One-loop Partition Function in AdS_3/CFT_2

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ITP-PKU

1st East Asia Joint Workshop on Fields and Strings,
May 28-30, 2016, USTC, Hefei

Based on the work with Jie-qiang Wu, arXiv:1509.02062

Outline

AdS_3/CFT_2 correspondence

- Semiclassical AdS_3 gravity
- 1-loop partition function

The proof in CFT

- Sewing prescription
- Large c CFT
- higher genus partition function

Conclusion

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A new window to study AdS/CFT without resorting to string theory

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More precisely they found that under appropriate boundary conditions the asymptotic symmetry group (ASG) of AdS_3 Einstein gravity is generated by two copies of Virasoro algebra with central charges

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In modern understanding: quantum gravity in AdS_3 is dual to a 2D CFT at AdS boundary

AdS_3/CFT_2: a perfect platform

AdS_3 gravity is solvable: all classical solutions are quotients of AdS_3 such that a path-integral is possible in principle E. Witten (1988) ...
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However, it is not clear

1. how to define the quantum AdS_3 gravity?
2. what is the dual CFT?

Semiclassical AdS_3 gravity

Let us focus on the semiclassical gravity, which corresponds to the CFT at the large central charge limit

$$c = \frac{3l}{2G}$$

- The partition function gets contributions from the saddle points

- For each classical solution, its regularized on-shell action $\propto 1/G \sim c$
- 1-loop determinant of the fluctuations around the configurations $\propto O(1)$

Semiclassical solutions

$$R_{\mu\nu} = -\frac{2}{l^2} g_{\mu\nu},$$

- All solutions are locally AdS_3
- More precisely, all classical solutions could be obtained as the quotients of global AdS_3 by the Kleinian group, a discrete subgroup of $PSL(2, C)$

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- At the boundary $r \to 0$, we have the Riemann sphere Ω
- The action of $SL(2, C)$ on Ω is a Mobius transformation

$$z \to \frac{az+b}{cz+d}, \qquad a, b, c, c \in C, \qquad ad - bc = 1$$

Handlebody solutions

Among all the solutions, the so-called handle-body solutions are of particular importance, and have been best understood.
The handlebody solution is homeomorphic to a domain enclosed by the closed surface
The non-handlebody solutions are much less understood (unstable?)

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For the handlebody solutions, the subgroup Γ is a Schottky group, a finitely generated free group, such that all nontrivial elements are loxodromic

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \sim \begin{pmatrix} p^{1/2} & 0 \\ 0 & p^{-1/2} \end{pmatrix}, \quad 0 < |p| < 1$$

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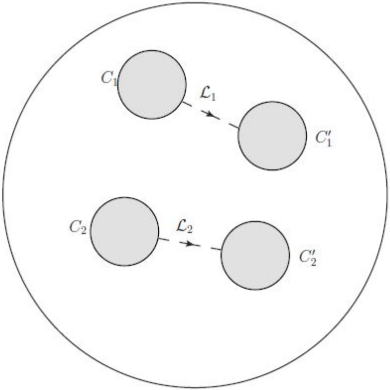
On the boundary, there is a compact Riemann surface, which could be determined by the Schottky uniformization "Retrosection theorem" by Koebe (1914)

Schottky group



The loxodromic element $\mathcal{L}_i(\mathcal{L}_i^{-1})$ maps C_i to C_i' such that the outer(inner) part of C_i is mapped to the inner(outer) part of $C_i^{'}$. The elements $\{\mathcal{L}_i\}$ freely generate the Schottky group

On-shell regularized action

- With respect to a Schottky group, there is a handle-body gravitational solution
- The essential point is that the on-shell regularized bulk action of gravitational configurations in pure AdS_3 gravity is a Liouville type action defined on the fundamental region K. Krasnov (2000), Zograf and Takhtadzhyan (1988)

$$S_{ZT}[\phi_s] = -\frac{c}{24\pi} \int \int_D \frac{i}{2} dz \wedge d\bar{z} \left(4\partial_z \phi_s \partial_{\bar{z}} \phi_s + \frac{1}{2} e^{2\phi_s} \right) + \text{boundary terms.}$$

- It is remarkable that the ZT action captures the whole leading contribution in the CFT partition function on boundary Riemann surface in the large c limit.

1-loop correction

- Consider the fluctuations around the gravitational configuration and compute their functional determinants
- 1-loop partition function Giombi et.al. 0804.1773, Yin 0710.2129

$$Z^{1-loop} = \prod_{\gamma \in \mathcal{P}} \prod_{s} \prod_{m=s}^{\infty} \frac{1}{|1 - q_\gamma^m|}.$$

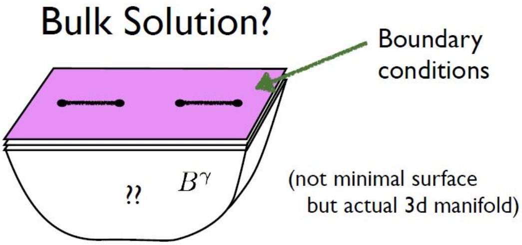
  Here the product over s is with respect to the spins of massless fluctuations and $\mathcal{P}$ is a set of representatives of primitive conjugacy classes of the Schottky group Γ. q_γ is called the multiplier of $\gamma \in \Gamma$, whose two eigenvalues are $q_\gamma^{\pm 1/2}$ with $|q_\gamma| < 1$.
- $\mathcal{P} = \{$non-repeated words up to conjugation$\}$, e.g.

$$\mathcal{P} = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_1^{-1}, \mathcal{L}_2^{-1}, \mathcal{L}_1\mathcal{L}_2 \sim \mathcal{L}_2\mathcal{L}_1, ...\}$$

- Difficulty: infinite number of words
- This formula was first conjectured by Xi Yin, and later has been derived by using the heat kernels and the method of images.
- Our work is to prove this relation in the dual CFT.

Motivation

- Our motivation comes from the study of holographic Rényi entropy
- In 2D CFT, the Rényi entropy is determined by the partition function on a higher genus Riemann surface Σ_n.
- Holographically, from AdS_3/CFT_2 correspondence, the partition function is captured by the partition function of the corresponding gravitational configuration B^γ such that $\partial B^\gamma = \mathcal{M}_g$
- Picture: in the large c limit, the leading term in CFT partition function should be equal to the on-shell regularized action, and the next-to-leading terms should correspond to 1-loop correction



Motivation II

This picture turns out to be correct in the classical level, and leads to the proof of the Ryu-Takayanagi formula for the holographic entanglement entropy under some reasonable assumptions T. Hartman 1303.6955, T. Faulkner 1303.7221

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Accumulated evidence shows that this is also true beyond the classical level

1. double-interval case Barrella et.al. 1306.4682,BC et.al. 1312.5510
2. single interval on a torus Barrella et.al. 1306.4682, BC and J.q. Wu 1405.6254,1507.00183
3. large single interval on a torus BC and J.q. Wu 1506.03206

Project

Prove the 1-loop correction from dual CFT:

$$Z^{1-loop} = \prod_{\gamma \in \mathcal{P}} \prod_{s} \prod_{m=s}^{\infty} \frac{1}{|1 - q_\gamma^m|}.$$

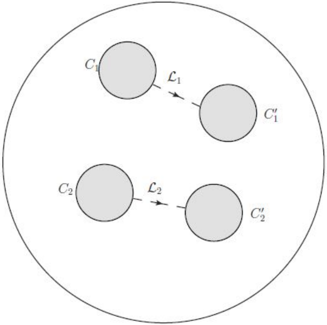
Not only for the configurations appearing in the computation of HRE, but for any handle-body solutions.

Partition function on a genus-g RS

It can be computed using the sewing prescription, following Segal's approach to CFT. It is defined to be the summation of $2g$-point functions on the Riemann sphere M.R. Gaberdiel et.al. 1002.3371

$$Z_g = \sum_{\phi_i, \psi_i \in \mathcal{H}} \prod_{i=1}^{g} G^{-1}_{\phi_i \psi_i} \langle \prod_{i=1}^{g} \phi_i[C_i] \psi_i[C'_i] \rangle_D ,$$

ϕ_i, ψ_i are the states in the Hilbert space $\mathcal{H}$, and $\phi_i[C_i]$ denote the states associated with the boundary circle C_i. $G_{\phi\psi}$ is the metric on the space of the states



Partition funciton on Σ_g

Via state-operator correspondence, the states can be transformed to the vertex operators inserted at the fixed points. With the vertex operators, the partition function is changed to the summation over $2g$-point functions of the vertex operators inserted at $2g$ fixed points

$$Z_g = \sum_{\phi_i,\psi_i\in\mathcal{H}} \prod_{i=1}^{g} G^{-1}_{\phi_i\psi_i} \langle \prod_{i=1}^{g} V(U(\gamma_i)p_i^{L_0}\phi_i, a_i) V(U(\gamma_i\hat{\gamma})\psi_i, r_i)\rangle,$$

This relation could be understood in the following way: one can insert a complete set of states in the Hilbert space at each pair of the circles C_i and C_i', which are related by the Schottky generator $\mathcal{L}_i$, and compute all the possible $2g$-point functions of corresponding vertex operators on the Riemann sphere.

This prescription could be applied to any CFT, but is most effective to read the next-to-leading terms in the large c CFT.

Vacuum module

In the CFT dual to pure AdS_3 gravity, the vacuum module dominates the partition function in the large c limit T. Hartman 1303.6955,...

The vacuum module is generated by the Virasoro generators acting on the vacuum.

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Let's focus on the holomorphic sector

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n},$$

The vacuum module

$$...L_{-n}^{r_n}...L_{-3}^{r_3}L_{-2}^{r_2} \mid 0\rangle,$$

Vacuum module in the large c limit I

In the large c limit, the vacuum module is simplified significantly.
We can renormalize the operators

$$\hat{L}_m = |\frac{12}{cm(m^2-1)}|^{\frac{1}{2}} L_m \quad \text{for } |m| \geq 2,$$

A general state in the vacuum module could be of the form

$$\prod_{m=2}^{\infty} \hat{L}_{-m}^{r_m} \mid 0\rangle,$$

with only finite number of r_m's being non-zero.
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We may define the "particle number" for such a state to be $r = \sum r_m$.
The physical reason behind this definition is that each single-particle state $\hat{L}_{-m}|0>$ corresponds to one graviton.
The single particle state is of particular importance in the following discussion

Vertex operators

For a single-particle state $\hat{L}_{-m}|0>$, its corresponding vertex operator is of the following forms at the origin and the infinity respectively

$$
\begin{aligned}
V_m &= \left(\frac{12}{cm(m^2-1)}\right)^{\frac{1}{2}} \frac{1}{(m-2)!} \partial^{m-2} T(z) \,|_{z=0}, \\
\bar{V}_m &= \left(\frac{12}{cm(m^2-1)}\right)^{\frac{1}{2}} \frac{1}{(m-2)!} (-z^2\partial_z)^{m-2} (z^4 T(z)) \,|_{z\to\infty} \quad \text{for } m \geq 2.
\end{aligned}
$$

At the origin, the normalized vertex operator for the particle-r state reads

$$
\hat{O} =: \left(\prod_{j=1}^{r} V_{m_j}\right) :
$$

In other words, the vertex operator of a multi-particle state is just the normal ordered product of the vertex operators for the single-particle states.

The important point is that this fact is even true for the states on the circle not around the origin.

Partition function on Σ_g in the large c limit

Recall that the partition function on Σ_g is

$$Z_g \mid_z = \sum_{m_1, m_2, \ldots m_g} \langle\, ^{\mathcal{L}_1}\bar{O}^{(1)}_{m_1} O^{(1)}_{m_1}\; ^{\mathcal{L}_2}\bar{O}^{(2)}_{m_2} O^{(2)}_{m_2} \;\ldots\; ^{\mathcal{L}_g}\bar{O}^{(g)}_{m_g}\; O^{(g)}_{m_g} \rangle,$$

where $m_1, m_2, \ldots m_g$ denote the summation of all of the states on the circles $C_1, C_2, \ldots C_g$ and $C'_1, C'_2, \ldots C'_g$.

In the large c limit, the leading contribution in the correlation functions is of order c^0

Moreover, it is dominated by the product of two-point functions of single-particle states

Holographically, this means that we can ignore the interaction of the gravitons, and have a free theory of the gravitons

Every $2g$-point function in the partition function could be decomposed into the summation of the products of g two-point functions in all possible ways.

Genus-1 partition function: revisited

- For the large c CFT, the genus-1 partition function is

$$Z_1 = \prod_{m=2}^{\infty} \frac{1}{1-q^m},$$

  where q is the modular parameter of the torus.
- In the torus case, the Schottky group is generated by only one $SL(2,C)$ element $\mathcal{L}$.
- The genus-1 partition function could be read from

$$Z_1 \quad = \quad \sum_{r=0}^{\infty} \frac{1}{r!} \sum_{\{m_j\}} \langle : (\prod_{j=1}^{r} {}^{\mathcal{L}}\bar{V}_m(r_1)) : \quad : (\prod_{j=1}^{r} V_m(r_1)) : \rangle + O(\frac{1}{c})$$

Genus-1 partition function: I

- For $r = 0$ term, the contribution from the vacuum is 1.
- For $r = 1$ term

$$Z^{(1)} = \sum_{m=2}^{\infty} \langle^{\mathcal{L}} \bar{V}_m(r_1) V_m(a_1) \rangle = \mathrm{Tr}_{\mathcal{H}_1} q^{L_0} = \sum_{m=2}^{\infty} q^m,$$

- For $r > 1$ case, the expectation value equals to

$$\begin{aligned} & \frac{1}{r!} \sum_{m_i=2}^{\infty} \langle :{}^{\mathcal{L}} \bar{V}_{m_1}(r_1) {}^{\mathcal{L}} \bar{V}_{m_2}(r_1) \ldots {}^{\mathcal{L}} \bar{V}_{m_r}(r_1) :: V_{m_1}(a_1) V_{m_2}(a_1) \ldots V_{m_r}(a_1) : \rangle \\ = \quad & \frac{1}{r!} \sum_{m_1=2}^{\infty} \sum_{m_2=2}^{\infty} \ldots \sum_{m_r=2}^{\infty} \sum_{\{P\}} \langle^{\mathcal{L}} \bar{V}_{m_{P_1}}(r_1) V_{m_1}(a_1) \rangle \\ & \cdot \langle^{\mathcal{L}} \bar{V}_{m_{P_2}}(r_1) V_{m_2}(a_1) \rangle \ldots \langle^{\mathcal{L}} \bar{V}_{m_{P_r}}(r_1) V_{m_r}(a_1) \rangle + O(c^{-1}), \end{aligned}$$

There is no two-point function between two V operators or two $\bar{V}$ operators at the same fixed point because of normal ordering.

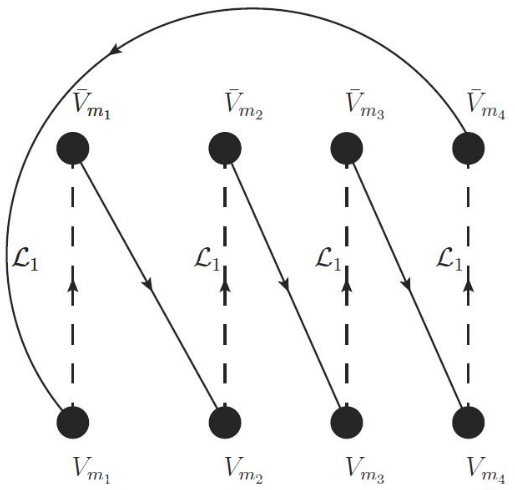


Figure: The link formed by the product of four two-point functions. It corresponds to the conjugacy class $\mathcal{L}^4$. Due to the normal ordering, the only possible connected link is the one in the diagram.

Diagram language

- To classify the possible combination of two-point functions in the summation clearly, we define a diagram language.
- The dotted vertices denote the fixed points, where the operators are inserted: the lower ones are the $V_m(a_1)$'s, while the upper ones are the ${}^{\mathcal{L}}\bar{V}_m(r_1)$'s.
- The dashed lines denote the summations over m_i's and the solid line denotes the correlation between two vertex operators.
- The dashed and solid lines may form a closed contour, which will be called a link.
- In short, a link is defined by certain product of two-point functions of single-particle operators.
- The expectation value of a link is reduced to a two-point function, which is determined by the multiplier of a Schottky group element.
- It is convenient to assign a direction on the dashed line indicating the flow between V to $\bar{V}$.

Genus 1 partition function: continued

For the partition function, we just need to sum over all the contributions from different combinations of the links

$$Z_1 = \sum_{t=0}^{\infty}\prod_{s=1}^{\infty}\frac{1}{s^t}\frac{1}{t!}(\sum_{r=2}^{\infty}q^{sr})^t = \exp\sum_{r=2}^{\infty}-\log(1-q^r) = \prod_{r=2}^{\infty}\frac{1}{1-q^r}.$$

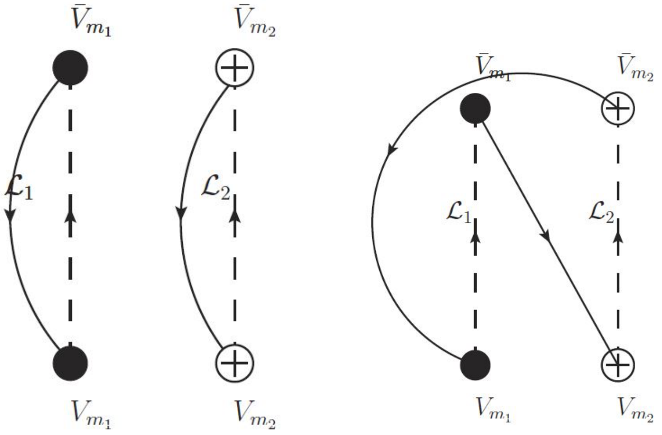
This is the genus-1 partition function found by Maloney and Witten (2007).

Genus 2 case

- The partition function could be written as

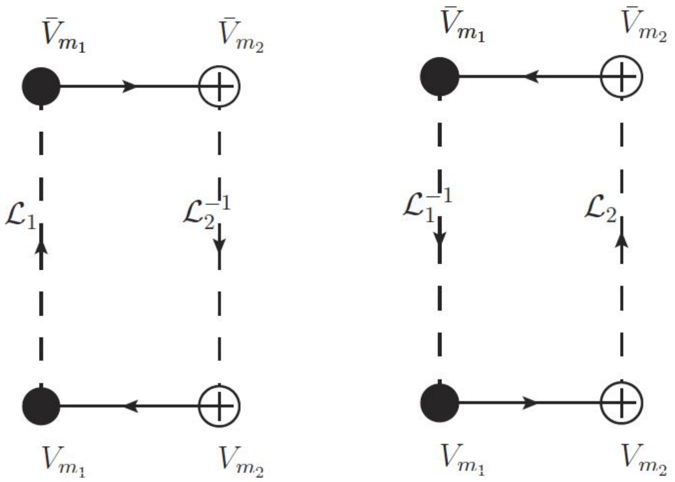
$$Z_2 = \sum_{m_1, m_2} \langle \,{}^{\mathcal{L}_1}\bar{O}^{(1)}_{m_1} O^{(1)}_{m_1} \,{}^{\mathcal{L}_2}\bar{O}^{(2)}_{m_2} O^{(2)}_{m_2} \rangle$$

  where m_1, m_2 are over all possible states in the vacuum module.
- For the multi-particle states, every operator O_{m_i} could be decomposed into the product of the operators corresponding to the single-particle states.
- However, there are now more possibility for the operators to combine.



(a) The link corresponds to $\mathcal{L}_1$ and $\mathcal{L}_2$. (b) The link corresponds to $\mathcal{L}_1\mathcal{L}_2$.

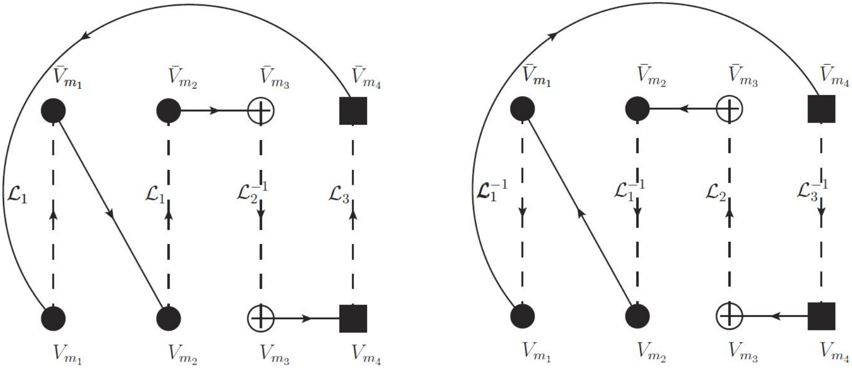
Figure: In the diagram, the same type of vertices means that the operators are in the fixed points of the pairwise circles in the Schottky uniformization. The two-point function between the operators on the same type of vertices just give the simplest link. The one between the operators on different types of vertices may lead to more complicated links.



(a) The link corresponds to $\mathcal{L}_1\mathcal{L}_2^{-1}$. (b) The link corresponds to $\mathcal{L}_2\mathcal{L}_1^{-1}$.

Figure: Two links with opposite orientations. The corresponding conjugacy classes are inverse to each other, but they have the same multiplier.

More complicated links



(a) The link corresponds to $\mathcal{L}_1^2\mathcal{L}_2^{-1}\mathcal{L}_3$.

(b) The link corresponds to $\mathcal{L}_3^{-1}\mathcal{L}_2\mathcal{L}_1^{-2}$.

Figure: More complicated links with three generators.

Links and conjugacy classes

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- The expectation value of a link is determined by the multiplier of the conjugacy class.
- A primitive conjugate element is the one which cannot be written as the positive power of another element, i.e. $\mathcal{L}^{(primary)} \neq (\mathcal{L}')^n, n \in N$. It corresponds to the primitive link which cannot be written as the positive powers of a shorter link.

1-loop partition function from CFT

- First of all, there is an one-to-one correspondence between the primitive link and primitive conjugacy class in the Schottky group. By considering all possible links, there is no missing in counting the primitive elements.
- Moreover, notice that the 1-loop partition function could be expanded

$$Z_{1-loop} = \prod_{\gamma} Z_{\gamma} = \prod_{\gamma} \left(\prod_{m=2}^{\infty} \frac{1}{1-q_{\gamma}^{m}} \right),$$

  and furthermore

$$\prod_{m=2}^{\infty} \frac{1}{1-q_{\gamma}^{m}} = \sum_{t=0}^{\infty} \frac{1}{t!} \prod_{s=1}^{\infty} \frac{1}{s^t} (\sum_{m=2}^{\infty} q_{\gamma}^{sm})^t.$$

- Therefore the 1-loop partition function could be expanded into a summation of the contribution from all possible links.

One subtlety

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- Mismatch?

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- The above discussion is focused on the holomorphic sector.
- The anti-holomorphic sector should give the same contribution.
- Mismatch?
- However, the computation in the CFT cannot distinguish the link with different orientation, though we may set up the one-to-one correspondence between the oriented links and conjugacy classes.
- On the other hand, $q_\gamma^{-1/2}$ should be the larger values of the conjugacy element so that it is actually the same for both γ and γ^{-1}.
- Therefore a more precise relation is

$$Z_g|_{holomorphic} = \prod_\gamma (Z_\gamma)^{\frac{1}{2}}$$

- This saves us from double counting.
- The full partition function

$$Z_g = \prod_\gamma |Z_\gamma|.$$

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- The partition function is encoded by the $2g$-point functions on the Riemann sphere.
- These multi-point functions are at most of order c^0. At leading order every $2g$-point function could be reduced to the products of the two-point functions of single-particle operators.

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- These multi-point functions are at most of order c^0. At leading order every $2g$-point function could be reduced to the products of the two-point functions of single-particle operators.
- By considering all possible ways to contract the operators and form the links, the 1-loop partition function has been reproduced.

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- Certainly, this should be the case since the 1-loop gravitational partition function is only given by the functional determinant of the free massless graviton.

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- Two-point function of single-particle states dominates, correspondingly the massless graviton is freely propagating and the interaction among gravitons can be ignored.
- Certainly, this should be the case since the 1-loop gravitational partition function is only given by the functional determinant of the free massless graviton.
- As an implication, we proved that the next-to-leading term in Rényi entropy is captured by the 1-loop quantum correction to the corresponding gravitational configuration

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- How about the non-handlebody configurations?

Thanks for your attention!