $\mathrm{d}\hat{\mu}_{\mathrm{L}}$ does not depend on v. As a result, although the integral $\int \mathrm{dv}$ is infinite, it will get cancelled in the probability calculations as it shows up both in the denominator and the numerator.
- Therefore, the measure for the space of physically distinct solutions can be taken as

$$\mathrm{d}\omega^{\mathrm{A}} = \left\{-2\left[\hat{\mathcal{H}}^{\mathrm{A}}_{\mathrm{grav}}(\mathrm{b}_0) + \mathrm{V}(\phi)\right]\right\}^{1/2} \mathrm{d}\phi.$$

- The 2D phase space $\hat{\Gamma}$ is reduced further to an interval $\mathbb{S}_0 = \{\phi : \phi \in (\phi_{\mathrm{min}}, \phi_{\mathrm{max}})\}$.
- It should be noted that such a defined measure depends explicitly on b_0, a choice that is arbitrary. However, in LQC there exists a preferred one, that is,

$$\mathrm{b}_0 = \mathrm{b}\left(\mathrm{t}_{\mathrm{B}}\right). \tag{9}$$

4.3 Probability of Slow-Roll Inflation in mLQCs (Cont.)

- Thus, the probability of the occurrence of an event E becomes

$$\mathrm{P(E)} = \frac{1}{\mathcal{D}} \int_{\mathcal{I}(\mathrm{E})} \left\{-2\left[\hat{\mathcal{H}}^{\mathrm{A}}_{\mathrm{grav}}(\mathrm{b}_0) + \mathrm{V}(\phi)\right]\right\}^{1/2} \mathrm{d}\phi,$$

$\mathcal{I}(\mathrm{E})$: the interval on the ϕ_{B}-axis, which corresponds to the physically distinct initial conditions in which the event E happens, and $\mathcal{D}$ is the total measure

$$\mathcal{D} \equiv \int_{\phi_{\mathrm{min}}}^{\phi_{\mathrm{max}}} \left\{-2\left[\hat{\mathcal{H}}^{\mathrm{A}}_{\mathrm{grav}}(\mathrm{b}_0) + \mathrm{V}(\phi)\right]\right\}^{1/2} \mathrm{d}\phi.$$

- Therefore, the probability for the desired slow-roll not to happen in mLQC-I with a chaotic potential is

$$\mathrm{P}^{\mathrm{I}}(\text{not realized}) \lesssim \frac{\int_{-5.158}^{0.917} \mathrm{d}\omega^{\mathrm{I}}}{\int_{-\phi^{\mathrm{I}}_{\text{max}}}^{\phi^{\mathrm{I}}_{\text{max}}} \mathrm{d}\omega^{\mathrm{I}}} \simeq 1.12 \times 10^{-5}.$$

4.3 Probability of Slow-Roll Inflation in mLQCs (Cont.)

- In mLQC-II, the probability for the desired slow-roll to not happen is

$$\mathrm{P}^{\mathrm{II}}(\text{not realized}) \lesssim 2.62 \times 10^{-6}.$$

- It is interesting to note that in LQC, the probability for desired slow roll inflation to not occur is [38],

$$\mathrm{P}^{\mathrm{LQC}}(\text{not realized}) \lesssim 2.74 \times 10^{-6},$$

  which is smaller than the result for mLQC-I and slightly larger than that for mLQC-II.

- Therefore, the desired slow-roll inflation is attractive and favorable in both LQC and mLQCs!

[38] A. Ashtekar and D. Sloan, GRG43, 3619 (2011); PLB694, 108 (2011).

Table of Contents

5. Conclusions

- We study pre-inflationary dynamics in the frameworks of LQC and modified LQC's and find that the replacement of the big bang singularity by a quantum bounce is a robust feature against the quantization ambiguities.

- The slow-roll inflation is still an attractor and favorable.

- In addition, for initially kinetic energy dominated models ($\dot{\phi}_B^2/2 \gg V(\phi_B)$), the evolution of the universe is also always divided into three different phases [39]:

(1) Bouncing (2) transition (3) slow-roll inflation

[39] B.-F. Li, P. Singh, AW, arXiv:1906.01001.

5. Conclusions (Cont.)

- The evolution of the expansion factor is universal during the bouncing phase, and can be well approximated by [40],

$$a(t) \simeq a_B \left[1 + \left(\frac{t}{t_0^A}\right)^2\right]^{1/6}, \quad t_0^A \equiv \left(24\pi G\rho_c^A\right)^{-1/2} \tag{10}$$

  because the corrections in mLQC's are all of the second order $\bar{\mu}$. We also obtained analytically higher-order corrections of (10).

- In LQC, the evolutions of the background and linear perturbations are universal (independent of the initial conditions and potentials of the scalar field), and can be well approximated by analytical solutions, as long as initially (at the quantum bounce) the kinetic energy of the scalar field dominates, $\frac{1}{2}\dot{\phi}_B^2 \gg V(\phi_B)$.

[40] B.-F. Li, P. Singh, AW, arXiv:1906.01001.

5. Conclusions (Cont.)

- Once the background is known, we can also study the linear scalar and tensor perturbations in the two modified LQC models, and compare with the *numerical* ones obtained recently by Ivan [41], by using, for example, the analytical method developed in [42].

[41] I. Agullo, Gen. Rel. Grav. 50 (2018) 91.

[42] T. Zhu, A. Wang, G. Cleaver, K. Kirsten and Q. Sheng, PLB773, 196 (2017); PRD96, 083520 (2017).

Thank You!