Towards the simplest EFT

Yu-tin Huang
w
Massimo Bianchi, Andrea L. Guerrieri, Chao-Jung Lee, Congkao Wen

National Taiwan University

2016 May 29, East Asia Joint Workshop on Fields and Strings @USTC

Preulde

Our world up to now looks perturbative (S-matrix exists)

What can we expect in the UV?

- Continues to be perturbative, with IR degrees of freedom still present in the UV (Four fermi $\rightarrow$ Electro Weak) S-matrix exists
- Becomes non-perturbative, with IR degrees of freedom still present in the UV (Quantum Gravity) S-matrix may exists
- Becomes non-perturbative, with IR degrees of freedom emerging as bound state (Pions $\rightarrow$ QCD) S-matrix exists
- Becomes a CFT S-matrix does not exists, even non-lagrangian

Preulde

Becomes a CFT S-matrix does not exists, even non-lagrangian

- Bootstrap approach (see Heng-Yu's talk)
- Vacuum manifold $\rightarrow$ spontaneous symmetry breaking $\rightarrow$ Goldstone bosons (EFT) S-matrix does exists

What is the space of consistent EFT (from CFT)?

Preulde

In a EFT we have an infinite set of irrelevant operators

$$\mathcal{L}_{EFT} = \mathcal{L}_{marginal} + \sum_i c_i \mathcal{O}_i(\partial, \phi)$$

In general $c_i \to c_i(g, N)$

- For non-lagrangian theories c_i is simply a number!
- For theories with S-duality, $c_i(g, N)$ is constrained
- With SUSY some c_i are determined exactly

How much constraint can we impose in the IR on $\mathcal{L}_{EFT}$?

Prelude

The existence of a UV completion → c_i of higher dimension operators must be Positive
Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi

- DBI

$$\mathcal{L} = -f^4\sqrt{1-(\partial y)^2} = f^4\Big[-1+\frac{(\partial y)^2}{2}+\frac{(\partial y)^4}{8}+\ldots\Big]$$

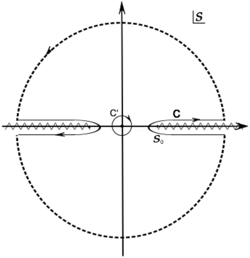
- String theory

$$\mathcal{M}^{\text{Regge}}(s,t\to 0) = -\psi_2(1)\,s^4+\frac{-\psi_4(1)}{192}\,s^6+\frac{-\psi_6(1)}{92160}\,s^8+\ldots$$

Prelude

Unitarity: The parameters enter into M_4:

$$M(s,t\to 0)=\sum_{n=0}^{\infty} f_n s^n+\mathcal{O}(s^2/t).$$

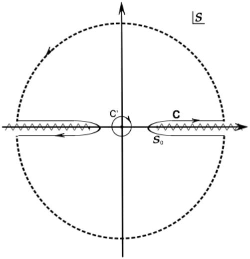


$$f_n=\frac{1}{2\pi i}\oint_{\mathcal{C}}\frac{\mathrm{d}s}{s^{n+1}}\left[M(s,t\to 0)+\mathcal{O}(s^2/t)\right]$$

Prelude

Unitarity: The parameters enter into M_4:

$$M(s,t\to 0)=\sum_{n=0}^{\infty} f_n s^n + \mathcal{O}(s^2/t).$$

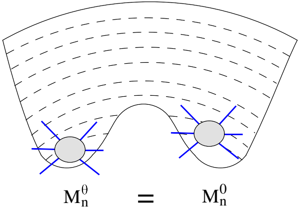


$$f_n=\frac{2}{\pi}\int_{s_0}^{\infty} \mathrm{d}s\frac{\sigma(s)}{s^n}>0,$$

Preulde

The D.O.F. for $\mathcal{L}_{EFT}$ are Goldstone bosons $\rightarrow$ Adler's zero

$$M_n(\pi_1 \cdots)|_{p_1 \to 0} = 0$$



$$e^{\theta G}|0\rangle = |\theta\rangle, \quad M_n^0 \equiv \langle 0| \cdots |0\rangle$$

$$M_n^\theta \equiv \langle \theta| \cdots |\theta\rangle = M_n^0 + M_{n+\pi}^0 + M_{n+\pi+\pi}^0 + \cdots$$

The U(1) goldstone bosons are derivatively coupled: $\mathcal{L}(\partial\phi)$ (Non-abelian extension see I. Low 14)

Preulde

Space-time symmetry breaking are different

- The generators have non-trivial commutator with P

$$[P, K] \sim D$$

  The Goldstone modes of the broken generators are derivatively related One dilaton
- For sCFT, there will be associated broken internal symmetries pions

There are multiple Goldstone modes for spontaneous space-time symmetry breaking

What does this imply for the effective action?

Outline

- New soft theorems for spacetime symmetry breaking
- Perturbative and non-perturbative checks
- Constraints on the effective action
- Constraints from maximal SUSY
- Scale vs Conformal Symmetry

Soft theorems

Ward identity

$$\partial_\mu \langle J^\mu(x)\phi(x_1)\cdots\phi(x_n)\rangle = -\sum_i \delta(x-x_i)\langle\phi(x_1)\cdots\delta\phi(x_i)\cdots\phi(x_n)\rangle$$

Spontenous symmetry breaking implies $J^\mu|0\rangle = p^\mu|phys\rangle$

- LHS: performing LSZ reduction on $i = 1, \cdots, n \rightarrow \; M_n(\pi_1\cdots)|_{p_1\rightarrow 0} = 0$
- RHS: $\begin{cases} = 0 \text{ if } \delta\phi \neq |phys\rangle \\ \neq 0 \text{ if } \delta\phi = |phys\rangle \end{cases}$

Conventional spontaneous symmetry breaking: $\delta\phi = constant$ hence Adler's zero

Soft theorems

Spontaneous broken dilation and conformal boost generator leads to single dilaton,

$$[K, D] \sim K$$

The dilaton transforms linearly under the broken generator $\rightarrow$ non-vanishing soft-limits:
Boels, Wormsbecher, Y-t Wen, Di Vecchia, Marotta, Mojaza, Nohle

$$M_n\Big|_{p_n \to 0} = \left(\mathcal{S}_n^{(0)} + \mathcal{S}_n^{(1)}\right) M_{n-1} + \mathcal{O}(p_n^2)\,,$$

$(\mathcal{S}_n^{(0)}, \mathcal{S}_n^{(1)})$ are universal soft functions

$$\begin{aligned}
\mathcal{S}_n^{(0)} &= \sum_{i=1}^{n-1} \left(p_i \cdot \frac{\partial}{\partial p_i} + \frac{d-2}{2}\right) - d\,, \\
\mathcal{S}_n^{(1)} &= p_n^{\mu} \sum_{i=1}^{n-1} \left[p_i^{\nu} \frac{\partial^2}{\partial p_i^{\nu} \partial p_i^{\mu}} - \frac{p_{i\mu}}{2} \frac{\partial^2}{\partial p_{i\nu} \partial p_i^{\nu}} + \frac{d-2}{2} \frac{\partial}{\partial p_i^{\mu}}\right].
\end{aligned}$$

Soft theorems

There's more! In general CFTs with scalar moduli space has "flavor" symmetry, which will be spontaneously broken along with conformal symmetry $\to$ pions

Exp: $\mathcal{N}=4$ SYM on Coulomb branch, 6 massless scalars (1 dilaton φ, 5 SO(6)$\to$SO(5) GBs ϕ^I)

$$A_n(\phi_1, \cdots, \phi^I_n)\big|_{p_n \to 0} = \sum_i A_{n-1}(\cdots, \delta^I O, \cdots) + \mathcal{O}(p_n^1)\,.$$

where $\delta^I\varphi = \phi^I$ and $\delta^I\phi^J = -\delta^{IJ}\varphi$.

Soft theorems

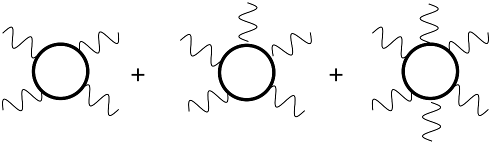
The soft theorems should be respected

- In the UV where massive D.o.F are present
- In the IR where massive D.o.F integrated away perturbatively
- In the IR where massive D.o.F integrated away non-perturbatively

Let's check

Perturbative Verifications

The one-loop effective action of $\mathcal{N} = 4$ SYM on the Coulomb branch, up to six fields



Derived from the integrand of SYM in D-dimensions (scalars: $\epsilon \cdot k_i = 0$, $\epsilon \cdot \ell = m$ for φ, $\epsilon \cdot \ell = 0$ for ϕ^I)

$$\mathcal{L}_{1-\text{loop}}^{\text{SU(4) singlet}} = \frac{g^4 N}{32 m^4 \pi^2} \left(\mathcal{O}_{F^4} + \frac{\mathcal{O}_{D^4 F^4}}{2^3 \times 15 m^4} - \frac{2\mathcal{O}_{D^2 F^6}}{15 m^6} + \frac{\mathcal{O}_{D^4 F^6}}{2^3 \times 21 m^8} - \frac{\mathcal{O}_{D^6 F^6}}{2 \times 15^2 m^{10}} + \cdots \right)$$

$$\mathcal{L}_{1-\text{loop}}^{\text{Sp(4)}} = \frac{\partial^4 \varphi^4}{16 m^4} + \frac{\partial^8 \varphi^4}{960 m^8} + \frac{\partial^4 \varphi^5}{4 m^6} + \frac{\partial^8 \varphi^5}{480 m^{10}} - \frac{5 \partial^4 \varphi^6}{4 m^6}$$
$$- \frac{\partial^8 \varphi^6}{480 m^{10}} + \frac{\partial^{10} \varphi^6}{2^{10} 3^5 m^{12}} + \frac{\partial^{12} \varphi^6}{2^{11} 3^2 m^{14}} + \frac{\partial^4 \varphi^2 \phi'^2}{8 m^4} - \frac{5 \partial^4 \varphi^2 \phi'^4}{4 m^6} + \frac{\partial^4 \varphi^4 \phi'^2}{4 m^6} + \ldots$$

Perturbative Verifications

$$\partial^4\varphi^m:\ \sum_{i<j} s_{ij}^2\,,\quad \partial^8\varphi^4:\ (\sum_{i<j} s_{ij}^2)^2\,,\quad \partial^8\varphi^5:\ (\sum_{i<j} s_{ij}^2)^2\,,$$

$$\partial^8\varphi^6:\ -\frac{b_1^{(4)}}{6}+\frac{5\,b_2^{(4)}}{768}+\frac{b_3^{(4)}}{36}-\frac{3\,b_4^{(4)}}{2}\,,$$

$$\partial^{10}\varphi^6:\ -\frac{48\,b_1^{(5)}}{7}+\frac{36}{35}b_2^{(5)}+\frac{108}{7}b_3^{(5)}+\frac{114}{35}b_4^{(5)}+\frac{60}{7}b_5^{(5)}\,,$$

$$\partial^{12}\varphi^6:\ \frac{433}{1350}b_1^{(6)}-\frac{58}{2025}b_2^{(6)}+\frac{20}{9}b_3^{(6)}+\frac{117}{35}b_4^{(6)}-\frac{184}{945}b_5^{(6)},$$

$$-\frac{74}{45}b_6^{(6)}+\frac{334}{315}b_7^{(6)}+\frac{177}{35}b_8^{(6)}-\frac{64}{63}b_9^{(6)}+\frac{104}{105}b_{10}^{(6)}$$

$$\partial^4\varphi^2\phi^2:\ s_{12}^2-s_{13}^2-s_{23}^2\,,\quad \partial^4\varphi^2\phi^4:\ b^{(2)}_{1,S_2\times S_4}-b^{(2)}_{2,S_2\times S_4}+b^{(2)}_{3,S_2\times S_4}-\frac{8}{5}b^{(2)}_{4,S_2\times S_4}$$

$$\partial^4\varphi^4\phi^2:\ b^{(2)}_{1,S_2\times S_4}-b^{(2)}_{2,S_2\times S_4}+b^{(2)}_{3,S_2\times S_4}-8b^{(2)}_{4,S_2\times S_4}$$

$$b_1^{(4)}=s_{12}^4+\mathcal{P}_6\,,\quad b_2^{(4)}=(s_{12}^2+\mathcal{P}_6)^2\,,\quad b_3^{(4)}=s_{12}^2s_{13}^2+\mathcal{P}_6\,,$$
$$b_4^{(4)}=s_{123}^4+\mathcal{P}_6\,,\quad b_1^{(5)}=s_{12}^5+\mathcal{P}_6\,,\quad b_2^{(5)}=s_{12}^2s_{123}^3+\mathcal{P}_6\,,$$
$$b_3^{(5)}=s_{12}^2s_{13}^3+\mathcal{P}_6\,,\quad b_4^{(5)}=s_{12}^2s_{34}^2+\mathcal{P}_6\,,\quad b_5^{(5)}=s_{123}^5+\mathcal{P}_6$$
$$b_1^{(6)}=s_{12}^6+\mathcal{P}_6\,,\quad b_2^{(6)}=s_{123}^6+\mathcal{P}_6\,,\quad b_3^{(6)}=s_{12}^4s_{13}^2+\mathcal{P}_6\,,$$
$$b_4^{(6)}=s_{12}^4s_{34}^2+\mathcal{P}_6\,,\quad b_5^{(6)}=s_{12}^3s_{13}^3+\mathcal{P}_6,\quad b_6^{(6)}=s_{12}^3s_{34}^3+\mathcal{P}_6\,,$$
$$b_7^{(6)}=s_{12}^2s_{123}^4+\mathcal{P}_6\,,\quad b_8^{(6)}=s_{14}^2s_{123}^4+\mathcal{P}_6\,,\quad b_9^{(6)}=s_{14}^4s_{123}^2+\mathcal{P}_6\,,$$
$$b_{10}^{(6)}=s_{123}^2s_{124}^2s_{135}^2+\mathcal{P}_6,\quad b^{(2)}_{1,S_2\times S_4}=s_{12}^2\,,\quad b^{(2)}_{2,S_2\times S_4}=s_{13}^2+\mathcal{P}_{\{12|3456\}}\,,$$
$$b^{(2)}_{3,S_2\times S_4}=s_{34}^2+\mathcal{P}_{\{12|3456\}}\,,\quad b^{(2)}_{4,S_2\times S_4}=s_{12}s_{13}+\mathcal{P}_{\{12|3456\}}$$

All soft theorems are satisfied

Non-Perturbative Verifications

The instanton effective action of $\mathcal{N} = 4$ SYM on the Coulomb branch, Massimo, Morales, Wen

$$S_{\rm eff}^{1-inst} = c' \frac{g^4}{\pi^6} e^{2\pi i\tau} \int \frac{d^4x\, d^8\theta \sqrt{\det_{4N} 2\bar{\Phi}_{Au,Bv}}}{\sqrt{\det_{2N}\left(\Phi^{AB}\bar{\Phi}_{AB} + \frac{1}{g}\bar{\mathcal{F}} + \frac{1}{\sqrt{2}g}\bar{\Lambda}_A(\Phi^{-1})^{AB}\bar{\Lambda}_B\right)_{\dot{\alpha}u,\dot{\beta}v}}} .$$

The $\mathcal{N} = 4$ on-shell superfields can be expanded in terms of the component fields $\{\phi^{AB}, \lambda^A_\alpha, F^-_{\alpha\beta}\}$. For just the scalars,

$$\bar{\Phi}_{AB} = \bar{\phi}_{AB}\,, \quad \bar{\Lambda}_{A\dot{\alpha}} = i\,\theta^{B\alpha}\partial_{\alpha\dot{\alpha}}\bar{\phi}_{AB}\,, \quad \bar{\mathcal{F}}_{\dot{\alpha}\dot{\beta}} = \frac{1}{2}\theta^{A\alpha}\theta^{B\beta}\partial_{\alpha\dot{\alpha}}\partial_{\beta\dot{\beta}}\bar{\phi}_{AB}$$

We obtain simple dilaton effective action

$$S_{\rm dilaton} = \int d^4x \left[(S_{\mu\nu}S^{\mu\nu})^2 - S_{\mu\nu}S^{\nu\rho}S_{\rho\sigma}S^{\sigma\mu}\right], \qquad S_{\mu\nu} = \frac{\partial_\mu\partial_\nu\varphi}{\varphi^2} - 2\frac{\partial_\mu\varphi\partial_\nu\varphi}{\varphi^3},$$

Non-Perturbative Verifications

But horrific vertices when expanded around $\varphi \to v + \varphi$

$$v^8\Gamma^{(4)}[\varphi]=(\partial_\mu\partial_\nu\varphi\,\partial^\mu\partial^\nu\varphi)^2-\partial_\mu\partial_\nu\varphi\,\partial^\nu\partial^\rho\varphi\,\partial_\rho\partial_\sigma\varphi\,\partial^\sigma\partial^\mu\varphi \equiv (\partial\partial\varphi\cdot\partial\partial\varphi)^2-(\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi) \tag{A.1}$$

$$\begin{aligned} v^9\Gamma^{(5)}[\varphi] &= -8\,\varphi(\partial\partial\varphi\cdot\partial\partial\varphi)^2 + 8\,\varphi(\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi) \\ &\quad - 8(\partial\partial\varphi\cdot\partial\partial\varphi)\,\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi + 8\,\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi - 2(\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\;\partial\varphi\cdot\partial\varphi) \end{aligned} \tag{A.2}$$

$$\begin{aligned} v^{10}\Gamma^{(6)}[\varphi] &= 36\,\varphi^2(\partial\partial\varphi\cdot\partial\partial\varphi)^2 - 36\,\varphi^2(\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi) \\ &\quad + 72\,\varphi(\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi)\,\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi - 72\,\varphi\,\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi \\ &\quad + 18\,\varphi(\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi) + 8(\partial\cdot\partial\partial\varphi\cdot\partial\varphi)^2 - 4\,\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi + 3(\partial\partial\varphi\cdot\partial\partial\varphi)\,(\partial\varphi\partial\varphi)^2 \end{aligned} \tag{A.3}$$

$$\begin{aligned} v^{11}\Gamma^{(7)}[\varphi] &= -120\,\varphi^3(\partial\partial\varphi\cdot\partial\partial\varphi)^2 + 120\,\varphi^3(\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi) \\ &\quad - 360\,\varphi^2(\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi)\,\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi + 360\,\varphi^2\,\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi \\ &\quad - 90\,\varphi^2(\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi) - 80\,\varphi(\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi)^2 + 40\,\varphi\,\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi \\ &\quad - 45\,\varphi\,(\partial\partial\varphi\cdot\partial\partial\varphi)\,(\partial\varphi\partial\varphi)^2 - 10\,\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi\,(\partial\varphi\partial\varphi)^2 \end{aligned} \tag{A.4}$$

$$\begin{aligned} v^{12}\Gamma^{(8)}[\varphi] &= 330\,\varphi^4(\partial\partial\varphi\cdot\partial\partial\varphi)^2 - 330\,\varphi^4(\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi) \\ &\quad + 1320\,\varphi^3(\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi)\,\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi - 1320\,\varphi^3\,\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi \\ &\quad + 330\,\varphi^3(\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi) + 440\,\varphi^2(\partial\cdot\partial\partial\varphi\cdot\partial\varphi)^2 - 220\,\varphi^2\,\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi \\ &\quad + \frac{495}{2}\,\varphi^2\,(\partial\partial\varphi\cdot\partial\partial\varphi)\,(\partial\varphi\partial\varphi)^2 + 110\,\varphi\,\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi\,(\partial\varphi\partial\varphi)^2 + \frac{15}{4}(\partial\varphi\partial\varphi)^4 \end{aligned} \tag{A.5}$$

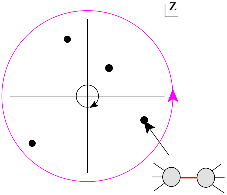
$$\begin{aligned} v^{13}\Gamma^{(9)}[\varphi] &= -792\,\varphi^5(\partial\partial\varphi\cdot\partial\partial\varphi)^2 + 792\,\varphi^5(\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi) \\ &\quad - 3960\,\varphi^4(\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi)\,\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi + 3960\,\varphi^4\,\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi \\ &\quad - 990\,\varphi^4(\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi) - 760\,\varphi^3(\partial\cdot\partial\partial\varphi\cdot\partial\varphi)^2 + 880\,\varphi^3\,\partial\varphi\cdot\partial\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi \\ &\quad - 990\,\varphi^3\,(\partial\partial\varphi\cdot\partial\partial\varphi)\,(\partial\varphi\partial\varphi)^2 - 660\,\varphi^2\,\partial\varphi\cdot\partial\partial\varphi\cdot\partial\varphi\,(\partial\varphi\partial\varphi)^2 - 45\,\varphi\,(\partial\varphi\partial\varphi)^4. \end{aligned}$$

All soft theorems are satisfied

Constraints on effective action

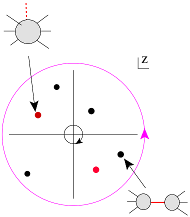
Using the fact that S-matrix are analytic functions, we start with: Britto, Cachazo, Feng, Witten

$$A_n(0) = \oint_{|z|=0} dz \frac{A_n(z)}{z} = -\oint_{|z|=z^*} dz \frac{A_n(z)}{z} \,,$$



The constraint from soft-theorems can be utilized via augmented recursion: Cheung, Kampf, Novotny, Shen, Trnka

$$A_n(0) = \oint_{|z|=0} dz \frac{A_n(z)}{zF(z)} = -\oint_{|z|=z^*} dz \frac{A_n(z)}{zF(z)} - \oint_{|z|=z^*} dz \frac{A_n(z)}{zF(z)},$$



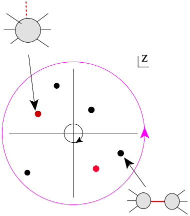
Constraints on effective action

Take

$$A(z) = A|_{p_i \to (1-za_i)p_i}, \quad F_n(z) = \prod_{i=1}^{n} [(1 - za_i)]^{d_i}$$

with $\sum_i a_i p_i = 0$

$$A_n(0) = \oint_{|z|=0} dz \frac{A_n(z)}{zF(z)} = -\oint_{|z|=z^*} dz \frac{A_n(z)}{zF(z)} - \oint_{|z|=z^*} dz \frac{A_n(z)}{zF(z)},$$



The residue of $F(z)$ is determined

$$A(z) \to A_0 + A_1 q + A_2 q^2 + \cdots A_d q^{d-1}$$

where $q = (1 - za_i)p_i$

Constraints on effective action

The residue of $F(z)$ is determined

$$A(z) \to A_0 + A_1 q + A_2 q^2 + \cdots A_d q^{d-1}$$

Since for the pure dilaton sector

$$M_n\Big|_{p_n \to 0} = \left(\mathcal{S}_n^{(0)} + \mathcal{S}_n^{(1)}\right) M_{n-1} + \mathcal{O}(p_n^2)\,,$$

we have $d = 2$.

The pure dilaton amplitude can be constructed using recursion

$$A_n(0) = \oint_{|z|=0} dz \frac{A_n(z)}{z \prod_i (1 - z a_i)^2}$$

The denominator $\sim z^{2n}$, while $A_n(z) \sim z^{2m}$ for order $\partial^{2m} \to$ we need $n > m$

Constraints on effective action

The pure dilaton sector is highly constrained:

s^n \ # of points	4	5	6	7	8	$\cdots$
2	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
3	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
4	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
5	$\checkmark$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
6	$\checkmark$	$\checkmark$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$
7	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$	$\checkmark$
8	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

At s^n, the EFT is determined up to coefficients for operators $\partial^{2n}\varphi^n$

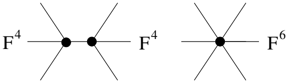
SUSY Constraints on effective action

Maximal SUSY is known to give exact results:

- s^2: F^4 operator one-loop exact $\lambda = \left(\frac{g^4 N}{8\pi^2 m^4}\right)$
- For the pure field-strengths Chen, Y-t, Wen

$$\mathcal{L}_{\text{eff}} = \sum_{p,q=1} c_0^{p,q} \frac{(F_+^2)^p (F_-^2)^q}{(M^2)^{2(p+q-1)}} + \sum_{m=1} \sum_{p,q=1} c_m^{p,q} \frac{D^{2m}(F_+^2)^p (F_-^2)^q}{(M^2)^{2(p+q-1)+m}} + \cdots$$

There are no local susy matrix elements that encode $F_-^2 F_+^{n-2}$ $\rightarrow$must have zero coefficient



One obtains an exact recursion formula

$$c_0^{1,q} = 4^{q-1}(c_0^{1,1})^q$$

SUSY Constraints on effective action

Assume D=4 maximal susy

$$\mathcal{A}_4 = \delta^8(Q)\frac{[12]^2}{\langle 34\rangle^2}\sum_k P_4^{(k)}(s_{ij}),$$

$$\mathcal{A}_5 = v\,\delta^8(Q)\frac{m^{(1)}_{1,2,3}m^{(2)}_{1,2,3}+m^{(3)}_{1,2,3}m^{(4)}_{1,2,3}}{\langle 45\rangle^2}\sum_k P_5^{(k)}(s_{ij}),$$

- s^2: F^4 operator one-loop exact $\lambda = \left(\frac{g^4 N}{8\pi^2 m^4}\right)$
- s^3: $A_4^{(3)} = A_5^{(3)} = 0$, and the first non-zero would be A_6

$$\begin{aligned} A_6^{(3)} &= a_1(s_{12}^3+\mathcal{P}_6)+a_2(s_{123}^3+\mathcal{P}_6) \\ &+ \lambda^2\left((s_{12}^2+s_{13}^2+s_{23}^2)\frac{1}{s_{123}}(s_{45}^2+s_{46}^2+s_{56}^2)+\mathcal{P}_6\right) \end{aligned}$$

soft theorem fixes $a_1 = 0\,,\quad a_2 = -\lambda^2 \rightarrow$ $A_n^{(3)}$ is two-loop exact

Up to six-derivatives, the effective action is identical to DBI in $AdS_5 \times S_5$

SUSY Constraints on effective action

- s^4: Recursion determines all $n > 4$ in terms of the four-point

$$\sum_{m\leq 8} \mathcal{L}_{\partial^m \phi^n} = \delta_{m,8}\, c_4^{(2)}(g,N)\mathcal{L}^{\ell=1}_{\partial^8\phi^n} + \sum_{m\leq 8} \mathcal{L}^{\text{DBI}}_{\partial^m\phi^n}\,,$$

- s^5:

$$P_4^{(3)}(s_{ij}) = c_4^{(3)}(g,N)\times(s_{12}^3+\mathcal{P}_4)\,,\quad P_5^{(3)}(s_{ij}) = c_5^{(3)}(g,N)\times(s_{12}^3+\mathcal{P}_5)\,.$$

Soft theorem determines $c_5^{(3)}(g,N) = -c_4^{(3)}(g,N)$

$$\mathcal{L}_{\partial^{10}\phi^n} = c_4^{(3)}(g,N)\mathcal{L}^{\ell=1}_{\partial^{10}\phi^n} + \lambda\times c_4^{(2)}(g,N)\mathcal{L}^{\ell=2}_{\partial^{10}\phi^n} + \mathcal{L}^{\text{DBI}}_{\partial^{10}\phi^n},$$

Maximal SUSY fixes the effective action up to 10 derivatives in terms of two unknown coefficients

Scale vs Conformal symmetry

$$M_n\big|_{p_n\to 0} = \left(\mathcal{S}_n^{(0)} + \mathcal{S}_n^{(1)}\right) M_{n-1} + \mathcal{O}(p_n^2)\,,$$

$$\mathcal{S}_n^{(0)} = \sum_{i=1}^{n-1}\left(p_i\cdot\frac{\partial}{\partial p_i} + \frac{d-2}{2}\right) - d\,, \leftarrow \text{Dilatation}$$

$$\mathcal{S}_n^{(1)} = p_n^{\mu}\sum_{i=1}^{n-1}\left[p_i^{\nu}\frac{\partial^2}{\partial p_i^{\nu}\partial p_i^{\mu}} - \frac{p_{i\mu}}{2}\frac{\partial^2}{\partial p_{i\nu}\partial p_i^{\nu}} + \frac{d-2}{2}\frac{\partial}{\partial p_i^{\mu}}\right] \leftarrow \text{Conformal Boost}.$$

"To what extent does the sub-leading soft theorem, due to broken conformal boost symmetry, follow from the leading behaviour stemming from broken dilation symmetry?"

- To all order in derivative coupling, the five point matrix elements satisfying leading soft automatically satisfies subleading soft theorems.
- At order s^n, all $2n$-point amplitudes can be recursively constructed via leading soft theorems. Explicit computation has shown that subleading soft theorems are again automatically satisfied.

Perturbative completion

So far we have consider

$$\mathcal{L}_{EFT} = \mathcal{L}_{marginal} + \sum_i c_i \mathcal{O}_i(\partial, \phi)$$

for higher insertions $n > 4$. Can we say more about $n = 4$?

$$M_4 = \sum_{p,q} c_{p,q} \sigma_2^p \sigma_3^q, \quad \sigma_2 = s^2 + t^2 + u^2, \ \sigma_3 = stu$$

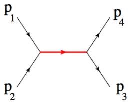
- Continues to be perturbative, with IR degrees of freedom still present in the UV (Four fermi $\rightarrow$ Electro Weak) S-matrix exists

We can employ unitarity

Perturbative completion

Let's assume a perturbative spectrum that is of integer spacing (why?)

- Arises in string theory, and many compactification scenarios
- Necessary for a chance of unitary M_4



$$\rightarrow A_3(\phi_1, \phi_2, h^\ell) \times A_3(h^\ell, \phi_3, \phi_4)$$

Since $A_3(\phi_1,\phi_2,h^\ell) \sim i c_\ell (p_1-p_2)^{\mu_1}(p_1-p_2)^{\mu_2}\cdots(p_1-p_2)^{\mu_\ell}\epsilon_{\mu_1\mu_2\cdots\mu_\ell}$ the residue must take the simple form:

$$[(p_1 - p_2)\cdot(p_3-p_4)]^{2n} = (t-u)^{2n} = (2t+s)^{2n}$$

the residue must be a definite positive function in t:

Perturbative completion

Let's assume a perturbative spectrum that is of integer spacing (why?)

- Arises in string theory, and many compactification scenarios
- Necessary for a chance of unitary M_4

$$M \sim \left. \frac{f(s,t)}{\color{red}{(s-m_1)}(t-m_2)\cdots} \right|_{s=m_1} \to \frac{f(m_1,t)}{(t-m_2)\cdots}$$

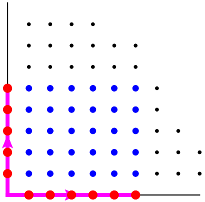
Unitarity requires the function $f(m_1, t)$ to have a zero when $t = m_2$, and all other t-channel poles.

Perturbative completion

Let's assume a perturbative spectrum that is of integer spacing (why?)

- Arises in string theory, and many compactification scenarios
- Necessary for a chance of unitary M_4
  $f(s,t)$ is a bounded polynomial function that has zero for each pair of $(s,t) = (m_i, m_j)$!

$$f(m_1, m_1) = f(m_1, m_2) = \cdots = f(m_i, m_j) = 0$$

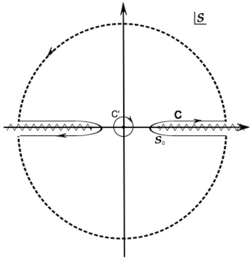


There are more zeros than poles, unless integers

Perturbative completion

Consider the amplitude for some fixed $t = t^*$, which we express in the form of a dispersion relation

$$M(s,t^*) = \int_{v=s} dv \frac{M(v,t^*)}{v-s} = -\frac{Res[M(v,t^*)]|_{v=v^*}}{v^*-s}$$



The residues in the complex s-plane lies on the real axis, where poles in the positive region are s-channel resonance, and negative region are from u-channel resonance. Due to permutation invariance, the residue of a given s-channel resonance, say $s = n$, there will be the opposite of the u-channel resonance in the negative s-branch, $s = -n - t^*$

$$M(s,t^*) = -\sum_{n=0}^{\infty} \frac{Res[M(v,t^*)]|_{v=n}(2n+t^*)}{(n-s)(n+t^*+s)}$$

Perturbative completion

$$M(s,t^*) = -\sum_{n=0}^{\infty} \frac{Res[M(v,t^*)]|_{v=n}(2n+t^*)}{(n-s)(n+t^*+s)}$$

Now, consider the case where $t^* = -2$, then we have:

$$\begin{aligned} M(s,-2) \quad = \quad & -\frac{Res[M(v,-2)]|_{v=0}(-2)}{(-s)(-2+s)} - \frac{Res[M(v,-2)]|_{v=1}(0)}{(1-s)(-1+s)} \\ & -\frac{Res[M(v,-2)]|_{v=2}(2)}{(2-s)(s)} - \sum_{n=3}^{\infty} \frac{Res[M(v,-2)]|_{v=n}(2n-2)}{(n-s)(n-2+s)} \,. \end{aligned} \quad (1)$$

There are no poles at $s = 0, 1$! For $t = -n$ the poles of $s = 0, 1, \cdots, n$ are missing

$$Res[M(s,t)]|_{s=0} = \prod_{i=1}^{\infty} (t+i)$$

But this is impossible for bounded high-energy behavior $\rightarrow$ The S-matrix must have zeros in the unphysical channel, at $s = -n$

Perturbative completion

The S-matrix must have zeros in the unphysical channel, at $s = -n$

$$M_4 \sim \frac{\prod_i (s+i)(t+i)(u+i)}{\prod_i (s-i)(t-i)(u-i)} \sim \frac{\Gamma[-s+1]\Gamma[-t+1]\Gamma[-u+1]}{\Gamma[s+1]\Gamma[t+1]\Gamma[u+1]}$$

Conclusion

- For spontaneously broken space-time symmetry, the broken symmetry mixes between various GB modes, leading to distinct soft features.
- Combined with analyticity and unitarity this imposes stringent constraint on the effective action: the entire action is determined by coefficient of $\partial^{2n}\varphi^n$.
- Maximal susy allows us to push this up to ten-derivatives (the simplest EFT?)
- A new arena to explore the relation between scale vs conformal invariance.

Further directions

- Constraint from S-duality
- High-time to extract unitarity constraint beyond four-points (related to *a*-theorem)
- Is the massless S-matrix well defined at the origin?

$$S_1 = -\frac{Nc_3^2}{2\pi^2}\int \phi^4 \sqrt{-\det\left(\eta_{\mu\nu} + \frac{\partial_\mu\phi\cdot\partial_\nu\phi}{c_3\,\phi^4} + \sqrt{\pi/(g_s N)}\frac{F_{\mu\nu}}{c_3\,\phi^2}\right)}\, d^4x.$$

Compare

$$\langle\vec{\phi}\rangle = (v,0,0,0,0,0),\ vs,\ \langle\vec{\phi}\rangle = (v,v,v,v,v,v)$$

The latter has the usual Adler's soft theorem. Do the near origin limit agree? (Ratio functions)