D-term Triggered Dynamical Supersymmetry Breaking

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- latest work in progress on overall U(1) scalar mass
- **I)** most desirable to break $\mathcal{N}=1$ SUSY dynamically (DSB)
	- In the past, instanton generated superpotential e.t.c. $\langle F\rangle_{\rm nonpt}\to \langle D\rangle\neq 0$ In this talk, we will accomplish D-term DSB (DDSB) in a self-consistent **Hartree-Fock approximation**
	- based on the nonrenormalizable D-gaugino-matter fermion term which is present in generic $\mathcal{N}=1$ SUSY (effective) U(N) gauge action
	- the vac. is metastable, can be made long lived
	- requires the discovery of scalar gluons as well as the scalar in the overall $U(1)$ in nature, so that distinct from the previous proposals
	- 1 • no messenger field needed in application, overall U(1) the hidden sector DSB will be accomplished relatively easily and is robust

• **features of our work as Q. F. T.**

1) variational prob. of V_{eff} in QFT for multiple species of order parameters

2) beginning quantitative estimates of susy breaking order parameters and scalar gluon mass

Contents

- I) Introduction and punch lines
- II) action, assumptions and some properties
- III) effective potential in the Hartree-Fock approximation
- IV) stationary conditions and gap equation

<u>• action</u>

 $\mathcal{L} = \int d^4\theta K(\Phi^a, \bar{\Phi}^a) + (gauging) + \int d^2\theta \text{Im} \frac{1}{2}\tau_{ab}(\Phi^a) \mathcal{W}^{\alpha a} \mathcal{W}^b_{\alpha} + \left(\int d^2\theta W(\Phi^a) + c.c. \right)$ $=\mathcal{L}_{\text{Kähler}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{sup}}$

- K ; Kähler potential
- τ_{ab} ; gauge kinetic superfield from the second derivatives of a generic holomorphic function $\mathcal{F}(\Phi^a)$ W ; superpotential

We look at its component expansion

•

•

•

・**assumptions made**

- 1) general $\mathcal{N}=1$ action with adjoint $\Phi^a \& V^a$ with three input functions K, \mathcal{F}_{ab}, W
- 2) third derivatives of $\mathcal F$ at the scalar vev's nonvanishing
- 3) W at tree level preserves $\mathcal{N}=1$ susy
- 4) the gauge group $U(N)$, the vac. being in its unbroken phase

r supercurrent $\mathbf{L} \mathbf{D}^a$, F^a eqs

off-shell form of the ${\cal N}=1$ supercurrent

$$
\eta_1 \mathcal{S}^{(1)\mu} = \sqrt{2} g_{ab} \eta_1 \sigma^{\nu} \bar{\sigma}^{\mu} \psi^a \mathcal{D}_{\nu} \bar{\phi}^b + \sqrt{2} i g_{ab} \eta_1 \sigma^{\mu} \bar{\psi}^a F^b
$$

$$
- i \mathcal{F}_{ab} \eta_1 \sigma_{\nu} \bar{\lambda}^a F^{\mu \nu b} + \frac{1}{2} \mathcal{F}_{ab} \epsilon^{\mu \nu \rho \delta} \eta_1 \sigma_{\nu} \bar{\lambda}^a F^{\rho \delta b} - \frac{i}{2} \bar{\mathcal{F}}_{ab} \eta_1 \sigma^{\mu} \bar{\lambda}^a D^b
$$

$$
+ \frac{\sqrt{2}}{4} \left(\mathcal{F}_{abc} \psi^c \sigma^{\nu} \bar{\sigma}^{\mu} \lambda^b - \bar{\mathcal{F}}_{abc} \bar{\lambda}^c \bar{\sigma}^{\mu} \sigma^{\nu} \bar{\psi}^b \right) \eta_1 \sigma_{\nu} \bar{\lambda}^a
$$

 \Rightarrow NGF will be l.c. of $\lambda^0 \& \psi^0$

$$
\begin{aligned}\n\bullet \qquad D^a &= -\frac{1}{2} g^{ab} \mathfrak{D}_b - \frac{1}{2\sqrt{2}} g^{ab} \left(\mathcal{F}_{bcd} \psi^d \lambda^c + \bar{\mathcal{F}}_{bcd} \bar{\psi}^d \bar{\lambda}^c \right), \\
F^a &= -g^{ab} \overline{\partial_b W} - \frac{i}{4} g^{ab} \left(\mathcal{F}_{bcd} \psi^c \psi^d - \bar{\mathcal{F}}_{bcd} \bar{\lambda}^c \bar{\lambda}^d \right)\n\end{aligned}
$$

・ **reasoning to DDSB**

• look at

•

$$
-\frac{1}{2}(\lambda^a, \psi^a) \begin{pmatrix} 0 & -\frac{\sqrt{2}}{4} \mathcal{F}_{abc} D^b \\ -\frac{\sqrt{2}}{4} \mathcal{F}_{abc} D^b & \partial_a \partial_c W \end{pmatrix} \begin{pmatrix} \lambda^c \\ \psi^c \end{pmatrix} + (c.c.).
$$

the non-vanishing vev of $D^0 \Rightarrow$ Dirac mass to the fermions

$$
\langle D^0 \rangle = -\frac{1}{2\sqrt{2}} \langle g^{00} \left(\mathcal{F}_{0cd} \psi^d \lambda^c + \bar{\mathcal{F}}_{0cd} \bar{\psi}^d \bar{\lambda}^c \right) \rangle, \tag{namely,}
$$

condensation of the Dirac bilinear responsible for $\langle D^0 \rangle \neq 0$.

• holomorphic part of the mass matrix:

$$
M_{Fa} \equiv \begin{pmatrix} 0 & -\frac{\sqrt{2}}{4} \langle \mathcal{F}_{0aa} D^0 \rangle \\ -\frac{\sqrt{2}}{4} \langle \mathcal{F}_{0aa} D^0 \rangle & \langle \partial_a \partial_a W \rangle \end{pmatrix}.
$$
mixed Majorana-Dirac type
eigenvalues:
$$
\Lambda_{a11}^{(\pm)} = \frac{1}{2} \langle \partial_a \partial_a W \rangle \left(1 \pm \sqrt{1 + \frac{\langle \mathcal{F}_{0aa} D^0 \rangle^2}{2 \langle \partial_a \partial_a W \rangle^2}} \right).
$$

• however, the non-vanishing F^0 term induced as well, as the stationary value of the scalar fields gets shifted.

In particular, the holo. part of the complete mass matrix

$$
\mathcal{M}_a = \begin{pmatrix} -\frac{i}{2}g^{aa}\mathcal{F}_{0aa}F^0, & -\frac{\sqrt{2}}{4}\sqrt{g^{aa}(\text{Im}\mathcal{F})^{aa}}\mathcal{F}_{0aa}D^0\\ -\frac{\sqrt{2}}{4}\sqrt{g^{aa}(\text{Im}\mathcal{F})^{aa}}\mathcal{F}_{0aa}D^0, & g^{aa}\partial_a\partial_aW + g^{aa}g_{0a,a}\bar{F}^0 \end{pmatrix} = \begin{pmatrix} m_{\lambda\lambda}^a & m_{\lambda\psi}^a\\ m_{\psi\lambda}^a & m_{\psi\psi}^a \end{pmatrix}
$$

• suppress the indices as we work with the unbroken phase U(N) phase

$$
\Delta \equiv -\frac{2m_{\lambda\psi}}{m_{\psi\psi}}, \qquad f \equiv \frac{2im_{\lambda\lambda}}{\text{tr}\mathcal{M}}.
$$

The two eigenvalues of the holomorphic mass matrix

$$
\Lambda^{(\pm)} \equiv (\mathrm{tr} \mathcal{M}) \lambda^{(\pm)},
$$

where

$$
\lambda^{(\pm)} = \frac{1}{2} \left(1 \pm \sqrt{(1+if)^2 + \left(1 + \frac{i}{2}f\right)^2 \Delta^2} \right).
$$

- The masses for the two species of SU(N) fermions ⇒
- Q: how to determine the stationary values of the scalar fields, $D^{\,0}$ and $F^{\,0}$ (perturbatively induced)

Corresponding to
$$
\mathcal{N} = 1
$$
supergravity

\n**and super-Higgs mechanism**

\n
$$
\mathcal{L} = \int d^2 \Theta 2 \mathcal{E} \left[\frac{3}{8} (\bar{\mathcal{D}} \bar{\mathcal{D}} - 8 \mathcal{R}) \exp \left\{ -\frac{1}{3} [K(\Phi, \Phi^{\dagger}) + \Gamma(\Phi, \Phi^{\dagger}, V)] \right\} + \frac{1}{16g^2} \tau_{ab}(\Phi) W^{\alpha a} W_{\alpha}^b + W(\Phi) \right] + h.c.
$$

The field redefinition of the gravitino

$$
\psi'_{\mu} = \psi_{\mu} + i \frac{\sqrt{2}}{6W^*e^{K/2}} \sigma^{\mu} \bar{\psi}_{\text{NG}} + \frac{\sqrt{2}}{3W^{*2}e^{K}} \partial_{\mu} \bar{\psi}_{\text{NG}}
$$

that eliminates the mixing with λ and ψ :

$$
\bar{\psi}_{\rm NG} \equiv i \frac{g}{\sqrt{2}} D_a \bar{\lambda}^a + e^{K/2} D_{a^*} W^* \bar{\psi}^a.
$$

• gravitino mass

$$
m_{3/2} \simeq e^{\langle K \rangle/2} \frac{\sqrt{|\langle D_a W \rangle|^2 + \frac{g^2}{2} \langle D^a \rangle^2}}{\sqrt{3}M_P^2}.
$$

・**special cases**

demand the Kähler function K to be special Kähler,

$$
\Rightarrow \qquad K = \text{ImTr } \overline{\Phi} \frac{\partial \mathcal{F}(\Phi)}{\partial \Phi},
$$

and $g_{ab} = \text{Im} \mathcal{F}_{ab}$ etc.

further, choose W such that the action possess the rigid $\mathcal{N}=2$ supersymmetry

⇒ tree vacua $\mathcal{N}=2\rightarrow\mathcal{N}=1$ spontaneously

> U(1): Antoniadis , Partouche, Taylor (1996) U(N): Fujiwara, H. I., Sakaguchi (2004)

change the notation for expectation values from $\langle \dots \rangle$ to \dots (\odot vev.= the stationary value in the variational analysis.)

・**point of the H.F. approximation**

spirit: tree ~ **1-loop in the** ℏ **expansion**

⇒ **optimal configuration, which is transcendental**

• three const. bkgd fields,

 $\varphi \equiv \varphi^0$ (complex), U(N) invariant scalar, $D \equiv D^0$ (real), $F \equiv F^0$ (complex).

• denote our effective potential by

$$
V = V^{\text{tree}} + V_{\text{c.t.}} + V_{1-\text{loop}}.
$$

after the elimination of the auxiliary fields denote by V_{scalar}

・**tree part & warm up**

all config. $U(N)$ inv. \Rightarrow suppress indices

$$
V^{\text{tree}}(D, F, \bar{F}, \varphi, \bar{\varphi}) = -gF\bar{F} - \frac{1}{2}(\text{Im}\mathcal{F}'')D^2 - FW' - \bar{F}\bar{W}'.
$$

• stationary conditions ⇒

all minus signs correct

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 $V_{\text{scalar}}^{\text{tree}}(\varphi,\bar{\varphi}) \equiv V^{\text{tree}}(\varphi,\bar{\varphi},D_{*}=0,F=F_{*}(\varphi,\bar{\varphi}),\bar{F}=\overline{F_{*}(\varphi,\bar{\varphi})})=g^{-1}(\varphi,\bar{\varphi})|W'(\varphi)|^{2}.$

$$
\left. \frac{\partial^2 V_{\text{scalar}}^{\text{tree}}(\varphi, \bar{\varphi})}{\partial \varphi \partial \bar{\varphi}} \right|_{\varphi_*, \bar{\varphi}_*} = g^{-1}(\varphi_*, \bar{\varphi}_*) \left| W''(\varphi_*) \right|^2,
$$

$$
m_s(\varphi, \bar{\varphi}) \equiv g^{-1}(\varphi, \bar{\varphi}) W''(\varphi),
$$

$$
m_{s*} = m_s(\varphi_*, \bar{\varphi}_*).
$$

$$
\Delta \equiv -2\frac{m_{\lambda\psi}}{m_{\psi\psi}} = \frac{\sqrt{2}}{2} \frac{\sqrt{g^{-1}(\text{Im}\mathcal{F}'')^{-1}} \mathcal{F}'''}{g^{-1}W'' + g^{-1}\partial g\bar{F}} \ D \equiv r(\varphi, \bar{\varphi}, F, \bar{F})D.
$$

$$
f_3 \equiv \frac{g^{-1}\mathcal{F}''F}{g^{-1}W'' + g^{-1}\partial g\bar{F}},
$$

• the mass scales of the problem:

 m_{s*} , the scalar gluon mass and $g^{-1}\overline{\mathcal{F}}''_{*}$, the third prepotential derivative, (and $g^{-1}\partial g$). SUSY breaking scale being essentially the geometric mean.

・**treatment of UV infinity**

UV scale and infinity reside in \mathcal{F} . The supersymmetric counterterm:

$$
V_{\text{c.t.}} = -\frac{1}{2} \text{Im} \int d^2 \theta \Lambda \mathcal{W}^{0\alpha} \mathcal{W}_{0\alpha} = -\frac{1}{2} (\text{Im} \Lambda) D^2.
$$

It is a counterterm associated with $\mathrm{Im}\mathcal{F}''$. A renormalization condition

$$
\left. \frac{1}{N^2} \frac{\partial^2 V}{(\partial D)^2} \right|_{D=0, \varphi=\varphi_*, \bar{\varphi}=\bar{\varphi}_*} = 2c,
$$

relate (or transmute) the original infinity of the dimensional reduction scheme with that of $\text{Im}\,\mathcal{F}''$.

・**the one-loop part**

$$
V_{1-\text{loop}} = \frac{N^2 |\text{tr}\mathcal{M}|^4}{32\pi^2} \left[A(\varepsilon, \gamma) \left(|\lambda^{(+)}|^4 + |\lambda^{(-)}|^4 - \left| \frac{m_s}{\text{tr}\mathcal{M}} \right|^4 \right) - |\lambda^{(+)}|^4 \log |\lambda^{(+)}|^2 - |\lambda^{(-)}|^4 \log |\lambda^{(-)}|^2 + \left| \frac{m_s}{\text{tr}\mathcal{M}} \right|^4 \log \left| \frac{m_s}{\text{tr}\mathcal{M}} \right|^4 \right].
$$

$$
A(\varepsilon, \gamma) = \frac{1}{2} - \gamma + \frac{1}{\varepsilon}, \qquad \varepsilon = 2 - \frac{d}{2}.
$$

IV) ・**variational analysis**

•

$$
\frac{\partial V}{\partial D} = 0
$$

$$
\frac{\partial V}{\partial F} = 0
$$
 and its complex conjugate

$$
\frac{\partial V}{\partial \varphi} = 0
$$
 and its complex conjugate

• work in the region where the strength $||F_*||$ small and can be treated perturbatively.

$$
\begin{aligned} \text{gap eq.} \qquad & \frac{\partial V(D,\varphi,\bar\varphi,F=0,\bar F=0)}{\partial D}=0,\\ \text{stationary cond.} \qquad & \frac{\partial V(D,\varphi,\bar\varphi,F=0,\bar F=0)}{\partial \varphi}=0 \end{aligned}
$$

stationary values $(D_*,\varphi_*,\bar{\varphi}_*)$

$$
\left. \frac{\partial V(D = D_*(0,0), \varphi = \varphi_*(0,0), \bar{\varphi} = \bar{\varphi}_*(0,0), F, \bar{F})}{\partial F} \right|_{D, \varphi, \bar{\varphi}, \bar{F} \text{ fixed}} = 0
$$

 $\Rightarrow \bar{F} = \bar{F}_*$ perturbatively

<u>•the analysis in the region $F_* \approx 0$ **</u>**

• explicit determination of $V(D, \phi, \bar{\phi}, F = 0, \bar{F} = 0)$: first solve the normalization condition

$$
2cN^2 = \left. \frac{\partial^2 V}{(\partial D)^2} \right|_{D=0, \Box}
$$

to obtain

$$
A = \frac{1}{2} + \frac{32\pi^2}{|m_{s*}|^4 (r_{0*}^2 + \bar{r}_{0*}^2)} \left(2c + \frac{\text{Im}\mathcal{F}''_*}{N^2} + \frac{\text{Im}\Lambda}{N^2}\right) \equiv \tilde{A}(c, \Lambda, \varphi_*, \bar{\varphi}_*).
$$

• r, Δ , complex in general, put sub. 0, as $F, \bar{F} \rightarrow 0$

$$
V_0 = V(D, \varphi, \bar{\varphi}, F = 0, \bar{F} = 0)
$$

$$
\frac{V_0}{N^2 |m_s|^4} = \left(\frac{1}{64\pi^2} + \tilde{c} - \tilde{\delta}(\varphi, \bar{\varphi})\right) \left(\frac{\Delta_0 + \bar{\Delta}_0}{2}\right)^2 + \frac{1}{32\pi^2} \tilde{A} \left(\frac{1}{8} |\Delta_0|^4 + f(\Delta_0, \bar{\Delta}_0)\right)
$$

$$
-\frac{1}{32\pi^2} \left(|\lambda_0^{(+)}|^4 \log |\lambda_0^{(+)}|^2 + |\lambda_0^{(-)}|^4 \log |\lambda_0^{(-)}|^2\right),
$$

$$
f(\Delta_0, \bar{\Delta}_0) = \frac{1}{2} \left(\sqrt{1 + \Delta_0^2} \sqrt{1 + \bar{\Delta}_0^2} - |\Delta_0|^2 - 1\right).
$$

• If Δ_0 real,

$$
\frac{V_0}{N^2|m_s|^4} = \left(\left(c' + \frac{1}{64\pi^2} \right) - \delta \right) \Delta_0^2 + \frac{1}{32\pi^2} \left[\frac{\tilde{A}}{8} \Delta_0^4 - \lambda_0^{(+)^4} \log \lambda_0^{(+)^2} - \lambda_0^{(-)^4} \log \lambda_0^{(-)^2} \right]
$$

From now on, Δ_0 real case only,

• gap equation
\n
$$
\frac{\partial V_0}{\partial D}\Big|_{\varphi,\bar{\varphi}} = 0.
$$
\n
$$
0 = \Delta_0 \left[2 \left(\left(c' + \frac{1}{64\pi^2} \right) - \delta \right) + \frac{1}{32\pi^2} \left\{ \frac{\tilde{A}}{2} \Delta_0^2 - \frac{1}{\sqrt{1 + \Delta_0^2}} \left(\lambda_0^{(+)3} \left(2 \log \lambda_0^{(+)2} + 1 \right) - \lambda_0^{(-)3} \left(2 \log \lambda_0^{(-)2} + 1 \right) \right) \right\} \right],
$$
\n
$$
\delta_* = 0
$$
\nIMaru1

• stationary cond.

$$
\frac{\partial V_0}{\partial \varphi}\Big|_{D,\bar{\varphi}} = 0
$$

2 $\partial (\ln |m_s|^2) \frac{V_0}{N^2 |m_s|^4} = \left(\frac{\partial \delta}{\partial \varphi}\right) \Delta_0^2 - \frac{\partial \Delta_0}{\partial \varphi} \frac{\partial}{\partial \Delta_0} \left(\frac{V_0}{N^2 |m_s|^4}\right)$

using the gap eq.

$$
\frac{V_0}{N^2|m_s|^4} = \frac{\frac{\partial \delta}{\partial \varphi}}{2\partial(\ln|m_s|^2)}\Delta_0^2
$$

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 $(\Delta_{0*}, \varphi_* = \bar{\varphi}_*)$ determined as the point of intersection of the two real curves in the $(\Delta_0, \varphi = \bar{\varphi})$ plane

Schematically,

DSB HAS BEEN REALIZED

・**numerical study**

• the minimal choice for DDSB:

$$
\mathcal{F} = \frac{c}{2N} \text{tr}\varphi^2 + \frac{1}{3!MN} \text{tr}\varphi^3 \equiv \frac{1}{2} c\varphi^2 + \frac{1}{3!M} \varphi^3,
$$

$$
W = \frac{m^2}{N} \text{tr}\varphi + \frac{d}{3!N} \text{tr}\varphi^3 \equiv m^2 \varphi + \frac{d}{3!} \varphi^3,
$$

• consistency check: $\left|\frac{F_*}{D_*}\right|$

$$
\left. \frac{F_*}{D_*} \right| \ll 1, \ |f_{3*}| \ll 1
$$

• samples:

・**Metastability of our false vacuum**

 $\langle D \rangle = 0$ tree vacuum is not lifted

 \Rightarrow check if our vacuum $\langle D \rangle \neq 0$ is sufficiently long-lived

Decay rate of
$$
\propto \exp\left[-\frac{\langle \Delta \phi \rangle^4}{\langle \Delta V \rangle}\right] = \exp\left[-\frac{(\Delta_0 \Lambda)^2}{m_s^2}\right] \ll 1 \quad \Delta_0 \Lambda \gg m_s
$$

・**second variation and mass of scalar gluons**

 $V_{\text{scalar}} = V(D = D_*(\varphi, \bar{\varphi}), F = F_*(\varphi, \bar{\varphi}) \approx 0, \bar{F} = \bar{F}_*(\varphi, \bar{\varphi}) \approx 0, \varphi, \bar{\varphi})$

at the stationary point $(D_*(\varphi_*,\bar{\varphi}_*),0,0,\varphi_*,\bar{\varphi}_*).$

separate $V(D, F, F, \varphi, \bar{\varphi})$ into two parts:

$$
V = \mathcal{V} + V_0
$$

Here

$$
\mathcal{V}(F,\bar{F},\varphi,\bar{\varphi}) \approx -gF\bar{F} - FW' - \bar{F}\bar{W}' + (\partial_F V_{1-\text{loop}})_*F + (\partial_{\bar{F}} V_{1-\text{loop}})_*\bar{F}
$$

+
$$
\frac{1}{2}(\partial_F^2 V_{1-\text{loop}})_*F^2 + \frac{1}{2}(\partial_{\bar{F}}^2 V_{1-\text{loop}})_*F^2 + (\partial_F \partial_{\bar{F}} V_{1-\text{loop}})_*F\bar{F},
$$

$$
V_0(D,\varphi,\bar{\varphi}) = V(D,\varphi,\bar{\varphi},F=0,\bar{F}=0).
$$

make exploit "implicit fn derivatives"

$$
\begin{aligned}\n\text{e.g.} \quad & \text{For } \mathcal{V}, \, \vec{y}_L = (F, \bar{F}), \vec{y}_R = (\varphi, \bar{\varphi}), \\
& \delta^2 \mathcal{V}_* \approx \frac{1}{2} \delta \vec{y}_R^t M_{RL_*} (-M_{LL_*}^{-1}) M_{LR_*} \delta \vec{y}_R \qquad M_{RR_*} \approx 0\n\end{aligned} \qquad\n\text{20}
$$

• scalar gluon mass: $\frac{1}{g} |W'' - (\partial \partial_F V_{1-\text{loop}})|_*^2$

in the region $|(\partial_F \partial_{\bar{F}} V)_0|_*, |(\partial_F^2 V)_0|_*, \ll g_*,$

consistency checked

• samples:

Summary

- A dynamical mechanism of DDSB proposed
- A nontrivial solution to the gap eq. for $\langle D \rangle$ found in a self-consistent HF approx.
- Our vacuum is metastable & can be made long-lived
- Numerical examples found