Hadronic molecules with heavy quarks

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Elementary particles in the Standard Model

**QUARKS**
- **up** (u) $2.3^{+0.7}_{-0.5}$ MeV
- **charm** (c) $1275 \pm 25$ MeV
- **top** (t) $\approx 173$ GeV
- **down** (d) $4.8^{+0.5}_{-0.3}$ MeV
- **strange** (s) $95 \pm 5$ MeV
- **bottom** (b) $4180 \pm 30$ MeV

**GUAGE BOSONS**
- **electron** (e) $0.51$ MeV
- **muon** (μ) $105.7$ MeV
- **tau** (τ) $1776.8 \pm 0.2$ MeV
- **Z boson** (Z) $\approx 91.2$ GeV
- **Higgs boson** (H) $\approx 126$ GeV

**LEPTONS**
- **electron neutrino** ($\nu_e$) $< 2.2$ MeV
- **muon neutrino** ($\nu_\mu$) $< 0.17$ MeV
- **tau neutrino** ($\nu_\tau$) $< 15.5$ MeV
- **W boson** ($W^{\pm}$) $\approx 80.4$ GeV
Origin of the mass of the visible universe: strong interaction!

\[ M_{\text{diamond}} \simeq \sum M_{C\text{atom}} \]

\[ M_{C\text{atom}} \simeq 6 \left( M_{\text{proton}} + M_{\text{neutron}} + M_{\text{electron}} \right) \]

\[ M_{\text{proton}} \simeq \sum_q \sigma_q m_q + (900 \text{ MeV}) \]

938 MeV

Lattice QCD results by the BMW Collaboration
Origin of the mass of the visible universe: strong interaction!

photo taken during Z. Fodor's talk at the MENU2016 Conference, Kyoto
Low-energy QCD is difficult

At low energies,

- nonperturbative
  lattice QCD, effective field theory (EFT), models

- color confinement
  quarks and gluons are confined in color-neutral hadrons
Ordinary and exotic hadrons

- In quark model notation
  - Ordinary mesons and baryons
  - Exotic hadrons: multiquark states, hybrids and glueballs

- Hadronic molecules: extended, loosely bound states composed of asymptotic hadrons \((\text{distance} \gg \text{hadron size})\), analogues of deuteron and other light nuclei

- Once the same quantum numbers, always mix \(\Rightarrow\) source of difficulties/confusions
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Hadronic molecules (I)

- Hadronic molecule: dominant component is a composite state of 2 or more hadrons
- Concept at large distances, so that can be approximated by system of multi-hadrons at low energies

Consider a 2-body bound state with a mass \( M = m_1 + m_2 - E_B \)

size: \( R \sim \frac{1}{\sqrt{2\mu E_B}} \gg r_{\text{hadron}} \)

- Only narrow hadrons can be considered as components of hadronic molecules, \( \Gamma_h \ll 1/r, r: \) range of forces

Filin et al., PRL105(2010)019101; FKG, Meißner, PRD84(2011)014013
Hadronic molecules (II)

• Why are hadronic molecules interesting?
  - one realization of color-neutral objects, analogue of light nuclei
  - important information for hadron-hadron interaction
  - understanding the $X Y Z$ states
  - model-independent statements can be made

for $S$-wave, compositeness $(1 - Z)$ related to measurable quantities

Weinberg, PR137(1965); Baru et al., PLB586(2004); Hyodo, IJMPA28(2013)1330045; ... 
see also, e.g., Weinberg's books: QFT Vol.I, Lectures on QM

$$g_{NR}^2 \approx (1 - Z) \frac{2\pi}{\mu^2} \sqrt{2\mu E_B} \leq \frac{2\pi}{\mu^2} \sqrt{2\mu E_B}$$

$$a \approx -\frac{2(1 - Z)}{(2 - Z)\sqrt{2\mu E_B}}, \quad r_e \approx \frac{Z}{(1 - Z)\sqrt{2\mu E_B}}$$

scale separation: EFT can be applied
Beginning of the story in 2003: discovery of $D_{s0}^*(2317)$

- Charm-strange $c\bar{s}$ mesons $D_{s0}^*(2317)$ and $D_{s1}(2460)$

  - $D_{s0}^*(2317): 0^+$  
    BaBar (2003)  
    $M = (2317.7 \pm 0.6)$ MeV,  
    $\Gamma < 3.8$ MeV  
    The only hadronic decay: $D_s \pi$

  - $D_{s1}(2460): 1^+$  
    CLEO (2003)  
    $M = (2459.5 \pm 0.6)$ MeV,  
    $\Gamma < 3.5$ MeV

- Notable features:
  masses are much lower than the quark model predictions for $c\bar{s}$ mesons
  
  $M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)} \simeq M_{D^*} - M_D + 1$ MeV
Beginning of the story in 2003: discovery of $X (3872)$

- $X (3872)$: Discovered in $B^\pm \rightarrow K^\pm J/\psi \pi \pi$, mass extremely close to the $D^0 \bar{D}^{*0}$ threshold
  
  $M_X = (3871.69 \pm 0.17)$ MeV

  $M_{D^0} + M_{D^{*0}} - M_X = (0.12 \pm 0.19)$ MeV

- $\Gamma < 1.2$ MeV

- $J^{PC} = 1^{++}$

  $\Rightarrow$ $S$-wave coupling to $D \bar{D}^*$

- Observed in the $D^0 \bar{D}^{*0}$ mode as well

  BaBar, PRD77(2008)011102

- Large coupling to $D^0 \bar{D}^{*0}$:

  $\mathcal{B}(X \rightarrow D^0 \bar{D}^{*0}) > 24\%$  

  PDG2014

- Large isospin breaking:

  \[
  \frac{\mathcal{B}(X \rightarrow \omega J/\psi)}{\mathcal{B}(X \rightarrow \pi^+ \pi^- J/\psi)} = 0.8 \pm 0.3
  \]
Beginning of the story in 2003: discovery of $X(3872)$

- $X(3872)$ Belle, PRL91(2003)262001

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- $J^{PC} = 1^{++}$ LHCb PRL110(2013)222001

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More exotic structures: $Z_c^\pm$ and $Z_b^\pm$ with hidden $Q\bar{Q}$

- $Z_c^\pm$, $Z_b^\pm$: charged structures in heavy quarkonium mass region, $Q\bar{Q}d\bar{u}, Q\bar{Q}u\bar{d}$
  - $Z_c(3900)$, $Z_c(4020)$, $Z_c(4200)$, $Z_c(4430)$, ...
- $Z_b(10610)$ and $Z_b(10650)$:
  - observed in $\Upsilon(10860) \rightarrow \pi^\mp [\pi^\mp \Upsilon(1S, 2S, 3S)/h_b(1P, 2P)]$
  - also in $\Upsilon(10860) \rightarrow \pi^\mp [B^*(\bar{B}^*)]^\pm$

More exotic structures: $Z_c^\pm$ and $Z_b^\pm$ with hidden $Q\bar{Q}$

- $Z_c^\pm$, $Z_b^\pm$: charged structures in heavy quarkonium mass region, $Q\bar{Q}\bar{d}u$, $Q\bar{Q}\bar{u}d$
  $Z_c(3900)$, $Z_c(4020)$, $Z_c(4200)$, $Z_c(4430)$, 
- $Z_b(10610)$ and $Z_b(10650)$: 

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also in $\Upsilon(10860) \to \pi^\mp [B^*(\bar{B}^*)^\pm$
$Z_c^\pm$ and $Z_b^\pm$ with hidden $Q\bar{Q}$ (II)

- $Z_c(3900/3885)^\pm$: structure around 3.9 GeV seen in $J/\psi\pi$ by BESIII and Belle in $Y(4260) \rightarrow J/\psi\pi^+\pi^-$, BESIII, PRL110(2013)252001; Belle, PRL110(2013)252002. And in $D\bar{D}^*$ by BESIII in $Y(4260) \rightarrow \pi^\pm (D\bar{D}^*)^\mp$ BESIII, PRD92(2015)092006.

- can be attributed to the same state Aldaladejo et al., PLB755(2016)337
Charmonium spectrum

Charmonium spectrum in Godfrey-Isgur quark model

- $3^3D_1$
- $4^3S_1$
- $2^3D_1$
- $3^1S_0$
- $3^3S_1$
- $1^3D_1$
- $2^1S_0$
- $2^3S_1$
- $1^1P_1$
- $1^3P_0$
- $1^3P_1$
- $1^3P_2$
- $1^1S_0$
- $1^3S_1$
- $2^1P_1$
- $2^3P_0$
- $2^3P_1$
- $2^3P_2$
- $1^3D_2$
- $2^3D_2$

Mass (MeV)

$J^{PC} = 0^{--}$ $1^{--}$ $1^{--}$ $0^{++}$ $1^{++}$ $2^{++}$ $2^{--}$

Note: $X(3915)$ is probably just the $\chi_{c2}(2P)$

Z.-Y. Zhou et al., PRL 115 (2015) 022001

Feng-Kun Guo (ITP)
Charmonium spectrum

Experimental status of charmonium spectrum
(before 2003)

Mass (MeV)

4500
4000
3500
3000

J/ψ
η_c(1S)
ψ(2S)
η_c(2S)
ψ(3770)
ψ(4040)
ψ(4160)
ψ(4415)

J^PC = 0^+ 1^- 1^+ 0^{++} 1^{++} 2^{++} 2^{--}

Godfrey-Isgur quark model
discovered before 2003

Note: \( \chi_c^2 (2P) \) is probably just the \( \chi_c \) with \( 2^{++} \).
Charmonium spectrum

Note: $X(3915)$ is probably just the $\chi_{c2}(2P)$ with $2^{++}$ \cite{Zhou2015}.
Charmonium spectrum

Note: $X(3915)$ is probably just the $\chi_{c2}(2P)$ with $2^{++}$ Z.-Y. Zhou et al., PRL115(2015)022001
Compositeness (I)

Weinberg, PR137(1965); Baru et al., PLB586(2004); ...
see also, e.g., Weinberg’s books: QFT Vol.I, Lectures on QM

Model-independent result for $S$-wave loosely bound composite states:

Consider a system with Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + V$$

$\mathcal{H}_0$: free Hamiltonian, $V$: interaction potential

- Compositeness:

  the probability of finding the physical state $|B\rangle$ in the 2-body continuum $|q\rangle$

  $$1 - Z = \int \frac{d^3q}{(2\pi)^3} |\langle q|B\rangle|^2$$

- $Z = |\langle B_0|B\rangle|^2$, $0 \leq (1 - Z) \leq 1$

  - $Z = 0$: pure composite state
  - $Z = 1$: pure elementary state
Compositeness : $1 - Z = \int \frac{d^3q}{(2\pi)^3} |\langle q|B\rangle|^2$

- Schrödinger equation

$$(\mathcal{H}_0 + V)|B\rangle = -E_B|B\rangle$$

multiplying by $\langle q|$ and using $\mathcal{H}_0|q\rangle = \frac{q^2}{2\mu}|q\rangle$

$$\langle q|B\rangle = -\frac{\langle q|V|B\rangle}{E_B + q^2/(2\mu)}$$

- $S$-wave, small binding energy so that $R = 1/\sqrt{2\mu E_B} \gg r$, $r$: range of forces

$$\langle q|V|B\rangle = g_{NR} \left[ 1 + O(r/R) \right]$$

- Compositeness:

$$1 - Z = \int \frac{d^3q}{(2\pi)^3} \frac{g_{NR}^2}{[E_B + q^2/(2\mu)]^2} \left[ 1 + O\left(\frac{r}{R}\right) \right] = \frac{\mu^2 g_{NR}^2}{2\pi \sqrt{2\mu E_B}} \left[ 1 + O\left(\frac{r}{R}\right) \right]$$
Coupling constant measures the compositeness for an $S$-wave bound state with a small binding energy (model-independent)

\[ g_{NR}^2 \approx (1 - Z) \frac{2\pi}{\mu^2} \sqrt{2\mu E_B} \leq \frac{2\pi}{\mu^2} \sqrt{2\mu E_B} \]

$Z$ can be measured directly from observables, such as scattering length $a$ and effective range $r_e$ (Weinberg (1965))

\[
a = -\frac{2R(1 - Z)}{2 - Z} \left[ 1 + \mathcal{O} \left( \frac{r}{R} \right) \right], \quad r_e = \frac{RZ}{1 - Z} \left[ 1 + \mathcal{O} \left( \frac{r}{R} \right) \right]
\]

Example: deuteron as $pn$ bound state. Exp.: $E_B = 2.2$ MeV, $a_{3S_1} = -5.4$ fm

\[
a_{Z=1} = 0 \text{ fm}, \quad a_{Z=0} = (4.3 \pm 1.4) \text{ fm}
\]
Heavy quark spin symmetry (HQSS):

- define \( \vec{s}_\ell \equiv \vec{J} - \vec{s}_Q \): total angular momentum of the light quark system
  - \( \vec{J} \): total angular momentum, \( \vec{s}_Q \): heavy quark spin
  - E.g., for \( D \) and \( D^* \): \( s^P_\ell = \frac{1}{2}^- \)
- \( s_\ell \) is a good quantum number
- spin multiplet: \( (D, D^*), (\eta_c, J/\psi) \)
Interaction between heavy hadron and light hadron is independent of heavy quark spin for $m_Q \to \infty$.

- $D^{*0}(2317)$: dominantly $DK$ molecule

  Barnes, Close, Lipkin (2003); van Beveren, Rupp (2003); Kolomeitsev, Lutz (2004); FKG, Shen, Chiang, Ping, Zou (2006); Gamermann et al. (2007); ...

- HQSS $\Rightarrow$ spin partner: a $D^*K$ molecule

  natural consequence: $M_{D^{*0}_s(2317)} \sim M_{D^*} - M_D$

- For hadronic molecules of $Q\bar{Q}+$light hadron

  exchange at least two gluons, LO: chromo-electric

  spin multiplet with approximately the same splitting as that for $Q\bar{Q}$

  Example:

  if $Y(4660)$ is a $\psi'f_0(980)$ state, then there would be an $\eta'_c f_0(980)$ state

  the same for hadro-quarkonium

• Consider $S$-wave interaction between a pair of $s^P = \frac{1}{2}^-$ (anti-)heavy mesons:

\[
\begin{align*}
0^{++} & : & D \bar{D}, & D^* \bar{D}^* \\
1^{+-} & : & \frac{1}{\sqrt{2}} (D \bar{D}^* + D^* \bar{D}), & D^* \bar{D}^* \\
1^{++} & : & \frac{1}{\sqrt{2}} (D \bar{D}^* - D^* \bar{D}), & D^* \bar{D}^* \\
2^{++} & : & & D^* \bar{D}^*
\end{align*}
\]

• Heavy quark spin irrelevant $\Rightarrow$ interaction matrix elements:

\[
\left\langle s_{\ell_1}, s_{\ell_2}, s_L \left| \hat{H} \right| s'_{\ell_1}, s'_{\ell_2}, s_L \right\rangle
\]

For each isospin, 2 independent terms

\[
\left\langle \frac{1}{2}, \frac{1}{2}, 0 \left| \hat{H} \right| \frac{1}{2}, \frac{1}{2}, 0 \right\rangle, \quad \left\langle \frac{1}{2}, \frac{1}{2}, 1 \left| \hat{H} \right| \frac{1}{2}, \frac{1}{2}, 1 \right\rangle
\]

$\Rightarrow$ 6 pairs grouped in 2 multiplets with $s_L = 0$ and 1, respectively
• Consider $S$-wave interaction between a pair of $s_\ell^P = \frac{1}{2}^-$ (anti-)heavy mesons:

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0^{++} & : \ D\bar{D}, \ D^*\bar{D}^* \\
1^{+-} & : \ \frac{1}{\sqrt{2}} \left( D\bar{D}^* + D^*\bar{D} \right), \ D^*\bar{D}^* \\
1^{++} & : \ \frac{1}{\sqrt{2}} \left( D\bar{D}^* - D^*\bar{D} \right) \\
2^{++} & : \ D^*\bar{D}^*
\end{align*}
\]

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\langle s_{\ell 1}, s_{\ell 2}, s_L | \hat{H} | s'_{\ell 1}, s'_{\ell 2}, s_L \rangle
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\langle \frac{1}{2}, \frac{1}{2}, 0 | \hat{H} | \frac{1}{2}, \frac{1}{2}, 0 \rangle, & \quad \langle \frac{1}{2}, \frac{1}{2}, 1 | \hat{H} | \frac{1}{2}, \frac{1}{2}, 1 \rangle
\end{align*}
\]

$\Rightarrow$ 6 pairs grouped in 2 multiplets with $s_L = 0$ and 1, respectively
HQSS for $XYZ$ (II)

- In the limit $m_c \to \infty$, $D$ and $D^*$ degenerated, convenient to use the basis of states: $s_{PC}^L \otimes s_{PC}^{c\bar{c}}$
  - $S$-wave: $s_{PC}^L, s_{PC}^{c\bar{c}} = 0^{--}$ or $1^{--}$
  - multiplet with $s_L = 0$:
    
    $0^{-+}_L \otimes 0^{--}_{c\bar{c}} = 0^{++}, \quad 0^{-+}_L \otimes 1^{---}_{c\bar{c}} = 1^{+-}$
  - multiplet with $s_L = 1$:
    
    $1^{--}_L \otimes 0^{--}_{c\bar{c}} = 1^{+-}, \quad 1^{--}_L \otimes 1^{---}_{c\bar{c}} = 0^{++} \oplus 1^{++} \oplus 2^{++}$

$\Rightarrow$ if $X(3872)$ is a $1^{++} D\bar{D}^*$ molecule, then its $s_L = 1$ \quad Voloshin, PLB604(2004)69

- Multiplets in strict heavy quark limit:
  - $X(3872)$ has three partners with $0^{++}, 2^{++}$ and $1^{+-}$
  - might be 6 molecules: $Z_b, Z'_b$ and $W_{b0}, W'_{b0}, W_{b1}$ and $W_{b2}$ (for $I = 1$)

Hidalgo-Duque et al., PLB727(2013)432; Baru et al., arXiv:1605.09649

Bondar et al., PRD84(2011)054010; Voloshin, PRD84(2011)031502; Mehen, Powell, PRD84(2011)114013
HQSS for $XYZ$ (II)

- In the limit $m_c \to \infty$, $D$ and $D^*$ degenerated, convenient to use the basis of states: $s_{LPC} \otimes s_{Pc\bar{c}}$
  
  - $S$-wave: $s_{LPC}^P, s_{Pc\bar{c}}^P = 0^{--}$ or $1^{--}$
  
  - multiplet with $s_L = 0$:
    
    $$0^{++}_L \otimes 0^{--}_{c\bar{c}} = 0^{++}, \quad 0^{--}_L \otimes 1^{--}_{c\bar{c}} = 1^{+-}$$
  
  - multiplet with $s_L = 1$:
    
    $$1^{--}_L \otimes 0^{--}_{c\bar{c}} = 1^{+-}, \quad 1^{--}_L \otimes 1^{--}_{c\bar{c}} = 0^{++} \oplus 1^{++} \oplus 2^{++}$$

  $\Rightarrow$ if $X(3872)$ is a $1^{++} D \bar{D}^*$ molecule, then its $s_L = 1$  
  
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  - might be 6 molecules: $Z_b$, $Z_b'$ and $W_{b0}$, $W'_{b0}$, $W_{b1}$ and $W_{b2}$ (for $I = 1$)
    
    Bondar et al., PRD84(2011)054010; Voloshin, PRD84(2011)031502; Mehen, Powell, PRD84(2011)114013
Calculations using physical $D$ and $D^*$ masses in nonrelativistic EFT

the effective Lagrangian for the LO interaction between spin multiplets,

\[ \mathcal{L}_{4H} = C_A \text{Tr} \left[ \bar{H}_a(Q) H_a(Q) \gamma_\mu \right] \text{Tr} \left[ H_b(Q) \bar{H}_b(Q) \gamma^\mu \right] \]

\[ + C_A^{(\tau)} \text{Tr} \left[ \bar{H}_a(Q) \vec{\tau}_{ab} H_b(Q) \gamma_\mu \right] \text{Tr} \left[ H_c(Q) \vec{\tau}_{cd} \bar{H}_d(Q) \gamma^\mu \right] \]

\[ + C_B \text{Tr} \left[ \bar{H}_a(Q) H_a(Q) \gamma_\mu \gamma_5 \right] \text{Tr} \left[ H_b(Q) \bar{H}_b(Q) \gamma^\mu \gamma_5 \right] \]

\[ + C_B^{(\tau)} \text{Tr} \left[ \bar{H}_a(Q) \vec{\tau}_{ab} H_b(Q) \gamma_\mu \gamma_5 \right] \text{Tr} \left[ H_c(Q) \vec{\tau}_{cd} \bar{H}_d(Q) \gamma^\mu \gamma_5 \right] \]

\[ \vec{\tau}: \text{Pauli matrices in isospin space; } \]

\[ H_a(Q): D, D^*; \quad H_a(Q): \bar{D}, \bar{D}^* \]

Isospin $I = 0$ or 1 $\Rightarrow$ 4 independent terms:

$C_{0A}, C_{0B}; C_{1A}, C_{1B}$: linear combinations of $C_{A,B}^{(\tau)}$

\[ C_{0\phi} = C_\phi + 3C_\phi^{(\tau)}, \quad C_{1\phi} = C_\phi - C_\phi^{(\tau)}, \quad \text{for } \phi = A, B \]
HQSS for \(XYZ\) (IV)

- Some channels have the same linear combination of contact terms

\[
V(D\bar{D}^*, 1^{++}) = V(D^*\bar{D}^*, 2^{++}) = C_{IA} + C_{IB}
\]

\[
V(D\bar{D}^*, 1^{-+}) = V(D^*\bar{D}^*, 1^{-+}) = C_{IA} - C_{IB}
\]

\(D\bar{D}\) does not have the same: \(V(D\bar{D}, 0^{++}) = C_{IA}\)

- This would suggest spin multiplets. Good candidates:

- \(X(3872)\) and \(X_2(4013)\) (not observed yet!); \(Z_c(3900)\) and \(Z_c(4020)\)
  

\[
M_{X_2(4013)} - M_{X(3872)} \approx M_{Z_c(4020)} - M_{Z_c(3900)} \approx M_{D^*} - M_D
\]

- \(Z_b(10610)\) and \(Z_b(10650)\):
  
Bondar et al., PRD84(2011)054010; . . .

\[
M_{Z_b(10650)} - M_{Z_b(10610)} \approx M_{B^*} - M_B
\]
HQSS for $XYZ$ (IV)

- Some channels have the same linear combination of contact terms

\[
V(D\bar{D}^*, 1^{++}) = V(D^*\bar{D}^*, 2^{++}) = C_{IA} + C_{IB}
\]
\[
V(D\bar{D}^*, 1^{+-}) = V(D^*\bar{D}^*, 1^{+-}) = C_{IA} - C_{IB}
\]

$D\bar{D}$ does not have the same:

\[
V(D\bar{D}, 0^{++}) = C_{IA}
\]

- This would suggest spin multiplets. Good candidates:

- $X(3872)$ and $X_2(4013)$ (not observed yet!); $Z_c(3900)$ and $Z_c(4020)$
  
  Nieves, Valderrama, PRD86(2012)056004; ...  

\[
M_{X_2(4013)} - M_{X(3872)} \approx M_{Z_c(4020)} - M_{Z_c(3900)} \approx M_{D^*} - M_D
\]

- $Z_b(10610)$ and $Z_b(10650)$:

  Bondar et al., PRD84(2011)054010; ...

\[
M_{Z_b(10650)} - M_{Z_b(10610)} \approx M_{B^*} - M_B
\]
Inputs and predictions (I)

- Solve Lippmann–Schwinger equation regularized with a Gaussian form factor, bound states appear as poles in the first Riemann sheet below threshold

- Inputs:
  
  - **Mass of** \( X(3872) \) \( \Rightarrow C_{0A} + C_{0B} \)
  
  - **Mass of** \( Z_b(10610) \) \( \Rightarrow C_{1A} - C_{1B} \)

- Predicted many partners of \( X(3872) \) and \( Z_b(10610) \) \( \text{FKG et al., PRD88}(2013) \)
  
  - **Partners of** \( X(3872) \) \([1^{++}]:\)

<table>
<thead>
<tr>
<th>( I(J^{PC}) )</th>
<th>States</th>
<th>Thresholds</th>
<th>Masses (( \Lambda = 0.5 ) GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0(1^{++})</td>
<td>( \frac{1}{\sqrt{2}} (D \bar{D}^* - D^* \bar{D}) )</td>
<td>3875.87</td>
<td>( 3871.68 ) (input)</td>
</tr>
<tr>
<td>0(2^{++})</td>
<td>( D^* \bar{D}^* )</td>
<td>4017.3</td>
<td>( 4012^{+4}_{-5} )</td>
</tr>
<tr>
<td>0(1^{++})</td>
<td>( \frac{1}{\sqrt{2}} (B \bar{B}^* - B^* \bar{B}) )</td>
<td>10604.4</td>
<td>( 10580^{+9}_{-8} )</td>
</tr>
<tr>
<td>0(2^{++})</td>
<td>( B^* \bar{B}^* )</td>
<td>10650.2</td>
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<td>0(2^{+})</td>
<td>( D^* B^* )</td>
<td>7333.7</td>
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Here \( V(D \bar{D}^*) = V(\bar{B}B^*) \left[ 1 + \mathcal{O} \left( \frac{\Lambda_{QCD}}{m_c} \right) \right] \) \( \ldots \) assumed
Inputs and predictions (II)

**Partners of $Z_b(10610)$ [$1^{+-}$]:**

<table>
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<tr>
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<th>Thresholds</th>
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Two virtual states in charm sector, could correspond to $Z_c(3900)$ and $Z_c(4020)$

- So far, the assignments of the predicted states to the observed ones only based on masses, $\Rightarrow$ decays and productions
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- So far, the assignments of the predicted states to the observed ones only based on masses, $\Rightarrow$ decays and productions
Essential point of in the spirit of effective field theory: scale separation
to include all relevant d.o.f. at the given scale
to study near-threshold structures, one has to take into account the corresponding channel, unless the coupling is weak, no matter what structure was used as the starting point
⇒ for $X(3872)$: has to consider $D\bar{D}^*$

Hadronic molecular structure: a long-distance concept
⇒ not all processes are sensitive to it!
• For processes dominated by **long-distance** physics:
  calculable with controlled uncertainties using low-energy EFT
  Examples:
  \[ X(3872) \rightarrow D^0 \bar{D}^0 \pi^0, \ X(3872) \rightarrow D^0 \bar{D}^0 \gamma \]
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  Examples:
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What can we say about the decays and productions (II)

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**X(3872), Y(4260) and Z_c(3900) (I)**

- Suppose that the $X(3872)$, $Y(4260)$ and $Z_c(3900)$ are hadronic molecules:

  - $X(3872)$: $J^{PC} = 1^{++}$, $D \bar{D}^*$  
    Törquist, PLB590(2004)209; ...

  - $Y(4260)$: $J^{PC} = 1^{--}$, $D_1(2420) \bar{D}$ [two $D_1$'s, should be the narrow one]  
    Wang, Hanhart, Zhao, PRL111(2013)132003

  - $Z(3900)$: $J^{PC} = 1^{+-}$, $D \bar{D}^*$  
    Wang et al. (2013); FKG et al., PRD88(2013)054007; ...

- First lattice result for the $X(3872)$ ($L \sim 2$ fm), no conclusion on its nature was made, but they gave

  
  $a_{D \bar{D}^*} = (-1.7 \pm 0.4)$ fm, \hspace{1cm} $r_{D \bar{D}^*} = (0.5 \pm 0.1)$ fm

  
  these values correspond to \hspace{1cm} $1 - Z \gtrsim 0.7$

  \Rightarrow lattice evidence for $X(3872)$ being dominantly a $D \bar{D}^*$ hadronic molecule

- So far negative lattice results for $Z_c(3900)$

  Chen et al., PRD89(2014)094506; Prelovsek et al., PRD91(2015)014504

  but

  Albaladejo et al., EPJC76(2016)576
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Production of $X(3872)$ and $Z_c(3900)$ in $Y(4260)$ decays

Large couplings

Loops are enhanced when the binding energies are small:

$$A \sim \mathcal{O} \left( \frac{v^5}{(v^2)^3} \right) V_{D_1 D^*}(q \pi/\gamma) = \mathcal{O} \left( \frac{1}{v} \right) V_{D_1 D^*}(q \pi/\gamma)$$

$S$-wave vertices for couplings to $Y(4260)$, $X(3872)$ and $Z_c(3900)$

Intermediate mesons are nonrelativistic. $v \approx 0.06$: velocity

Power counting: three-momentum $\sim \mathcal{O}(v)$, energy $\sim \mathcal{O}(v^2)$

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- Power counting: three-momentum $\sim \mathcal{O}(v)$, energy $\sim \mathcal{O}(v^2)$

Loop integral measure: $v^5$, propagator: $\frac{1}{v^2}$
If these three states are hadronic molecules, then

- the $Z_c(3900)$ can be easily produced in the $Y(4260)$ decays, in line with the BESIII and Belle observations
- Prediction:

  the $X(3872)$ can be easily produced in $Y(4260) \rightarrow \gamma X(3872)$

BESIII observation of $e^+e^- \rightarrow \gamma X(3872) \rightarrow \gamma J/\psi \pi^+\pi^-$ at $\sqrt{s} = 4.26$ GeV:
Decays: $X(3872) \rightarrow \psi\gamma$

The ratio

$$\frac{\mathcal{B}(X(3872) \rightarrow \psi'\gamma)}{\mathcal{B}(X(3872) \rightarrow J/\psi\gamma)} = 2.46 \pm 0.64 \pm 0.29$$

is insensitive to the molecular component of the $X(3872)$:

- loops are sensitive to unknown couplings $g_{\psi DD}/g_{\psi' DD}$
- loops are divergent, needs a counterterm (short-distance physics)!

FKG, Hanhart, Kalashnikova, Meißner, Nefediev, PLB742(2015)394

LHCb, NPB886(2014)665

see also Mehen, Springer, PRD83(2011)094009; Molnar et al., arXiv:1601.03366
Summary

- Hadronic molecule structure should be detected in processes sensitive to long-distance physics.
- Heavy quark symmetry can provide interesting predictions/insights to hadronic molecules.
- Should go beyond the mass spectrum because important structure information is contained in the coupling, leading to decays and productions.

Thank you!
Summary

- hadronic molecule structure should be detected in processes sensitive to long-distance physics
- heavy quark symmetry can provide interesting predictions/insights to hadronic molecules
- should go beyond the mass spectrum because important structure information is contained in the coupling

⇒ decays and productions

Thank you!
Backup slides
If the $Y(4260)$ and $Y(4360)$ are mixed hadro-charmonia with $h_c$ and $\psi'$ core,

implications of HQSS for hadro-charmonia:

Li, Voloshin, MPLA29(2014)1450060

Cleven et al., PRD92(2015)014005
Suppose the scattering length is very large, the $S$-wave scattering amplitude

$$f_0(k) = \frac{1}{k \cot \delta_0(k) - ik} \approx \frac{1}{-1/a - ik}$$

- bound state pole: $1/a = \kappa \equiv \sqrt{2\mu_b}$
- virtual state pole: $1/a = -\kappa$

- If the same “binding” energy, cannot be distinguished above threshold ($k$ is real):

$$|f_0(k)|^2 \sim \frac{1}{\kappa^2 + k^2}$$
A bound state and virtual state with a 5 MeV binding energy, a width to the inelastic channel is allowed.

Cleven et al., EPJA47(2011)120
Decays: $X(3872) \rightarrow D^0 \bar{D}^0 \pi^0$

- A long-distance process, thus can be studied in nonrelativistic EFT
- Already studied by many authors

Voloshin (2004); Fleming et al (2007); Braaten, Lu (2007); Hanhart et al (2007); …

Our new insight:

If there is a near-threshold $D \bar{D}$ hadronic molecule $\Rightarrow$ a large impact

**Problem**: one unknown contact term $C_{0A}$

- grey band: tree-level result (consistent with Fleming et al., PRD76(2007))
- vertical line: a $D \bar{D}$ bound state at threshold
Phenomenology: $X_2(4013) \rightarrow D\bar{D}/D\bar{D}^*$

- $X_2(4013)$: $2^{++}$, above $D\bar{D}$, $D\bar{D}^*$ thresholds
  - dominant decay modes: $D\bar{D}$ and $D\bar{D}^* + c.c.$ in $D$-wave

- Order-of-magnitude estimate: $\Gamma(X_2) \sim$ a few MeV–tens of MeV

<table>
<thead>
<tr>
<th>[MeV]</th>
<th>Without pion-exchange FF</th>
<th>With pion-exchange FF</th>
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<tbody>
<tr>
<td></td>
<td>$\Lambda = 0.5$ GeV</td>
<td>$\Lambda = 1$ GeV</td>
</tr>
<tr>
<td>$\Gamma(D^+D^-)$</td>
<td>$3.3_{-1.4}^{+3.4}$</td>
<td>$7.3_{-2.1}^{+7.9}$</td>
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<tr>
<td>$\Gamma(D^0\bar{D}^0)$</td>
<td>$2.7_{-1.2}^{+3.1}$</td>
<td>$5.7_{-1.8}^{+7.8}$</td>
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- Fresh calculation with nonperturbative pion: $\Gamma(X_2) \sim 50$ MeV

Baru et al., arXiv:1605.09649